Simultaneous equations with matrices

AS Core Pure Mathematics | Edexcel 8FM0 | 12 questions, 68 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 68**

1 Use a matrix method to solve the simultaneous equations

$3x + 2y = 7$

$5x + 4y = 9$

(3 marks)

Answer _____

2 Two lines have equations

$4x - 6y = 5$

$6x - 9y = k$

where k is a constant.

a. Show that the matrix of coefficients is singular.

b. Find the value of k for which the equations have infinitely many solutions, and interpret this geometrically.

c. Describe the two lines for any other value of k

(4 marks)

Answer _____

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3 Three equations are

$x + 2y - z = -3$

$2x - y + 3z = 14$

$x + y + z = 4$

a. Write the equations in the form **Ax** = **b**

b. Find $\mathbf{A}^{-1}$

c. Hence solve the equations.

(5 marks)

Answer _____

4 The equations

$2x + y + kz = 1$

$x - y + 2z = 4$

$3x + z = 2$

where k is a constant, do not have a unique solution.

Find the value of k

(3 marks)

Answer _____

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5

$$\mathbf{M} = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 3 & 2 \end{vmatrix}$$

a. Show that det $\mathbf{M} = 1$ and use algebra to find $\mathbf{M}^{-1}$

b. Hence solve

$x + 2y + z = 3$
$2x + 5y + 2z = 5$
$x + 3y + 2z = 4$

c. Interpret your answer to part (b) geometrically.

(6 marks)

Answer

6

Three planes have equations

Π_1: $x + 2y - z = 3$
Π_2: $2x - y + 3z = 1$
Π_3: $4x + 3y + z = 7$

a. Show that the matrix of coefficients is singular.

b. Show that the three equations are consistent, and find the general solution in terms of a parameter t, where $z = t$

c. Describe the geometrical arrangement of the three planes.

(6 marks)

Answer

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7 Three planes have equations

$x - y + 2z = 1$

$3x + y - z = 4$

$5x - y + 3z = k$

where k is a constant.

a. Show that the planes do not meet at a single point.

b. Find the value of k for which the equations are consistent.

c. Describe the arrangement of the planes when k takes this value, and when it takes any other value.

(6 marks)

Answer

8 Three planes have equations

$\Pi_1: 2x - y + 3z = 4$

$\Pi_2: 4x - 2y + 6z = c$

$\Pi_3: x + y + z = 2$

where c is a constant.

a. Explain why the equations have no solution when $c = 5$, and describe the arrangement of the planes.

b. Describe the arrangement of the planes when $c = 8$, and state the number of solutions.

(4 marks)

Answer

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9 Consider the equations

$$x - 2y + 3z = 4$$
$$2x + y - z = 3$$
$$kx + 3y - 4z = 5$$

where k is a constant.

a. Find the value of k for which the matrix of coefficients is singular.

b. For this value of k, show that the equations are inconsistent.

c. Interpret part (b) geometrically.

(6 marks)

Answer

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10 A café in Leeds sells tea, coffee and slices of cake. Each item has a fixed price. On three days the café recorded these sales.

	Teas	**Coffees**	**Cakes**	**Takings**
Monday	12	20	15	£130.40
Tuesday	18	25	10	£145.10
Wednesday	9	30	22	£174.40

The price of a tea is £t, a coffee £c and a slice of cake £s

a. Write down a matrix equation for t, c and s

b. Use your calculator to find the inverse of the matrix and hence find the price of each item.

c. On Thursday the café sold 10 of each item and took £78.50. Use your answers to comment on the assumption that each item has a fixed price.

(7 marks)

Answer

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11 Ola invests a total of £20 000 in three savings accounts, £x in account X, £y in account Y and £z in account Z. The accounts pay simple interest of 2%, 3.5% and 5% per year respectively.

After one year Ola has earned £640 in interest in total. She invested twice as much in account X as in account Z.

a. Write down three equations in x, y and z

b. Use a matrix method to find how much Ola invested in each account.

c. Her friend Kwame suggests replacing the third condition with "the total interest is 3.2% of the total invested". Explain why the three equations would then not give a unique answer.

(8 marks)

Answer

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12 Three planes have equations

$$x + y + z = 2$$
$$2x + ky + 3z = 5$$
$$-x + 3y + kz = 2$$

where k is a constant.

a. Show that the matrix of coefficients is singular when $k^2 - k - 6 = 0$

b. For $k = 3$, show that the planes meet in a line and find the general solution.

c. For $k = -2$, show that the equations are inconsistent and describe the arrangement of the planes.

d. State the arrangement of the planes for any other value of k

(10 marks)

Answer

End of questions | Total 68 marks

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