Areas and tangents in polar coordinates

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	$19\pi/2$	4
2	$2\pi^3/3$	3
3	2π	4
4	Area = $\frac{1}{4}\int \theta(1 + \cos 2\theta)\, d\theta$ from 0 to $\pi/2 = \pi^2/32 - 1/8$	6
5	$3\pi/4 + 9\sqrt{3}/32$	8
6	**a)** $dy/d\theta = 7\cos\theta + 3\cos 2\theta = 6\cos^2\theta + 7\cos\theta - 3 = (3\cos\theta - 1)(2\cos\theta + 3) = 0$ gives $\cos\theta = 1/3$ **b)** $r = 8$; $r\sin\theta = 16\sqrt{2}/3$ **c)** $32\sqrt{2}/3$	7
7	**a)** (9, 0), (1, π), (5/2, 2.25) and (5/2, 4.04) **b)** $x = -25/16$ **c)** At $\theta = \pi$ the curve is at $x = -1$, but the curve reaches $x = -25/16$ further left on either side, so it bends back in at $\theta = \pi$: a dimple	7
8	**a)** (3, π/3) and (3, 5π/3) **b)** Area = $\int(-3 + 8\cos\theta + 2\cos 2\theta)\, d\theta$ from 0 to $\pi/3 = 9\sqrt{3}/2 - \pi$	9
9	**a)** A(3/2, π/3), B(3/2, -π/3) **b)** $5\pi/4$	9
10	**a)** $dx/d\theta = e^{\theta}(\cos\theta - \sin\theta) = 0$ gives $\tan\theta = 1$, $\theta = \pi/4$, $r = e^{\pi/4}$ **b)** Area = triangle ONA - $\frac{1}{2}\int e^{2\theta}\, d\theta$ from 0 to $\pi/4 = e^{\pi/2}/4 - (e^{\pi/2} - 1)/4 = 1/4$	10
11	**a)** $y = (p + 1)\sin\theta - 2\sin^3\theta$, $dy/d\theta = \cos\theta(p + 1 - 6\sin^2\theta) = 0$ **b)** $\cos\theta = 0$ always gives 2 points; $\sin^2\theta = (p + 1)/6$ gives 4 more points exactly when $0 < (p + 1)/6 < 1$, i.e. $p < 5$ (at $p = 5$ they coincide with $\theta = \pi/2, 3\pi/2$) **c)** (1, π/2), (1, 3π/2), (2, π/4), (2, 3π/4), (2, 5π/4), (2, 7π/4) **d)** $9\pi/2$	11
12	**a)** $r(0) + r(\pi) = 2(a + 0.4) = 5.6$, so $a = 2.4$ **b)** $\frac{1}{2}\int(2.4 + 0.4\cos 4\theta)^2\, d\theta$ from 0 to $2\pi = 5.84\pi$ **c)** 370 minutes (about 6 hours 10 minutes) **d)** For example: the flow rate from the hose may not be constant, or the pond would not be filled to the brim	10

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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