Calculus of hyperbolic functions

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 93 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 93**

1 Differentiate each of the following with respect to x.

a. $y = x^2 \cosh 3x$

b. $y = \tanh(x^2)$

c. $y = \ln(\cosh 2x)$

(5 marks)

Answer _____

2 a. Find $\int (3 \cosh 2x - 4 \sinh x)\, dx$

b. Find $\int \tanh 2x\, dx$

c. Using the identity $\cosh 2x \equiv 1 + 2 \sinh^2 x$, show that

$$\int_0^{\ln 3} \sinh^2 x\, dx = \frac{10}{9} - \frac{1}{2} \ln 3$$

(8 marks)

Answer _____

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3

a. Given that $y = \text{arsinh}\, x$, prove that $\frac{dy}{dx} = \frac{1}{\sqrt{(1 + x^2)}}$

b. Hence differentiate $\text{arsinh}(3x)$ with respect to x.

c. Given that $\frac{d}{dx}(\text{arcosh}\, x) = \frac{1}{\sqrt{(x^2 - 1)}}$, find $\frac{d}{dx}(\text{arcosh}(x^2))$ for $x > 1$

(7 marks)

Answer

4

$f(x) = \frac{1}{\sqrt{(9x^2 + 16)}}$

a. Show that $\int f(x)\, dx = A\, \text{arsinh}(Bx) + c$, where A and B are constants to be found and c is an arbitrary constant.

b. Hence find the exact value of $\int_0^1 f(x)\, dx$, giving your answer as a multiple of ln 2

c. Find the exact mean value of $f(x)$ over the interval $[0, \frac{4}{3}]$, giving your answer in the form $p \ln(1 + \sqrt{q})$.

(7 marks)

Answer

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5 In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

Find the exact value of

$$\int_{18}^{23} \frac{1}{\sqrt{(x^2 - 6x - 135)}}\,dx$$

giving your answer in the form $\ln k$, where k is a rational number. (6 marks)

Answer

6 The curve C has equation

$y = 6\cosh x - \sinh 2x, \quad x \in \mathbb{R}$

a. Show that the x coordinates of the stationary points of C satisfy

$2\sinh^2 x - 3\sinh x + 1 = 0$

b. Hence find the exact coordinates of the stationary points of C, giving the x coordinates as natural logarithms.

c. Determine the nature of each stationary point. (9 marks)

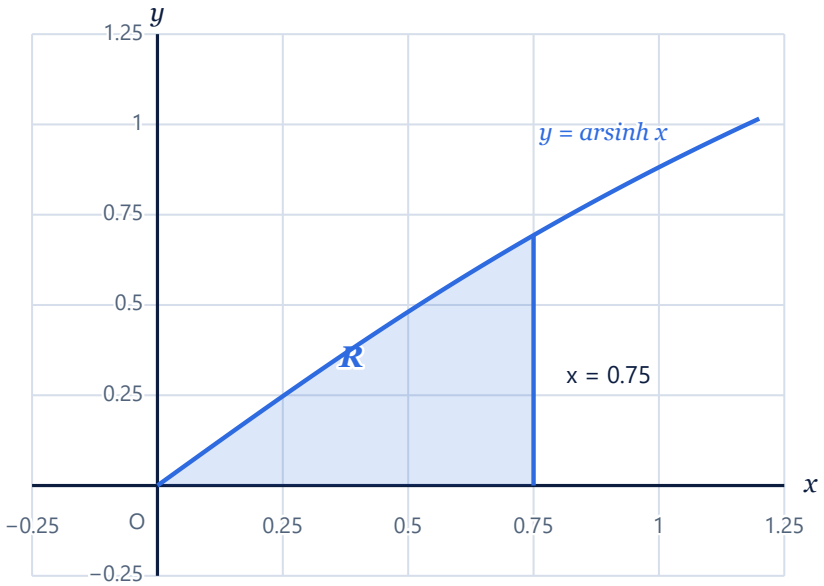
Answer

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7 The diagram shows part of the curve with equation $y = \text{arsinh}\, x$, $x \geq 0$

The finite region R, shown shaded, is bounded by the curve, the x-axis and the line with equation $x = \frac{3}{4}$

Use integration by parts to find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are rational numbers.



(6 marks)

Answer _____

8 Use the substitution $x = 2 \sinh u$ to show that

$\int_0^2 \sqrt{(x^2 + 4)}\, dx = 2\sqrt{2} + 2 \ln(1 + \sqrt{2})$

(7 marks)

Answer _____

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9

a. Find $\int \frac{x + 6}{\sqrt{(x^2 + 9)}} \, dx$

b. Hence find the exact value of

$\int_0^4 \frac{x + 6}{\sqrt{(x^2 + 9)}} \, dx$

giving your answer in the form $a + b \ln 3$, where a and b are integers.

(7 marks)

Answer

10

In this question $-\frac{\pi}{2} < x < \frac{\pi}{2}$

a. Show that $\frac{d}{dx}(\text{arsinh}(\tan x)) = \sec x$

b. Hence find the exact value of $\int_0^{\pi/4} \sec x \, dx$, giving your answer as a natural logarithm.

c. Given that $\frac{d}{dx}(\text{artanh } x) = \frac{1}{1 - x^2}$, show that $\frac{d}{dx}(\text{artanh}(\sin x)) = \sec x$

d. Hence prove that $\text{artanh}(\sin x) = \text{arsinh}(\tan x)$

(9 marks)

Answer

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11 A skydiver jumps from a hovering helicopter. Her speed v m s^{-1}, t seconds after she jumps, is modelled by

$v = 40 \tanh(\frac{t}{4}), \quad t \geq 0$

a. State the speed that the skydiver approaches according to the model.

b. Show that her acceleration a m s^{-2} is given by $a = 10 - \frac{v^2}{160}$, and hence find her acceleration when her speed is 30 m s^{-1}

c. Find the exact time at which her speed reaches 30 m s^{-1}, and give this time to 3 significant figures.

d. Find, to 3 significant figures, the distance she falls in this time.

(11 marks)

Answer

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12 The diagram shows part of the curve H with equation $x^2 - y^2 = 1$, $x \geq 1$, $y \geq 0$, which meets the x-axis at $A(1, 0)$.

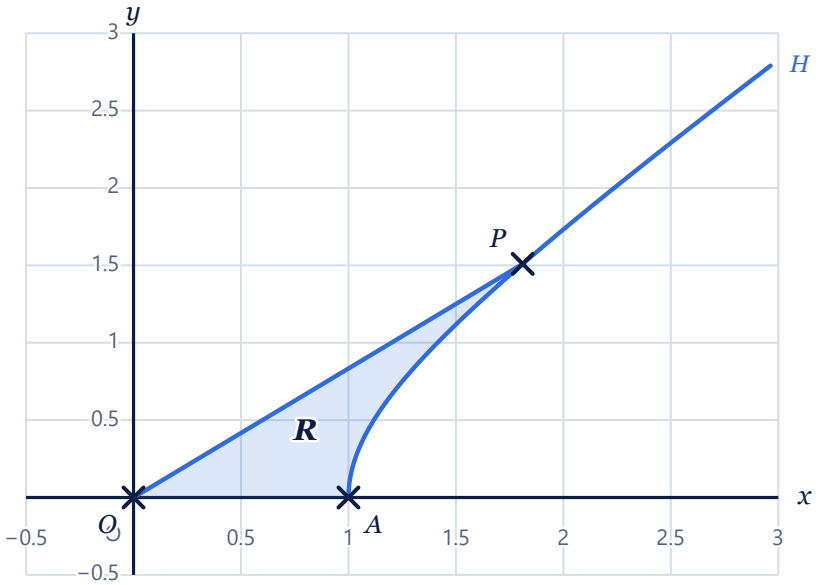
The point $P(\cosh t, \sinh t)$, where $t > 0$, lies on H. The region R, shown shaded, is bounded by the line segment OP, the curve H from A to P, and the line segment OA.

a. Use the substitution $x = \cosh u$ to show that

$$\int \sqrt{(x^2 - 1)}\, dx = \frac{1}{2}x\sqrt{(x^2 - 1)} - \frac{1}{2}\operatorname{arcosh} x + c$$

b. Hence show that the area of R is $\frac{t}{2}$

c. Find the exact coordinates of P when the area of R is $\ln 2$



(11 marks)

Answer

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End of questions | Total 93 marks

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