# Coupled differential equations

Answers

**Further Maths topic practice**

This topic online

| Q | Answer | Marks |
|---|---|---|
| 1 | **a)** shown **b)** $x = Ae^{t} + Be^{5t}$ | 5 |
| 2 | **a)** $y = Ae^{3t} - (3/2)Be^{-2t}$ **b)** $x = 3e^{3t} + 2e^{-2t}$, $y = 3e^{3t} - 3e^{-2t}$ | 5 |
| 3 | **a)** shown **b)** $y = Ae^{2t} + Be^{3t}$, $x = Ae^{2t} + 2Be^{3t}$ | 6 |
| 4 | **a)** shown **b)** $k < -1$ **c)** $x = 2\cos 2t + \sin 2t$, $y = \sin 2t$ | 9 |
| 5 | **a)** shown **b)** $x = Ae^{t} + Be^{-t} - 3t$ **c)** $x = 2e^{t} - e^{-t} - 3t$, $y = -2e^{t} + 3e^{-t} + 6t - 3$ | 10 |
| 6 | **a)** shown **b)** $x = Ae^{t} + Be^{2t} - 3e^{-t}$ **c)** $y = Ae^{t} + (2/3)Be^{2t} - 5e^{-t}$<br>**d)** $x = 6e^{t} + 3e^{2t} - 3e^{-t}$, $y = 6e^{t} + 2e^{2t} - 5e^{-t}$ | 11 |
| 7 | **a)** shown **b)** $x = Ae^{2t} + Be^{-t}$ **c)** $y = (1/2)Ae^{2t} + 2Be^{-t}$<br>**d)** $T = (1/3)\ln 2$ days = 5.55 hours | 11 |
| 8 | **a)** shown **b)** $x = e^{-t}(3\cos 2t + \sin 2t)$, $y = e^{-t}(\cos 2t - 3\sin 2t)$ **c)** shown<br>**d)** $T = (1/2)\ln 1000 = 3.45$ hours | 12 |
| 9 | **a)** $dx/dt = 0.3x - 0.2y$, $dy/dt = 0.4y - 0.1x$ **b)** shown<br>**c)** $x = 60e^{0.2t} - 10e^{0.5t}$, $y = 10e^{0.5t} + 30e^{0.2t}$<br>**d)** $T = (10/3)\ln 6 = 5.97$ years; about 29 700 grey squirrels<br>**e)** it predicts unlimited exponential growth of the grey squirrels, which is unrealistic as food and space are limited | 12 |
| 10 | **a)** shown **b)** $y = Ae^{-0.1t} + Be^{-0.5t} + 12$<br>**c)** $y = 12 - 15e^{-0.1t} + 3e^{-0.5t}$, $x = 8 - 5e^{-0.1t} - 3e^{-0.5t}$<br>**d)** 6.50 mg per litre; $x \to 8$ and $y \to 12$ mg per litre, assuming the river keeps supplying pollutant at the same rate | 12 |
| 11 | **a)** shown **b)** $x = Ae^{-t} + Be^{-4t} + 2.5$<br>**c)** $x = 2.5 - 2e^{-t} - 0.5e^{-4t}$, $y = 3.5 - 4e^{-t} + 0.5e^{-4t}$<br>**d)** $t = 0.609$ minutes; 1.37 kg in each tank | 12 |
| 12 | **a)** shown **b)** $x = e^{0.3t}(A\cos 0.2t + B\sin 0.2t)$<br>**c)** $x = e^{0.3t}(30\cos 0.2t + 35\sin 0.2t)$, $y = e^{0.3t}(100\cos 0.2t - 25\sin 0.2t)$<br>**d)** $T = 5\arctan 4 = 6.63$ years; about 301 barn owls<br>**e)** the model still has the owl population increasing when their food has run out, which is unrealistic; it is not valid beyond this time | 12 |


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Full worked solutions, written out by hand, are on the solutions sheet for this topic.


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