Coupled differential equations

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 117 marks

Further Maths topic practice

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1 The variables x and y satisfy the differential equations

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = 3x + 4y$$

a. Show that $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 5x = 0$

b. Find a general solution for x.

(5 marks)

Answer _____

2 The variables x and y satisfy the differential equations

$$\frac{dx}{dt} = x + 2y$$
$$\frac{dy}{dt} = 3x$$

A general solution for x is $x = Ae^{3t} + Be^{-2t}$, where A and B are constants.

a. Find a general solution for y.

b. Given that $x = 5$ and $y = 0$ when $t = 0$, find the particular solutions for x and y.

(5 marks)

Answer _____

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3 The variables x and y satisfy the differential equations

$$\frac{dx}{dt} = 4x - 2y$$
$$\frac{dy}{dt} = x + y$$

a. Show that $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$

b. Find general solutions for y and for x.

(6 marks)

Answer

4 The variables x and y satisfy the differential equations

$$\frac{dx}{dt} = x + ky$$
$$\frac{dy}{dt} = x - y$$

where k is a non-zero constant.

a. Show that $\frac{d^2x}{dt^2} = (k + 1)x$

b. Find the set of values of k for which x oscillates.

Given that $k = -5$, and that $x = 2$ and $y = 0$ when $t = 0$,

c. find x and y in terms of t.

(9 marks)

Answer

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5 The variables x and y satisfy the differential equations

$$\frac{dx}{dt} = 2x + y$$

$$\frac{dy}{dt} = -3x - 2y + 3t$$

a. Show that $\frac{d^2x}{dt^2} - x = 3t$

b. Find a general solution for x.

c. Given that $x = 1$ and $y = -2$ when $t = 0$, find the particular solutions for x and y. (10 marks)

Answer

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6 The variables x and y satisfy the differential equations

$$\frac{dx}{dt} = 4x - 3y$$

$$\frac{dy}{dt} = 2x - y + 6e^{-t}$$

a. Show that $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = -18e^{-t}$

b. Find a general solution for x.

c. Find a general solution for y.

d. Given that $x = 6$ and $y = 3$ when $t = 0$, find the particular solutions for x and y. (11 marks)

Answer

7 Two species of algae, P and Q, compete in a pond. At time t days, the population of P is x thousand and the population of Q is y thousand. The populations are modelled by the differential equations

$$\frac{dx}{dt} = 3x - 2y$$
$$\frac{dy}{dt} = 2x - 2y$$

a. Show that $\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 0$

b. Find a general solution for x.

c. Find a general solution for y.

Initially there are 200 thousand of P and 250 thousand of Q. The model predicts that, at time T days, the two populations are equal.

d. Determine the value of T, giving your answer in hours to 3 significant figures. (11 marks)

Answer

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8 Two greenhouses are heated by a shared system. At time t hours after a fault is repaired, the temperatures in the two greenhouses are x °C and y °C above their target temperatures. The temperatures are modelled by the differential equations

$$\frac{dx}{dt} = -x + 2y$$
$$\frac{dy}{dt} = -2x - y$$

a. Show that $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$

When $t = 0$, $x = 3$ and $y = 1$

b. Find x and y in terms of t.

c. Show that $x^2 + y^2 = 10e^{-2t}$

d. Hence find a value of T such that, according to the model, both temperatures are within 0.1 °C of their targets for all $t > T$.

(12 marks)

Answer

9 A woodland contains red squirrels and grey squirrels. At time t years, there are x hundred red squirrels and y hundred grey squirrels. An ecologist models the populations as follows.

- The rate of change of x is 0.3 times x minus 0.2 times y.
- The rate of change of y is 0.4 times y minus 0.1 times x.

a. Write down a pair of differential equations for this model.

b. Show that $10\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 7\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0$

Initially there are 5000 red squirrels and 4000 grey squirrels.

c. Find x and y in terms of t.

d. Find the time at which, according to the model, the red squirrels die out, and the number of grey squirrels at that time.

e. Comment on the model for the grey squirrels.

(12 marks)

Answer

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10 Two lakes, P and Q, are joined by a channel. A river carrying a pollutant flows into lake P. At time t weeks after the pollution starts, the concentration of the pollutant is x mg per litre in lake P and y mg per litre in lake Q. The concentrations are modelled by

$$\frac{dx}{dt} = 2 - 0.4x + 0.1y$$
$$\frac{dy}{dt} = 0.3x - 0.2y$$

Initially there is no pollutant in either lake.

a. Show that $20\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + y = 12$

b. Find a general solution for y.

c. Find the particular solutions for y and for x.

d. Find the concentration in lake Q after 10 weeks, and describe what the model predicts for each lake in the long term.

(12 marks)

Answer

11 Two tanks, A and B, are connected by pipes, and salt solution flows into each tank. At time t minutes, tank A contains x kg of salt and tank B contains y kg of salt. The amounts are modelled by

$$\frac{dx}{dt} = -3x + y + 4$$
$$\frac{dy}{dt} = 2x - 2y + 2$$

Initially both tanks contain pure water.

a. Show that $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 10$

b. Find a general solution for x.

c. Find the particular solutions for x and for y.

d. Show that the tanks next contain equal amounts of salt when $u = e^{-t}$ satisfies $u^3 + u^2 + u - 1 = 0$, and hence find this time and the amount of salt in each tank then.

(12 marks)

Answer

12 On an island, barn owls hunt field voles. At time t years after a study begins, there are x barn owls and y hundred field voles. The populations are modelled by

$$\frac{dx}{dt} = 0.2x + 0.1y$$
$$\frac{dy}{dt} = -0.5x + 0.4y$$

a. Show that $100\frac{d^2x}{dt^2} - 60\frac{dx}{dt} + 13x = 0$

b. Find a general solution for the number of barn owls at time t years.

At the start of the study there are 30 barn owls and 10 000 field voles.

c. Find the particular solutions for x and y.

d. Find the time at which, according to the model, the field voles die out, and the number of barn owls at that time.

e. Comment on the model in the light of your answer to part (d).

(12 marks)

Answer _____

End of questions | Total 117 marks

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