De Moivre's theorem

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** $-1/2 + (\sqrt{3}/2)i$ **b)** $\cos 8\theta + i \sin 8\theta$ $(n = 8)$	4
2	**a)** $64 - 64\sqrt{3}\, i$ **b)** $-32768i$	4
3	True for $n = 1$; assuming true for $n = k$, multiplying by $\cos\theta + i \sin\theta$ and using the compound angle formulae gives $\cos(k + 1)\theta + i \sin(k + 1)\theta$, so true for all positive integers n	5
4	**a)** $\cos 4\theta = \text{Re}(c + is)^4 = c^4 - 6c^2 s^2 + s^4 = 8c^4 - 8c^2 + 1$ **b)** $-1/2$	5
5	$z^n - z^{-n} = (\cos n\theta + i \sin n\theta) - (\cos n\theta - i \sin n\theta) = 2i \sin n\theta$; $(z - 1/z)^3 = -8i \sin^3\theta = 2i \sin 3\theta - 6i \sin\theta$, so $\sin^3\theta = (3\sin\theta - \sin 3\theta)/4$	6
6	**a)** $\tan 3\theta = (3c^2 s - s^3)/(c^3 - 3cs^2) = (3t - t^3)/(1 - 3t^2)$ **b)** $\theta = \pi/12$ gives $\tan 3\theta = 1$, so $3t - t^3 = 1 - 3t^2$, i.e. $t^3 - 3t^2 - 3t + 1 = 0$ **c)** $\tan(\pi/12) = 2 - \sqrt{3}$ (it must lie between 0 and 1)	8
7	**a)** $\cos 5\theta = c^5 - 10c^3 s^2 + 5cs^4 = 16c^5 - 20c^3 + 5c$ **b)** $x = 0.914, 0.669, -0.105, -0.5, -0.978$	10
8	**a)** $32i \sin^5\theta = 2i \sin 5\theta - 10i \sin 3\theta + 20i \sin\theta$, so $\sin^5\theta = (\sin 5\theta - 5\sin 3\theta + 10\sin\theta)/16$ **b)** $\theta = 0, \pi/4, 3\pi/4, \pi, 5\pi/4, 7\pi/4$	9
9	**a)** $a = 1/8$, $b = 1/2$, $c = 3/8$ **b)** $7\sqrt{3}/64 + \pi/8$	7
10	**a)** $-64 \sin^6\theta = 2\cos 6\theta - 12\cos 4\theta + 30\cos 2\theta - 20$, so $32\sin^6\theta = 10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta$ **b)** $V = (\pi/32)[10x - (15/2)\sin 2x + (3/2)\sin 4x - (1/6)\sin 6x]$ from 0 to $\pi = 5\pi^2/16$ cm^3 **c)** 308 g	11
11	**a)** Multiply top and bottom by $\cos\theta$: $(\cos\theta + i \sin\theta)/(\cos(-\theta) + i \sin(-\theta)) = \cos 2\theta + i \sin 2\theta$ **b)** $\theta = \pi/10, 3\pi/10$	8
12	**a)** $\cos 6\theta = c^6 - 15c^4 s^2 + 15c^2 s^4 - s^6 = 32c^6 - 48c^4 + 18c^2 - 1$ **b)** With $y = \cos^2\theta$ the cubic is $\cos 6\theta = 0$; $\theta = \pi/12, \pi/4, 5\pi/12$ give three different values of $\cos^2\theta$	12

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c) $(2 + \sqrt{3})/4$

d) $\cos^2(\pi/12)\cos^2(5\pi/12) = (1/32)/(1/2) = 1/16$, both cosines positive, so the product is 1/4

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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