De Moivre's theorem

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 89 marks

Further Maths topic practice

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1

a. Use de Moivre's theorem to express $(\cos \frac{\pi}{9} + i \sin \frac{\pi}{9})^6$ in the form $a + bi$, where a and b are exact.

b. Express $\frac{(\cos 2\theta + i \sin 2\theta)^3}{(\cos \theta - i \sin \theta)^2}$ in the form $\cos n\theta + i \sin n\theta$, where n is an integer to be found.

(4 marks)

Answer _____

2

In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

Use de Moivre's theorem to find, in the form $a + bi$ where a and b are exact,

a. $(1 - \sqrt{3}\, i)^7$

b. $(-2 + 2i)^{10}$

(4 marks)

Answer _____

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3 Prove by induction that, for all positive integers n,

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(5 marks)

Answer

4 a. Use de Moivre's theorem to show that

$\cos 4\theta \equiv 8\cos^4\theta - 8\cos^2\theta + 1$

b. Hence find the exact value of $8\cos^4\left(\frac{\pi}{12}\right) - 8\cos^2\left(\frac{\pi}{12}\right)$

(5 marks)

Answer

5 $z = \cos\theta + i\sin\theta$

a. Show that, for any positive integer n,

$z^n - \frac{1}{z^n} = 2i\sin n\theta$

b. Hence show that

$\sin^3\theta \equiv \frac{1}{4}(3\sin\theta - \sin 3\theta)$

(6 marks)

Answer

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6 a. Use de Moivre's theorem to show that

$\tan 3\theta \equiv \dfrac{3t - t^3}{1 - 3t^2}$

where $t = \tan\theta$.

b. By choosing a suitable value of θ, show that $\tan(\frac{\pi}{12})$ is a root of the equation

$t^3 - 3t^2 - 3t + 1 = 0$

c. Hence find the exact value of $\tan(\frac{\pi}{12})$, giving a reason for your choice of root.

(8 marks)

Answer

7 a. Use de Moivre's theorem to show that

$\cos 5\theta \equiv 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

b. Hence find the distinct roots of the equation

$32x^5 - 40x^3 + 10x + 1 = 0$

giving your answers to 3 decimal places where appropriate.

(10 marks)

Answer

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8 $z = \cos\theta + i\sin\theta$, and you may use without proof that $z^n - \frac{1}{z^n} = 2i\sin n\theta$.

a. Show that

$\sin^5\theta \equiv \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$

b. Hence, making your reasoning clear, solve

$\sin 5\theta - 5\sin 3\theta + 6\sin\theta = 0$

for $0 \le \theta < 2\pi$.

(9 marks)

Answer

9 In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

a. Given that $z = \cos\theta + i\sin\theta$, use $z + \frac{1}{z} = 2\cos\theta$ and $z^n + \frac{1}{z^n} = 2\cos n\theta$ to express

$\cos^4\theta$ in the form

$a\cos 4\theta + b\cos 2\theta + c$

where a, b and c are rational constants to be found.

b. Hence find the exact value of

$\int_0^{\pi/3} \cos^4\theta\, d\theta$

(7 marks)

Answer

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10 a. Given that $z = \cos\theta + i\sin\theta$, show that

$32\sin^6\theta \equiv 10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta$

A glass bead is modelled by the solid formed when the region bounded by the curve with equation $y = \sin^3 x$, $0 \le x \le \pi$, and the x-axis is rotated through 2π radians about the x-axis. The units are centimetres.

b. Use algebraic integration and the result of part (a) to show that the volume of the bead is $\frac{5\pi^2}{16}$ cm^3.

c. The glass has a density of 2.5 grams per cm^3. Find, to 3 significant figures, the total mass of 40 of these beads.

(11 marks)

Answer

11

a. Show that, for $\cos\theta \neq 0$,

$$\frac{1 + i\tan\theta}{1 - i\tan\theta} = \cos 2\theta + i\sin 2\theta$$

b. Hence solve the equation

$$\left(\frac{1 + i\tan\theta}{1 - i\tan\theta}\right)^5 = -1$$

for $0 < \theta < \frac{\pi}{2}$, giving your answers as exact multiples of π.

(8 marks)

Answer

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12 In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

a. Use de Moivre's theorem to show that

$\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$

b. By considering the equation $\cos 6\theta = 0$, explain why the roots of

$32y^3 - 48y^2 + 18y - 1 = 0$

are $\cos^2(\frac{\pi}{12})$, $\cos^2(\frac{\pi}{4})$ and $\cos^2(\frac{5\pi}{12})$.

c. Hence find the exact value of $\cos^2(\frac{\pi}{12})$.

d. Use the product of the roots of the cubic to show that $\cos(\frac{\pi}{12})\cos(\frac{5\pi}{12}) = \frac{1}{4}$

(12 marks)

Answer

End of questions | Total 89 marks

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