Exponential form of complex numbers

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** $4e^{5\pi i/6}$ **b)** $5\sqrt{2}\, e^{-\pi i/4}$	3
2	**a)** $-3 - 3\sqrt{3}\, i$ **b)** $-1 + i$	3
3	**a)** $6e^{5\pi i/12}$ **b)** $(3/2)e^{\pi i/12}$ **c)** $z^4 = 81e^{\pi i} = -81$	4
4	Shown: $e^{-i\theta} = \cos\theta - i\sin\theta$, so the sum is $2\cos\theta$ and the difference is $2i\sin\theta$; $\tan\theta = \sin\theta/\cos\theta = (e^{i\theta} - e^{-i\theta})/(i(e^{i\theta} + e^{-i\theta})) = (e^{2i\theta} - 1)/(i(e^{2i\theta} + 1))$	4
5	**a)** $\lvert z1z2\rvert = 2\sqrt{2}$, $\arg(z1z2) = \pi/3$ **b)** $n = 3$, $w^3 = -16\sqrt{2}$ **c)** $n = 6$, $w^6 = 512$	5
6	$z1 + z2 = 2\sqrt{3} + 6i$, with modulus $\sqrt{48} = 4\sqrt{3}$ and argument $\arctan(\sqrt{3}) = \pi/3$; OPRQ is a rhombus (all sides 4), and the diagonal OR of a rhombus bisects the angle POQ	5
7	**a)** (i) $z1$ is in the second quadrant, so $\arg z1 = \pi - \pi/4 = 3\pi/4$ (ii) for a quotient subtract the arguments: $e^{i(3\pi/4 - \pi/3)}$ **b)** $\sqrt{2}\, e^{5\pi i/12}$	4
8	**a)** $z^* = 2e^{-\pi i/3}$, $1/z = (1/2)e^{-\pi i/3}$, $iz = 2e^{5\pi i/6}$ **b)** $z^3 = 8e^{\pi i} = -8$, so $z^3 + 8 = 0$ **c)** $n = 6$, $(iz)^6 = -64$	6
9	**a)** $e^{4i\theta}/e^{-3i\theta} = e^{7i\theta} = \cos 7\theta + i\sin 7\theta$ **b)** $\theta = \pi/7, 2\pi/7, 3\pi/7, 4\pi/7, 5\pi/7, 6\pi/7$	5
10	**a)** $1 + e^{i\theta} = e^{i\theta/2}(e^{-i\theta/2} + e^{i\theta/2}) = 2\cos(\theta/2)\, e^{i\theta/2}$ **b)** $\lvert 1 + e^{i\theta}\rvert = 2\cos(\theta/2)$, $\arg(1 + e^{i\theta}) = \theta/2$ **c)** $(1 + e^{\pi i/3})^6 = (\sqrt{3}\, e^{\pi i/6})^6 = 27e^{\pi i} = -27$	7
11	**a)** If $z = re^{i\theta}$ then $ze^{i\alpha} = re^{i(\theta + \alpha)}$: same distance from O, argument increased by α **b)** $B = (2 - \sqrt{3}) + (1 + 2\sqrt{3})i$ **c)** $C = (2 + \sqrt{3}) + (1 - 2\sqrt{3})i$ **d)** $5\sqrt{3}$	8
12	**a)** Take out $e^{i(\alpha + \beta)/2}$ and use $e^{ix} \pm e^{-ix}$ **b)** $z1 + z2 = 5\sqrt{3}\, e^{\pi i/4}$, $z1 - z2 = 5e^{3\pi i/4}$ **c)** $\sqrt{3}\, e^{-\pi i/2} = -\sqrt{3}\, i$	9

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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