Exponential form of complex numbers

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 63 marks

Further Maths topic practice

Scan for the solutions

Name: _____ **Date:** _____ **Score:** _____ **/ 63**

1 Express each of these complex numbers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give r and θ as exact values.

a. $z = -2\sqrt{3} + 2i$

b. $w = 5 - 5i$

(3 marks)

Answer _____

2 Express each of these complex numbers in the form $a + bi$, where a and b are exact real numbers.

a. $6e^{-2\pi i/3}$

b. $\sqrt{2}\, e^{3\pi i/4}$

(3 marks)

Answer _____

3 $z = 3e^{\pi i/4}$ and $w = 2e^{\pi i/6}$

In parts (a) and (b), give your answers in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$.

a. Find zw

b. Find $\frac{z}{w}$

c. Show that z^4 is a real number and state its value.

(4 marks)

Answer _____

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

4 (a) Use $e^{i\theta} = \cos\theta + i\sin\theta$ to show that

(i) $e^{i\theta} + e^{-i\theta} = 2\cos\theta$

(ii) $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$

(b) Hence show that

$$\tan\theta = \frac{e^{2i\theta} - 1}{i(e^{2i\theta} + 1)}$$

(4 marks)

Answer

5 $z_1 = 2e^{\pi i/4}$ and $z_2 = \sqrt{2}\, e^{\pi i/12}$

(a) Write down the exact values of

(i) $|z_1 z_2|$

(ii) $\arg(z_1 z_2)$

Given that $w = z_1 z_2$ and that n is a positive integer,

(b) determine the smallest value of n for which w^n is a negative real number, and the value of w^n in this case,

(c) determine the smallest value of n for which $\arg(w^n) = 0$, and the value of w^n in this case.

(5 marks)

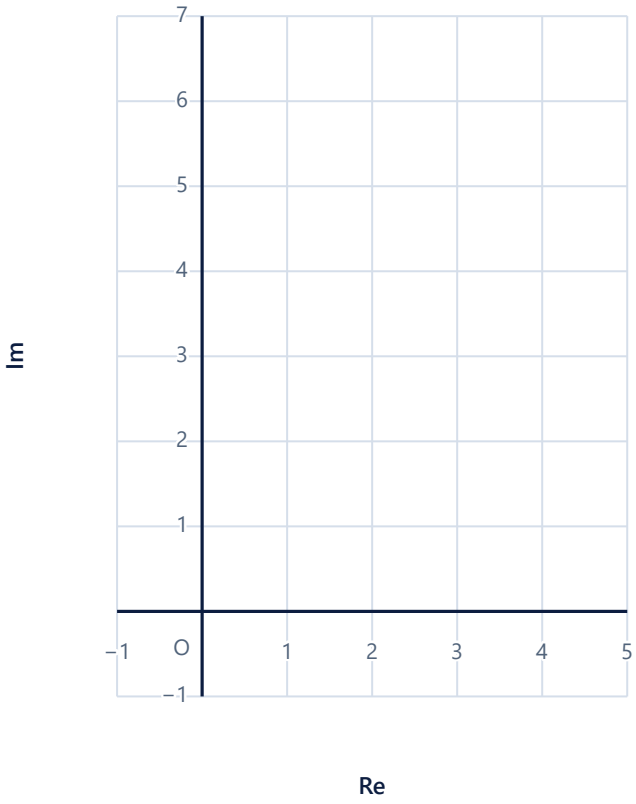
Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

6 $z_1 = 4e^{\pi i/6}$ and $z_2 = 4e^{\pi i/2}$

(a) Show that $z_1 + z_2 = 4\sqrt{3}\, e^{\pi i/3}$

(b) The points P, Q and R represent z_1, z_2 and $z_1 + z_2$ on an Argand diagram, and O is the origin. State the type of quadrilateral $OPRQ$ and explain why arg $(z_1 + z_2)$ lies exactly halfway between arg z_1 and arg z_2.



(5 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

7 A student was asked to write $\frac{z_1}{z_2}$ in the form $re^{i\theta}$, where $z_1 = -2 + 2i$ and $z_2 = 1 + \sqrt{3}\,i$. The student's attempt is shown below.

Line 1: $|z_1| = 2\sqrt{2}$, $\arg z_1 = \arctan(\frac{2}{-2}) = -\frac{\pi}{4}$

Line 2: $|z_2| = 2$, $\arg z_2 = \frac{\pi}{3}$

Line 3: $\frac{z_1}{z_2} = \frac{2\sqrt{2}}{2}\ e^{i(-\pi/4 + \pi/3)}$

Line 4: $\frac{z_1}{z_2} = \sqrt{2}\ e^{\pi i/12}$

The student made errors in Line 1 and Line 3.

a. Explain and correct the error in (i) Line 1 (ii) Line 3

b. Write down the correct value of $\frac{z_1}{z_2}$ in the form $re^{i\theta}$.

(4 marks)

Answer _____

8 $z = 2e^{\pi i/3}$

a. Write each of z^*, $\frac{1}{z}$ and iz in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$.

b. Show that $z^3 + 8 = 0$

c. Find the smallest positive integer n for which $(iz)^n$ is real, and find the value of $(iz)^n$ in this case.

(6 marks)

Answer _____

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

9

a. By writing each bracket in exponential form, show that

$$\frac{\cos 4\theta + i \sin 4\theta}{\cos 3\theta - i \sin 3\theta} = \cos 7\theta + i \sin 7\theta$$

b. Hence find all the values of θ, in the interval $0 < \theta < \pi$, for which $\frac{\cos 4\theta + i \sin 4\theta}{\cos 3\theta - i \sin 3\theta}$ is real.

(5 marks)

Answer

10

a. Show that $1 + e^{i\theta} = 2\cos(\frac{\theta}{2})\, e^{i\theta/2}$

b. Hence, for $0 < \theta < \pi$, write down the modulus and the argument of $1 + e^{i\theta}$ in terms of θ.

c. Hence show that $(1 + e^{\pi i/3})^6 = -27$

(7 marks)

Answer

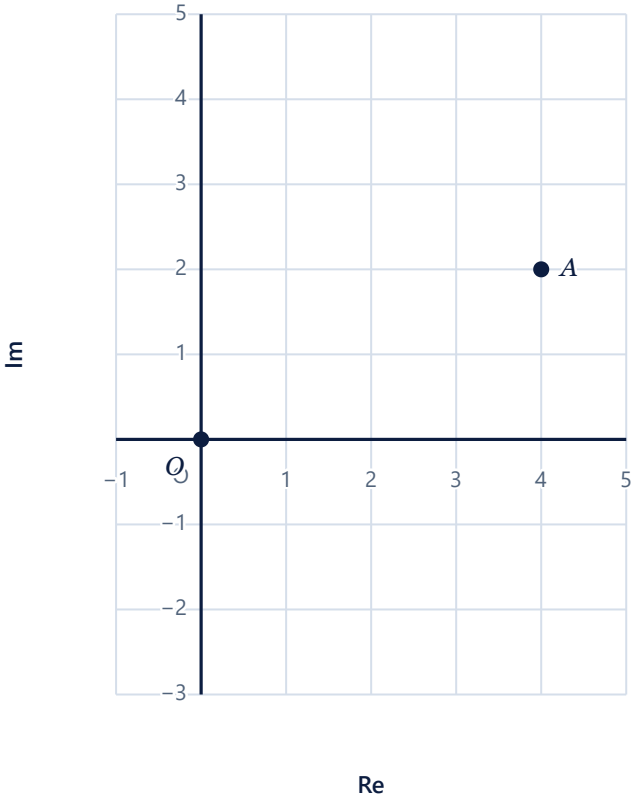
11 The point A on an Argand diagram represents the complex number $4 + 2i$, and O is the origin.

a. Explain why multiplying a complex number by $e^{i\alpha}$ rotates its point about O through an angle α anticlockwise.

b. The triangle OAB is equilateral, and B lies anticlockwise from A about O. Find the complex number represented by B, giving your answer in the form $p + qi$ where p and q are exact.

c. The triangle OAC is also equilateral, with C on the other side of OA from B. Find the complex number represented by C in the same form.

d. Find the exact area of triangle OAB.



(8 marks)

Answer

12 a. Show that, for real α and β,

$e^{i\alpha} + e^{i\beta} = 2\cos\left(\frac{\alpha - \beta}{2}\right) e^{i(\alpha + \beta)/2}$

and

$e^{i\alpha} - e^{i\beta} = 2i\sin\left(\frac{\alpha - \beta}{2}\right) e^{i(\alpha + \beta)/2}$

Given that $z_1 = 5e^{5\pi i/12}$ and $z_2 = 5e^{\pi i/12}$,

b. use part (a) to write $z_1 + z_2$ and $z_1 - z_2$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$,

c. hence show that $\frac{z_1 + z_2}{z_1 - z_2}$ is purely imaginary and find its value.

(9 marks)

Answer

End of questions | Total 63 marks

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.