Hyperbolic functions

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** 4/3 **b)** 3/5 **c)** $\cosh(\ln a) = (a^2 + 1)/(2a)$	5
2	$\cosh^2 x - \sinh^2 x = 1$, $\sinh 2x = 2 \sinh x \cosh x$ and $1 - \tanh^2 x = 1/\cosh^2 x$, all shown from the definitions	6
3	**a)** $y = \cosh x$: U-shaped, symmetrical in the y-axis, minimum at (0, 1). $y = \sinh x$: through the origin with rotational symmetry about O. The sinh curve lies below the cosh curve and the two get closer as x increases **b)** Shown: $-\cosh x < \sinh x < \cosh x$, so $-1 < \tanh x < 1$ **c)** $x = (1/2) \ln 3$	8
4	**a)** $\text{arcosh } x = \ln(x + \sqrt{(x^2 - 1)})$ shown **b)** $\ln 3$	6
5	$x = 0$ or $x = \ln 4 (= 2 \ln 2)$	5
6	**a)** $\cosh 2x = 1 + 2\sinh^2 x$ shown **b)** $x = \ln(2 + \sqrt{5})$ or $x = \ln((\sqrt{5} - 1)/2)$	8
7	**a)** $\sqrt{5}/2$ **b)** $1/\sqrt{5} = \sqrt{5}/5$ **c)** $\sinh 2\alpha = \sqrt{5}/2$ shown **d)** $\ln((3 + \sqrt{5})/2)$	7
8	**a)** $\cosh 3x = 4\cosh^3 x - 3\cosh x$ shown **b)** $x = \ln(2 + \sqrt{3})$ or $x = -\ln(2 + \sqrt{3})$	8
9	**a)** $f(x) = e^x + 2e^{-x}$ shown **b)** $x = 0$ or $x = \ln 2$ **c)** $k \geq 2\sqrt{2}$ **d)** minimum value $2\sqrt{2}$ at $x = (1/2) \ln 2$	10
10	**a)** $\cosh x = 4/3$ shown **b)** P$(\ln((4 + \sqrt{7})/3), \sqrt{7})$ and Q$(-\ln((4 + \sqrt{7})/3), -\sqrt{7})$ **c)** $-\ln((4 + \sqrt{7})/3)$ $\ln((4 + \sqrt{7})/3)$	9
11	**a)** 5 m **b)** $10(e^{1.5} + e^{-1.5}) - 15 = 32.0$ m **c)** $x = \pm 20 \ln 2$; 27.7 m **d)** 1.16 m (the parabola is higher)	10
12	**a)** $\text{artanh } x = (1/2) \ln((1 + x)/(1 - x))$ shown **b)** $\text{artanh}((x - 1)/(x + 1)) = (1/2) \ln x$ shown **c)** $x = 4$	10

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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