Hyperbolic functions

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 92 marks

Further Maths topic practice

Scan for the solutions

Name: _____ **Date:** _____ **Score:** _____ **/ 92**

1 Using the definitions of the hyperbolic functions in terms of exponentials,

a. find the exact value of sinh(ln 3),

b. find the exact value of tanh(ln 2),

c. show that, for $a > 0$, $\cosh(\ln a) = \frac{a^2 + 1}{2a}$

(5 marks)

Answer _____

2 Using the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials,

a. prove that $\cosh^2 x - \sinh^2 x \equiv 1$

b. prove that $\sinh 2x \equiv 2 \sinh x \cosh x$

c. Hence show that $1 - \tanh^2 x \equiv \frac{1}{\cosh^2 x}$

(6 marks)

Answer _____

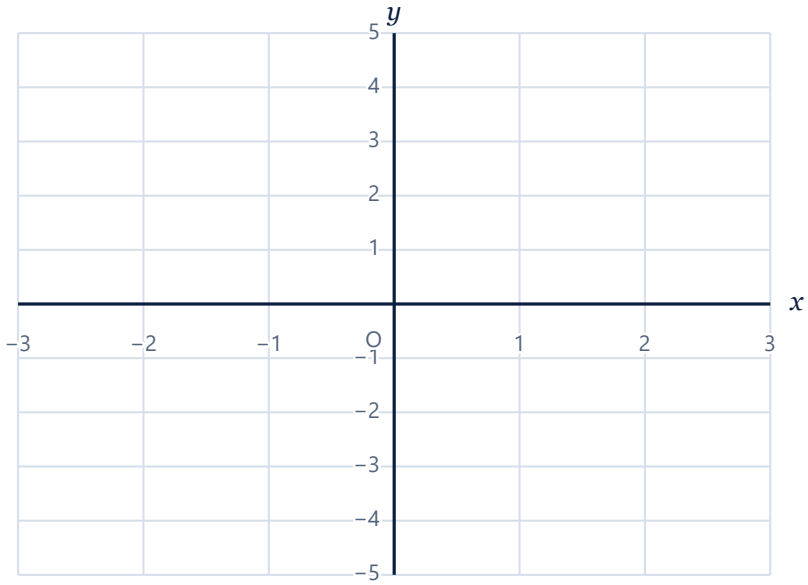
Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

3

a. On the axes below, sketch the graphs of $y = \cosh x$ and $y = \sinh x$, showing the coordinates of any points where the graphs meet the coordinate axes.

b. Using the definitions in terms of exponentials, show that $\cosh x - \sinh x = e^{-x}$ and $\cosh x + \sinh x = e^{x}$. Hence show that $-1 < \tanh x < 1$ for all real x.

c. Solve the equation $\tanh x = \frac{1}{2}$, giving your answer as an exact natural logarithm.



(8 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

4 a. Starting from the definition of cosh y in terms of exponentials, show that, for $x \geq 1$,

arcosh $x = \ln(x + \sqrt{(x^2 - 1)})$

explaining why the other root is rejected.

b. Hence find the exact value of $\text{arcosh}(\frac{5}{3})$ in its simplest form.

(6 marks)

Answer

5 In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

Solve the equation

$5 \cosh x - 3 \sinh x = 5$

giving your answers in exact form.

(5 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

6

a. Using the definitions of the hyperbolic functions in terms of exponentials, show that
$\cosh 2x \equiv 1 + 2\sinh^2 x$

b. Hence solve the equation
$\cosh 2x - 3\sinh x = 3$
giving your answers as exact natural logarithms.

(8 marks)

Answer

7

Let $\alpha = \text{arsinh}(\frac{1}{2})$.

a. Find the exact value of $\cosh \alpha$.

b. Find the exact value of $\tanh \alpha$.

c. Show that $\sinh 2\alpha = \frac{\sqrt{5}}{2}$

d. Hence, or otherwise, express $2\,\text{arsinh}(\frac{1}{2})$ as a single natural logarithm in the form $\ln(\frac{p + \sqrt{q}}{r})$, where p, q and r are integers.

(7 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

8

a. Using the definition of cosh x in terms of exponentials, prove that
cosh $3x \equiv 4\cosh^3 x - 3\cosh x$

b. Hence solve the equation
cosh $3x = 13\cosh x$
giving your answers as exact natural logarithms.

(8 marks)

Answer

9

$f(x) = 3\cosh x - \sinh x, \quad x \in \mathbb{R}$

a. Show that $f(x) = e^x + 2e^{-x}$

b. Solve the equation $f(x) = 3$, giving your answers in exact form.

c. Find the set of values of the constant k for which the equation $f(x) = k$ has real solutions.

d. Hence write down the minimum value of $f(x)$ and find the exact value of x at which it occurs.

(10 marks)

Answer

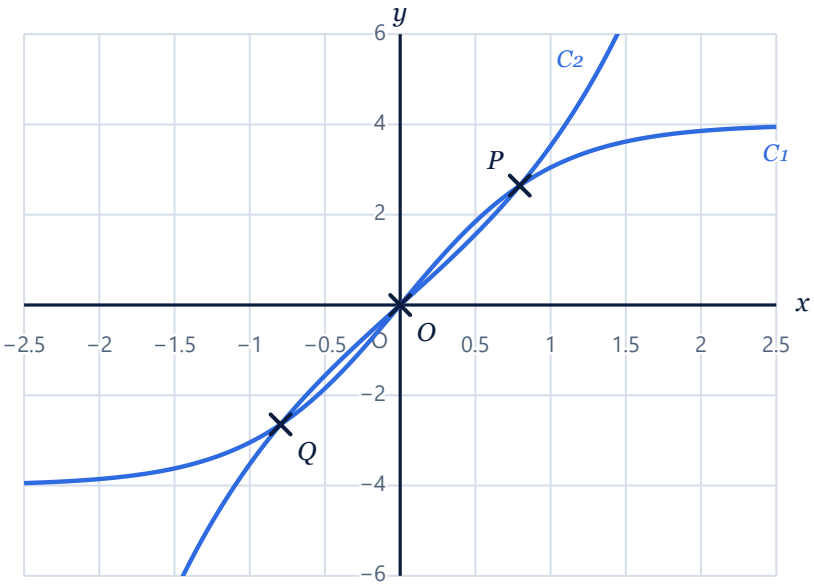
Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

10 The diagram shows the curves C_1 with equation $y = 4 \tanh x$ and C_2 with equation $y = 3 \sinh x$. The curves meet at the origin O and at the points P and Q.

a. Show that the x coordinates of P and Q satisfy $\cosh x = \frac{4}{3}$

b. Find the exact coordinates of P and of Q, giving the x coordinates as natural logarithms.

c. Hence find the set of values of x for which $3 \sinh x > 4 \tanh x$



(9 marks)

Answer

11 A power cable hangs between the tops of two vertical poles which stand on horizontal ground 60 m apart. The ground is taken as the x-axis and the point on the ground midway between the poles as the origin. The cable is modelled by the curve

$$y = 20\cosh\left(\frac{x}{20}\right) - 15, \quad -30 \le x \le 30$$

where x and y are measured in metres.

a. Find the height of the lowest point of the cable above the ground.

b. Find the height of each pole, giving your answer in terms of e and then to 3 significant figures.

c. Find the exact values of x at which the cable is 10 m above the ground. Hence find, to 3 significant figures, the horizontal distance between these two points.

d. A second engineer models the cable as a parabola $y = ax^2 + b$ with the same lowest point and passing through the tops of both poles. Find, to 3 significant figures, the difference between the heights predicted by the two models at $x = 20$

(10 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

12 a. Starting from the definition of tanh y in terms of exponentials, show that

$$\text{artanh } x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad |x| < 1$$

b. Show that, for $x > 0$,

$$\text{artanh}\left(\frac{x-1}{x+1}\right) = \frac{1}{2}\ln x$$

c. Hence solve the equation

$$\text{artanh}\left(\frac{x-1}{x+1}\right) = \text{arsinh}\left(\frac{3}{4}\right)$$

(10 marks)

Answer _____

End of questions | Total 92 marks

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.