Improper integrals

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	$2e^{-6}$	3
2	**a)** $2/\sqrt[3]{x}$ is not defined (it tends to infinity) at x = 0, which is a limit of the integral **b)** 12	4
3	**a)** 6 **b)** The integral from 4 to t is $12\sqrt{t} - 24$, which $\to \infty$ as $t \to \infty$, so it diverges	5
4	**a)** 12 **b)** The integral from a to 5 is $1/(a - 1) - 1/4$, which $\to \infty$ as $a \to 1$, so it diverges	6
5	1/4	6
6	**a)** The upper limit of the integral is infinite **b)** $1/(3x) - 2/(3(2x + 3))$ **c)** a = 1/3, b = 5/2	7
7	**a)** $(x + 3)^2 + 4$ **b)** π/2	6
8	**a)** ln x is not defined at x = 0, which is a limit of integration **b)** -4/9 shown	6
9	**a)** shown **b)** π/12	6
10	**a)** shown **b)** k = 1: ln t $\to \infty$; 0 < k < 1: $t^{1-k} \to \infty$; divergent in both cases **c)** k = 3/2	8
11	**a)** shown **b)** 160 litres **c)** 59.4% **d)** 160 > 120, so the model cannot be right for large t; it must break down before all the gas has escaped	9
12	**a)** The integral from 0 to t is $(3/2)\ln(2t + 1)$, which $\to \infty$ **b)** 9π/2 **c)** k = 49.5	10

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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