Improper integrals

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 76 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 76**

1 Evaluate the improper integral

$\int_2^{\infty} 6e^{-3x}\,dx$

giving your answer in exact form.

(3 marks)

Answer _____

2 $I = \int_0^8 \frac{2}{\sqrt[3]{x}}\,dx$

a. Explain why I is an improper integral.

b. Find the value of I.

(4 marks)

Answer _____

3 a. Evaluate $\int_4^{\infty} \frac{6}{x\sqrt{x}}\,dx$

b. Show that $\int_4^{\infty} \frac{6}{\sqrt{x}}\,dx$ is divergent.

(5 marks)

Answer _____

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4

a. Find the value of $\int_1^5 \frac{3}{\sqrt{(x-1)}}\,dx$

b. Show that $\int_1^5 \frac{1}{(x-1)^2}\,dx$ is divergent.

(6 marks)

Answer

5

Show that the improper integral

$\int_0^\infty xe^{-2x}\,dx$

converges, and find its exact value.

(6 marks)

Answer

6

a. Explain why $\int_1^\infty \frac{1}{x(2x+3)}\,dx$ is an improper integral.

b. Express $\frac{1}{x(2x+3)}$ in partial fractions.

c. Hence show that $\int_1^\infty \frac{1}{x(2x+3)}\,dx = a \ln b$, where a and b are rational numbers to be found.

(7 marks)

Answer _____

7

a. Write $x^2 + 6x + 13$ in the form $(x+p)^2 + q$, where p and q are integers.

b. Hence evaluate $\int_{-\infty}^\infty \frac{1}{x^2+6x+13}\,dx$, giving your answer as an exact multiple of π.

(6 marks)

Answer _____

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8 $I = \int_0^1 \sqrt{x} \ln x \, dx$

a. Explain why I is an improper integral.

b. Show that $I = -\frac{4}{9}$

You may use without proof the result that $x^k \ln x \to 0$ as $x \to 0$, for any constant $k > 0$ (6 marks)

Answer _____

9 a. Use the substitution $u = e^x$ to show that

$$\int \frac{e^x}{e^{2x} + 9} \, dx = \frac{1}{3} \arctan\left(\frac{e^x}{3}\right) + c$$

b. Hence find the exact value of $\int_{\ln 3}^{\infty} \frac{e^x}{e^{2x} + 9} \, dx$ (6 marks)

Answer _____

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10 k is a positive constant.

a. Show that, for $k > 1$, $\int_1^{\infty} x^{-k}\,\mathrm{d}x = \dfrac{1}{k-1}$

b. Explain, with working, what happens to $\int_1^{\infty} x^{-k}\,\mathrm{d}x$ when $k = 1$ and when $0 < k < 1$

c. Find the value of k for which $\int_1^{\infty} x^{-k}\,\mathrm{d}x = \int_0^{1} x^{-\frac{1}{2}}\,\mathrm{d}x$

(8 marks)

Answer

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11 A gas cylinder develops a leak. The rate at which gas escapes, R litres per hour, is modelled by

$R = 40te^{-\frac{t}{2}} \quad t \geq 0$

where t is the time in hours after the leak starts.

a. Show that, according to the model, the volume of gas that escapes in the first T hours is

$160 - (80T + 160)e^{-\frac{T}{2}}$ litres

b. Use an improper integral to find the total volume of gas that the model predicts will escape if the leak is never stopped. You must state clearly any limits you use.

c. Find the percentage of this total that escapes in the first 4 hours.

d. The cylinder held 120 litres of gas when the leak started. Comment on the model in the light of your answer to part (b).

(9 marks)

Answer

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12 The curve C has equation

$$y = \frac{3}{2x + 1} \quad x \geq 0$$

The region R is bounded by C, the x-axis and the y-axis, and extends without limit in the positive x direction.

a. Show that the area of R is not finite.

b. The region R is rotated through 2π radians about the x-axis. Show that the volume of the solid formed is finite, and find its exact value.

c. The part of R with $0 \leq x \leq k$ is rotated through 2π radians about the x-axis. The volume of this solid is 99% of the volume found in part (b). Find the value of k.

(10 marks)

Answer

End of questions | Total 76 marks

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