Inverse trigonometric functions in calculus

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 74 marks

Further Maths topic practice

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1 Differentiate with respect to x

a. arcsin $(4x)$

b. arctan (x^3)

(3 marks)

Answer _____

2 Find

a. $\int \frac{1}{25 + x^2} \, dx$

b. $\int \frac{3}{\sqrt{(4 - x^2)}} \, dx$

(3 marks)

Answer _____

3 a. Given that $y = \arccos x$, $-1 < x < 1$, show that

$$\frac{dy}{dx} = -\frac{1}{\sqrt{(1 - x^2)}}$$

b. Given that $y = \sin(\arccos 3x)$, $-\frac{1}{3} < x < \frac{1}{3}$, determine $\frac{dy}{dx}$, giving your answer in its simplest form.

(5 marks)

Answer _____

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4 Find the exact value of

a. $\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{3 + 4x^2}\,dx$

b. $\int_0^{\frac{1}{3}} \frac{1}{\sqrt{(4 - 9x^2)}}\,dx$

(6 marks)

Answer _____

5 $f(x) = (1 + x^2) \arctan x$

a. Find $f'(x)$.

b. Show that

$(1 + x^2) f''(x) - 2f(x) = kx$

where k is an integer to be found.

(6 marks)

Answer _____

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6 Show that

$$\int_0^4 \frac{4x - 3}{x^2 + 16}\,dx = a \ln 2 + b\pi$$

where a and b are rational numbers to be found.

(6 marks)

Answer

7 a. Write $8x - x^2 - 7$ in the form $a - (x - b)^2$, where a and b are integers.

b. Hence find the exact value of

$$\int_4^{\frac{11}{2}} \frac{1}{\sqrt{(8x - x^2 - 7)}}\,dx$$

(5 marks)

Answer

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8

$$f(x) = \frac{3x^2 + 4x + 11}{(x + 1)(x^2 + 4)}$$

a. Express $f(x)$ in partial fractions.

b. Hence show that

$\int_0^2 f(x)\,dx = p \ln q + r\pi$

where p, q and r are rational numbers to be found.

(8 marks)

Answer

9

The curve C has equation

$y = \arctan(2x) - x$

a. Find the exact coordinates of the stationary points of C.

b. Determine the nature of each stationary point.

(7 marks)

Answer

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10 The curve C has equation

$y = x \arcsin x \quad -1 < x < 1$

The tangent to C at the point where $x = \frac{1}{2}$ meets the y-axis at the point P.

Show that the coordinates of P are $(0, k\sqrt{3})$, where k is a rational number to be found. (6 marks)

Answer

11 The curve C has equation

$$y = \frac{e^x}{\sqrt{(9 - e^{2x})}} \quad x < \ln 3$$

a. Use the substitution $u = e^x$ to show that $\int y \, dx = \arcsin\left(\frac{e^x}{3}\right) + c$

b. Find the exact area of the region bounded by C, the x-axis and the lines $x = \ln\frac{3}{2}$ and $x = \ln\frac{3\sqrt{2}}{2}$

c. Prove that C has no stationary points. (9 marks)

Answer

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12 A mural is painted on a vertical wall. The bottom of the mural is 2 m above a visitor's eye level and the top of the mural is 5 m above the visitor's eye level. The visitor stands on level ground at a horizontal distance x metres from the wall.

The angle subtended at the visitor's eye by the mural is θ radians, where

$\theta = \arctan\left(\frac{5}{x}\right) - \arctan\left(\frac{2}{x}\right) \quad x > 0$

a. Show that $\frac{d\theta}{dx} = \frac{2}{x^2 + 4} - \frac{5}{x^2 + 25}$

b. Find the exact value of x for which $\frac{d\theta}{dx} = 0$

c. Show that this value of x gives a maximum value of θ, and find this maximum angle in degrees to 1 decimal place.

d. Give one limitation of the model.

(10 marks)

Answer

End of questions | Total 74 marks

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