Maclaurin series

Worked solutions

Further Maths topic practice

This topic online

1 $f(x) = \ln(2 + x), \quad x > -2$

a. Using differentiation, find the Maclaurin series for $f(x)$ in ascending powers of x, up to and including the term in x^3, giving each coefficient in its simplest form.

b. Use your series to find an approximation for ln 2.1, giving your answer to 6 decimal places. (5 marks)

a)

Differentiate and evaluate at $x = 0$

$f(x) = \ln(2 + x)$

$f(0) = \ln 2$

$f'(x) = (2 + x)^{-1}$

$f'(0) = \frac{1}{2}$

$f''(x) = -(2 + x)^{-2}$

$f''(0) = -\frac{1}{4}$

$f'''(x) = 2(2 + x)^{-3}$

$f'''(0) = \frac{2}{8} = \frac{1}{4}$

$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$

$\ln(2 + x) = \ln 2 + \frac{1}{2}x - \frac{1}{4} \times \frac{x^2}{2} + \frac{1}{4} \times \frac{x^3}{6} + \ldots$

$$\ln(2 + x) = \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{24}x^3 + \ldots$$

b)

$\ln 2.1 = \ln(2 + 0.1)$, so use $x = 0.1$

$\ln 2.1 \approx \ln 2 + 0.05 - 0.00125 + 0.0000416\ldots$

$\ln 2.1 \approx 0.693147\ldots + 0.048791\ldots$

$$\ln 2.1 \approx 0.741939 \text{ (6 d.p.)}$$

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2 Use the standard Maclaurin series to find, in ascending powers of x,

a. the first three non-zero terms of the series for $x \cos 2x$

b. the series for $\ln(1 - 3x)$, up to and including the term in x^3, stating the range of values of x for which the expansion is valid.

(5 marks)

a)

$\cos u = 1 - \frac{u^2}{2} + \frac{u^4}{24} - \ldots$, with $u = 2x$

$\cos 2x = 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{24} - \ldots$

$\cos 2x = 1 - 2x^2 + \frac{2}{3}x^4 - \ldots$

$$x \cos 2x = x - 2x^3 + \frac{2}{3}x^5 - \ldots$$

b)

$\ln(1 + u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \ldots$, with $u = -3x$

$\ln(1 - 3x) = (-3x) - \frac{(-3x)^2}{2} + \frac{(-3x)^3}{3} - \ldots$

$$\ln(1 - 3x) = -3x - \frac{9}{2}x^2 - 9x^3 - \ldots$$

$\ln(1 + u)$ is valid for $-1 < u \leq 1$

$-1 < -3x \leq 1$

$$\text{Valid for } -\frac{1}{3} \leq x < \frac{1}{3}$$

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3 a. Write down the Maclaurin series for $\sin x$ and for $\ln(1 + x)$, each up to and including the term in x^3

b. Hence, without differentiating, find the Maclaurin series for $\ln(1 + \sin x)$, in ascending powers of x up to and including the term in x^3, giving each coefficient in its simplest form.

(5 marks)

a)

$$\sin x = x - \frac{x^3}{6} + \ldots$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$$

b)

Let $u = \sin x = x - \frac{x^3}{6}$, keeping terms up to x^3

$$u^2 = x^2 + (\text{terms in } x^4 \text{ and higher})$$

$$u^3 = x^3 + (\text{terms in } x^5 \text{ and higher})$$

$$\ln(1 + \sin x) = u - \frac{u^2}{2} + \frac{u^3}{3} - \ldots$$

$$\ln(1 + \sin x) = \left(x - \frac{x^3}{6}\right) - \frac{x^2}{2} + \frac{x^3}{3} + \ldots$$

$$\ln(1 + \sin x) = x - \frac{x^2}{2} + \left(-\frac{1}{6} + \frac{1}{3}\right)x^3 + \ldots$$

$$\boxed{\ln(1 + \sin x) = x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots}$$

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4

a. Using the Maclaurin series for $\ln(1 + x)$, find the series expansion for $\ln\left(\frac{1 + 2x}{1 - x}\right)$ in ascending powers of x, up to and including the term in x^3, giving each coefficient in its simplest form.

b. State the range of values of x for which the expansion is valid.

c. By choosing a suitable value of x, use your answer to part (a) to find an approximation for ln 2, giving your answer as a fraction in its simplest form.

d. Find the percentage error in this approximation.

(8 marks)

a)

$$\ln(1 + 2x) = 2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \ldots$$

$$\ln(1 + 2x) = 2x - 2x^2 + \frac{8}{3}x^3 - \ldots$$

$$\ln(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots$$

Use the laws of logarithms and subtract

$$\ln\left(\frac{(1 + 2x)}{(1 - x)}\right) = \ln(1 + 2x) - \ln(1 - x)$$

$$\ln\left(\frac{(1 + 2x)}{(1 - x)}\right) = (2 + 1)x + (-2 + \frac{1}{2})x^2 + (\frac{8}{3} + \frac{1}{3})x^3 + \ldots$$

$$\boxed{\ln\left(\frac{(1 + 2x)}{(1 - x)}\right) = 3x - \frac{3}{2}x^2 + 3x^3 + \ldots}$$

b)

$\ln(1 + 2x)$ needs $-1 < 2x \leq 1$, so $-\frac{1}{2} < x \leq \frac{1}{2}$

$\ln(1 - x)$ needs $-1 < -x \leq 1$, so $-1 \leq x < 1$

$$\boxed{\text{Valid for } -\frac{1}{2} < x \leq \frac{1}{2}}$$

c)

Solve $\frac{(1 + 2x)}{(1 - x)} = 2$

$$1 + 2x = 2 - 2x$$

$$x = \frac{1}{4}$$

$x = \frac{1}{4}$ is in the valid range

$$\ln 2 \approx 3(\frac{1}{4}) - \frac{3}{2}(\frac{1}{4})^2 + 3(\frac{1}{4})^3$$

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$$\ln 2 \approx \frac{3}{4} - \frac{3}{32} + \frac{3}{64}$$

$$\boxed{\ln 2 \approx \frac{45}{64}}$$

d)

$$\%\ \text{error} = \frac{\left(\frac{45}{64} - \ln 2\right)/\ln 2}{} \times 100$$

$$\%\ \text{error} = \frac{(0.703125 - 0.693147\ldots)}{0.693147\ldots} \times 100$$

$$\boxed{1.44\%\ (3\ \text{s.f.})}$$

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5 $f(x) = \ln(\cos x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

a. Show that the Maclaurin series for $f(x)$, up to and including the term in x^4, is

$-\frac{1}{2}x^2 - \frac{1}{12}x^4$

b. Hence write down the first two non-zero terms of the Maclaurin series for $\ln(\cos 2x)$.

(6 marks)

a)

$f(x) = \ln(\cos x)$

$f'(x) = \frac{-\sin x}{\cos x} = -\tan x$

$f''(x) = -\sec^2 x$

$f'''(x) = -2\sec x \times \sec x \tan x = -2\sec^2 x \tan x$

Product rule

$f^{(4)}(x) = -4\sec^2 x \tan x \times \tan x - 2\sec^2 x \times \sec^2 x$

$f^{(4)}(x) = -4\sec^2 x \tan^2 x - 2\sec^4 x$

At $x = 0$, $\sec 0 = 1$ and $\tan 0 = 0$

$f(0) = 0$

$f'(0) = 0$

$f''(0) = -1$

$f'''(0) = 0$

$f^{(4)}(0) = -2$

$\ln(\cos x) = -\frac{1}{2!}x^2 - \frac{2}{4!}x^4 + \ldots$

$$\ln(\cos x) = -\frac{1}{2}x^2 - \frac{1}{12}x^4 + \ldots$$

b)

Replace x with $2x$

$\ln(\cos 2x) = -\frac{1}{2}(2x)^2 - \frac{1}{12}(2x)^4 + \ldots$

$$\ln(\cos 2x) = -2x^2 - \frac{4}{3}x^4 + \ldots$$

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6 Given that $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

a. use differentiation to find the first two non-zero terms of the Maclaurin series for arctan x, giving each coefficient in its simplest form.

b. By substituting $x = \frac{1}{\sqrt{3}}$ into your answer to part (a), find an approximation for π, giving your answer in the form $\frac{p\sqrt{3}}{q}$, where p and q are integers.

(6 marks)

a)

$f(x) = \arctan x$

$f(0) = 0$

$f'(x) = (1+x^2)^{-1}$

$f'(0) = 1$

$f''(x) = -2x(1+x^2)^{-2}$

$f''(0) = 0$

Product rule

$f'''(x) = -2(1+x^2)^{-2} + 8x^2(1+x^2)^{-3}$

$f'''(0) = -2$

$\arctan x = x + \frac{(-2)}{3!}x^3 + \ldots$

$$\arctan x = x - \frac{1}{3}x^3 + \ldots$$

b)

$\arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\frac{\pi}{6} \approx \frac{1}{\sqrt{3}} - \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^3$

$\frac{\pi}{6} \approx \frac{1}{\sqrt{3}} - \frac{1}{(9\sqrt{3})} = \frac{8}{(9\sqrt{3})}$

$\pi \approx \frac{48}{(9\sqrt{3})} = \frac{16}{(3\sqrt{3})}$

$$\pi \approx \frac{16\sqrt{3}}{9}$$

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7 $y = e^x \cos x$

a. Show that $\frac{d^4y}{dx^4} = ky$, where k is a constant to be found.

b. Hence find the first five non-zero terms of the Maclaurin series for y, giving each coefficient in its simplest form.

(8 marks)

a)

Product rule each time

$$\frac{dy}{dx} = e^x \cos x - e^x \sin x = e^x(\cos x - \sin x)$$

$$\frac{d^2y}{dx^2} = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x)$$

$$\frac{d^2y}{dx^2} = -2e^x \sin x$$

$$\frac{d^3y}{dx^3} = -2e^x \sin x - 2e^x \cos x = -2e^x(\sin x + \cos x)$$

$$\frac{d^4y}{dx^4} = -2e^x(\sin x + \cos x) - 2e^x(\cos x - \sin x)$$

$$\frac{d^4y}{dx^4} = -4e^x \cos x$$

$$\boxed{\frac{d^4y}{dx^4} = -4y, \text{ so } k = -4}$$

b)

Values at $x = 0$

$$y(0) = 1$$

$$y'(0) = 1$$

$$y''(0) = 0$$

$$y'''(0) = -2$$

Use $y^{(4)} = -4y$ and $y^{(5)} = -4y'$

$$y^{(4)}(0) = -4$$

$$y^{(5)}(0) = -4$$

$$y = 1 + x + 0x^2 - \frac{2}{3!}x^3 - \frac{4}{4!}x^4 - \frac{4}{5!}x^5 + \ldots$$

$$\boxed{e^x \cos x = 1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4 - \frac{1}{30}x^5 + \ldots}$$

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8

a. Use the Maclaurin series for e^x to find the series expansion of e^{-x^2} in ascending powers of x, up to and including the term in x^6

b. Use your answer to part (a) and calculus to find an approximation for

$\int_0^{0.6} e^{-x^2}\,dx$

giving your answer to 5 decimal places.

c. Use the integration function on your calculator to evaluate the integral, giving your answer to 5 decimal places.

d. Comment on the accuracy of your approximation in part (b).

(7 marks)

a)

$e^u = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \ldots$, with $u = -x^2$

$$e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2} + \frac{(-x^2)^3}{6} + \ldots$$

$$\boxed{e^{-x^2} = 1 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6 + \ldots}$$

b)

$$\int \left(1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6}\right) dx = x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + c$$

Evaluate $\left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42}\right]$ from 0 to 0.6

$$0.6 - \frac{0.6^3}{3} + \frac{0.6^5}{10} - \frac{0.6^7}{42} - 0$$

$$0.6 - 0.072 + 0.007776 - 0.0006665\ldots$$

$$\boxed{\int \text{from } 0 \text{ to } 0.6 \text{ of } e^{-x^2}\,dx \approx 0.53511}$$

c)

$$\boxed{0.53515}$$

d)

$$0.53515 - 0.53511 = 0.00004$$

$$\%\ \text{error} = \frac{0.00004}{0.53515} \times 100 \approx 0.008\%$$

$$\boxed{\text{Very accurate: it agrees to 3 decimal places}}$$

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9 a. Write down the Maclaurin series for e^{2x} and for $\sin x$, each up to and including the term in x^3

b. Hence find

$$\lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x \sin x}$$

c. Use Maclaurin series to find

$$\lim_{x \to 0} \frac{1 - \cos 3x}{x \ln(1 + x)}$$

(8 marks)

a)

$$e^{2x} = 1 + 2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{6} + \ldots$$

$$e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \ldots$$

$$\sin x = x - \frac{x^3}{6} + \ldots$$

b)

$$e^{2x} - 1 - 2x = 2x^2 + \frac{4}{3}x^3 + \ldots$$

$$x \sin x = x^2 - \frac{x^4}{6} + \ldots$$

Divide top and bottom by x^2

$$\frac{(e^{2x} - 1 - 2x)}{(x \sin x)} = \frac{(2 + \frac{4}{3}x + \ldots)}{(1 - \frac{x^2}{6} + \ldots)}$$

As $x \to 0$, the top tends to 2 and the bottom tends to 1

Limit = 2

c)

$$\cos 3x = 1 - \frac{(3x)^2}{2} + \frac{(3x)^4}{24} - \ldots$$

$$1 - \cos 3x = \frac{9}{2}x^2 - \frac{27}{8}x^4 + \ldots$$

$$x \ln(1 + x) = x(x - \frac{x^2}{2} + \ldots) = x^2 - \frac{1}{2}x^3 + \ldots$$

Divide top and bottom by x^2

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$$\frac{(1-\cos 3x)}{(x\ln(1+x))} = \frac{\left(\frac{9}{2}-\frac{27}{8}x^2+\ldots\right)}{\left(1-\frac{1}{2}x+\ldots\right)}$$

$$\boxed{\text{Limit} = \frac{9}{2}}$$

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10 Given that $\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$, $-1 < x < 1$

a. use the Maclaurin series for $\ln(1+x)$ and $\ln(1-x)$ to show that

$\operatorname{artanh} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots$

and state the range of values of x for which this expansion is valid.

b. Write down, in terms of r, the general term of the series.

c. Using the first three terms of the series with a suitable value of x, find an approximation for ln 3, giving your answer as a fraction in its simplest form.

d. Find the percentage error in your approximation.

(9 marks)

a)

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \ldots$$

Subtract: the even powers cancel and the odd powers double

$$\ln(1+x) - \ln(1-x) = 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \ldots$$

$$\operatorname{artanh} x = \frac{1}{2}\left(2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \ldots\right)$$

$$\boxed{\operatorname{artanh} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots}$$

$\ln(1+x)$ needs $-1 < x \leq 1$ and $\ln(1-x)$ needs $-1 \leq x < 1$

$$\boxed{\text{Valid for } -1 < x < 1}$$

b)

$$\boxed{\text{General term} = \frac{x^{2r+1}}{(2r+1)},\ r = 0, 1, 2, \ldots}$$

c)

$\ln 3 = 2\operatorname{artanh} x$ when $\frac{(1+x)}{(1-x)} = 3$

$$1 + x = 3 - 3x$$

$$x = \frac{1}{2}$$

$$\ln 3 \approx 2\left(\frac{1}{2} + \frac{\left(\frac{1}{2}\right)^3}{3} + \frac{\left(\frac{1}{2}\right)^5}{5}\right)$$

$$\ln 3 \approx 2\left(\frac{1}{2} + \frac{1}{24} + \frac{1}{160}\right)$$

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$$\ln 3 \approx 1 + \frac{1}{12} + \frac{1}{80}$$

$$\ln 3 \approx \frac{240}{240} + \frac{20}{240} + \frac{3}{240}$$

$$\boxed{\ln 3 \approx \frac{263}{240}}$$

d)

$$\%\ \text{error} = \frac{(\ln 3 - 263}{240)/\ln 3} \times 100$$

$$\%\ \text{error} = \frac{(1.098612... - 1.095833...)}{1.098612...} \times 100$$

$$\boxed{0.253\%\ (3\ \text{s.f.})}$$

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11 $y = e^{\arcsin x}$, $-1 < x < 1$

a. Show that $(1 - x^2)(\frac{dy}{dx})^2 = y^2$

b. Hence show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

c. Find the Maclaurin series for y, in ascending powers of x, up to and including the term in x^4, giving each coefficient in its simplest form.

(10 marks)

a)

Chain rule, with the derivative of arcsin x = $\frac{1}{\sqrt{(1-x^2)}}$

$$\frac{dy}{dx} = e^{\arcsin x} \times \frac{1}{\sqrt{(1-x^2)}}$$

$$\frac{dy}{dx} = \frac{y}{\sqrt{(1-x^2)}}$$

$$\sqrt{(1-x^2)}\frac{dy}{dx} = y$$

Square both sides

$$\boxed{(1-x^2)(\frac{dy}{dx})^2 = y^2}$$

b)

Differentiate implicitly (product rule on the left)

$$-2x(\frac{dy}{dx})^2 + 2(1-x^2)\frac{dy}{dx}\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}$$

Divide by $2\frac{dy}{dx}$, which is never 0 since $y > 0$

$$-x\frac{dy}{dx} + (1-x^2)\frac{d^2y}{dx^2} = y$$

$$\boxed{(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0}$$

c)

Differentiate again

$$-2x\frac{d^2y}{dx^2} + (1-x^2)\frac{d^3y}{dx^3} - \frac{dy}{dx} - x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

$$(1-x^2)\frac{d^3y}{dx^3} - 3x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$$

And again

$$-2x\frac{d^3y}{dx^3} + (1-x^2)\frac{d^4y}{dx^4} - 3\frac{d^2y}{dx^2} - 3x\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} = 0$$

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$$(1-x^2)\frac{d^4y}{dx^4} - 5x\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} = 0$$

Put $x = 0$

$$y(0) = e^0 = 1$$

$$y'(0) = \frac{1}{\sqrt{1}} = 1$$

$$y''(0) = 0 + y(0) = 1$$

$$y'''(0) = 2y'(0) = 2$$

$$y^{(4)}(0) = 5y''(0) = 5$$

$$y = 1 + x + \frac{1}{2!}x^2 + \frac{2}{3!}x^3 + \frac{5}{4!}x^4 + \ldots$$

$$e^{\arcsin x} = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{5}{24}x^4 + \ldots$$

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12 A power cable hangs between two pylons that are 60 m apart, as shown in the diagram. The lowest point of the cable is O, midway between the pylons. The height of the cable above O, y metres, at a horizontal distance x metres from O, is modelled by

$$y = 40\cosh\left(\frac{x}{40}\right) - 40, \quad -30 \le x \le 30$$

a. Using the Maclaurin series for e^x, show that

$$y \approx \frac{x^2}{80} + \frac{x^4}{k}$$

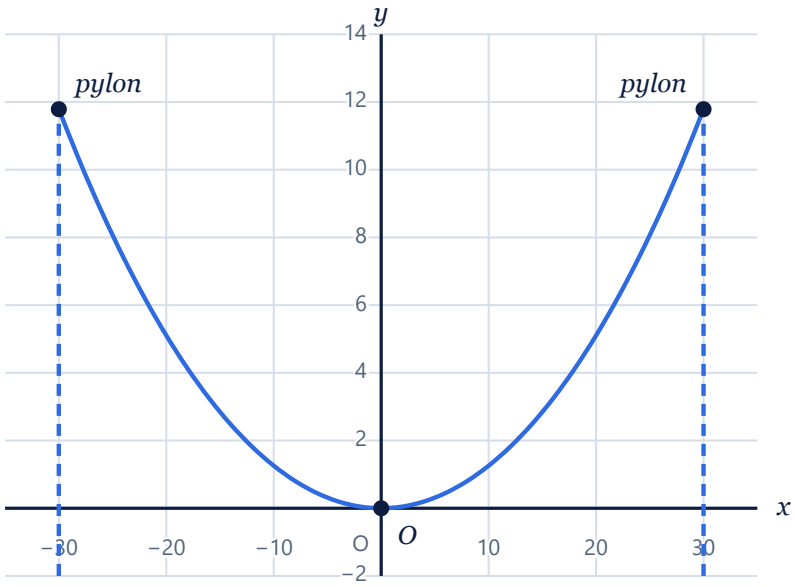
where k is a constant to be found.

An engineer uses the simpler model $y = \frac{x^2}{80}$

b. Estimate the height of the cable at a pylon, above O, using (i) the simpler model and (ii) the approximation in part (a). Give your answers to 4 significant figures.

c. Find the percentage error in each estimate, compared with the value given by the original model.

d. Explain why the simpler model becomes less accurate further from O.



(9 marks)

a)

$$\cosh u = \frac{(e^u + e^{-u})}{2}$$

$$e^u = 1 + u + \frac{u^2}{2} + \frac{u^3}{6} + \frac{u^4}{24} + \ldots$$

$$e^{-u} = 1 - u + \frac{u^2}{2} - \frac{u^3}{6} + \frac{u^4}{24} - \ldots$$

Add and halve: the odd powers cancel

$$\cosh u = 1 + \frac{u^2}{2} + \frac{u^4}{24} + \ldots$$

Let $u = \frac{x}{40}$

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$$40\cosh\left(\frac{x}{40}\right) = 40 + \frac{40x^2}{3200} + \frac{40x^4}{61440000} + \ldots$$

$$y \approx \frac{x^2}{80} + \frac{x^4}{1536000}$$

$\boxed{k = 1536000}$

b)

(i) $y = \frac{30^2}{80} = 11.25$

(ii) $y = 11.25 + \frac{30^4}{1536000} = 11.25 + 0.527\ldots = 11.777\ldots$

$\boxed{\text{(i) } 11.25 \text{ m, (ii) } 11.78 \text{ m}}$

c)

$$y = 40\cosh\left(\frac{30}{40}\right) - 40 = 11.787\ldots$$

$$\%\text{ error (i)} = \frac{(11.787\ldots - 11.25)}{11.787\ldots} \times 100 = 4.56\%$$

$$\%\text{ error (ii)} = \frac{(11.787\ldots - 11.777\ldots)}{11.787\ldots} \times 100 = 0.0847\%$$

$\boxed{4.56\% \text{ and } 0.0847\%}$

d)

The simpler model ignores the x^4, x^6, ... terms

These are tiny when x is small but grow quickly as $|x|$ increases

$\boxed{\text{So the error is largest near the pylons, furthest from O}}$

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