Maclaurin series

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 86 marks

Further Maths topic practice

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1 $f(x) = \ln(2 + x), \quad x > -2$

a. Using differentiation, find the Maclaurin series for $f(x)$ in ascending powers of x, up to and including the term in x^3, giving each coefficient in its simplest form.

b. Use your series to find an approximation for ln 2.1, giving your answer to 6 decimal places.

(5 marks)

Answer _____

2 Use the standard Maclaurin series to find, in ascending powers of x,

a. the first three non-zero terms of the series for $x \cos 2x$

b. the series for $\ln(1 - 3x)$, up to and including the term in x^3, stating the range of values of x for which the expansion is valid.

(5 marks)

Answer _____

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3 a. Write down the Maclaurin series for $\sin x$ and for $\ln(1 + x)$, each up to and including the term in x^3

b. Hence, without differentiating, find the Maclaurin series for $\ln(1 + \sin x)$, in ascending powers of x up to and including the term in x^3, giving each coefficient in its simplest form.

(5 marks)

Answer

4 a. Using the Maclaurin series for $\ln(1 + x)$, find the series expansion for $\ln\left(\frac{1 + 2x}{1 - x}\right)$ in ascending powers of x, up to and including the term in x^3, giving each coefficient in its simplest form.

b. State the range of values of x for which the expansion is valid.

c. By choosing a suitable value of x, use your answer to part (a) to find an approximation for ln 2, giving your answer as a fraction in its simplest form.

d. Find the percentage error in this approximation.

(8 marks)

Answer

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5 $f(x) = \ln(\cos x), \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

a. Show that the Maclaurin series for $f(x)$, up to and including the term in x^4, is

$-\frac{1}{2}x^2 - \frac{1}{12}x^4$

b. Hence write down the first two non-zero terms of the Maclaurin series for $\ln(\cos 2x)$. (6 marks)

Answer _____

6 Given that $\frac{d}{dx}(\arctan x) = \frac{1}{1 + x^2}$

a. use differentiation to find the first two non-zero terms of the Maclaurin series for $\arctan x$, giving each coefficient in its simplest form.

b. By substituting $x = \frac{1}{\sqrt{3}}$ into your answer to part (a), find an approximation for π, giving your answer in the form $\frac{p\sqrt{3}}{q}$, where p and q are integers.

(6 marks)

Answer _____

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7 $y = e^x \cos x$

a. Show that $\frac{d^4y}{dx^4} = ky$, where k is a constant to be found.

b. Hence find the first five non-zero terms of the Maclaurin series for y, giving each coefficient in its simplest form.

(8 marks)

Answer

8

a. Use the Maclaurin series for e^x to find the series expansion of e^{-x^2} in ascending powers of x, up to and including the term in x^6

b. Use your answer to part (a) and calculus to find an approximation for

$\int_0^{0.6} e^{-x^2} dx$

giving your answer to 5 decimal places.

c. Use the integration function on your calculator to evaluate the integral, giving your answer to 5 decimal places.

d. Comment on the accuracy of your approximation in part (b).

(7 marks)

Answer

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9

a. Write down the Maclaurin series for e^{2x} and for $\sin x$, each up to and including the term in x^3

b. Hence find

$$\lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x \sin x}$$

c. Use Maclaurin series to find

$$\lim_{x \to 0} \frac{1 - \cos 3x}{x \ln(1 + x)}$$

(8 marks)

Answer _____

10 Given that $\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad -1 < x < 1$

a. use the Maclaurin series for $\ln(1+x)$ and $\ln(1-x)$ to show that

$\operatorname{artanh} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots$

and state the range of values of x for which this expansion is valid.

b. Write down, in terms of r, the general term of the series.

c. Using the first three terms of the series with a suitable value of x, find an approximation for ln 3, giving your answer as a fraction in its simplest form.

d. Find the percentage error in your approximation.

(9 marks)

Answer

11 $y = e^{\arcsin x}, \quad -1 < x < 1$

a. Show that $(1 - x^2)(\frac{dy}{dx})^2 = y^2$

b. Hence show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

c. Find the Maclaurin series for y, in ascending powers of x, up to and including the term in x^4, giving each coefficient in its simplest form.

(10 marks)

Answer

12 A power cable hangs between two pylons that are 60 m apart, as shown in the diagram. The lowest point of the cable is O, midway between the pylons. The height of the cable above O, y metres, at a horizontal distance x metres from O, is modelled by

$$y = 40\cosh\left(\frac{x}{40}\right) - 40, \quad -30 \le x \le 30$$

a. Using the Maclaurin series for e^x, show that

$$y \approx \frac{x^2}{80} + \frac{x^4}{k}$$

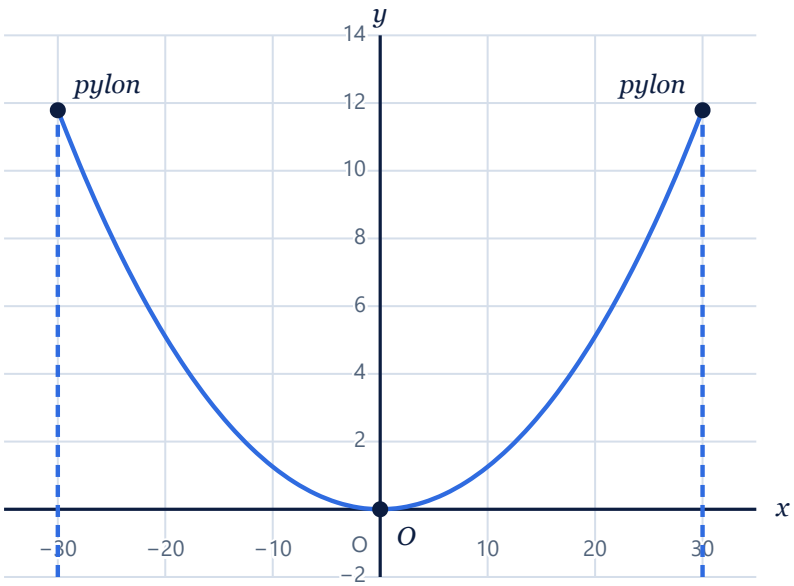
where k is a constant to be found.

An engineer uses the simpler model $y = \frac{x^2}{80}$

b. Estimate the height of the cable at a pylon, above O, using (i) the simpler model and (ii) the approximation in part (a). Give your answers to 4 significant figures.

c. Find the percentage error in each estimate, compared with the value given by the original model.

d. Explain why the simpler model becomes less accurate further from O.



(9 marks)

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Answer

End of questions | Total 86 marks

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