Mean value of a function

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 73 marks

Further Maths topic practice

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1 Find the mean value of $f(x) = 3x^2 - 2x + 1$ over the interval $[1, 4]$. (3 marks)

Answer _____

2 Show that the mean value of $f(x) = \frac{4}{2x + 1}$ over the interval $[0, 4]$ is $\ln 3$ (3 marks)

Answer _____

3 Find the exact mean value of $f(x) = \sin^2 x$ over the interval $[0, \frac{\pi}{3}]$ (4 marks)

Answer _____

4 $f(x) = xe^x$

a. Find the exact mean value of $f(x)$ over the interval $[0, 2]$.

b. Hence write down the exact mean value of $3f(x) - 4$ over the interval $[0, 2]$. (5 marks)

Answer _____

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5 $f(x) = \dfrac{1}{x^2 - 4x + 13}$

a. Write $x^2 - 4x + 13$ in the form $(x + p)^2 + q$, where p and q are integers.

b. Hence find $\int f(x)\,dx$

c. Find the exact mean value of $f(x)$ over the interval $[2, 5]$.

(6 marks)

Answer _____

6 Find the exact mean value of

$f(x) = \dfrac{1}{\sqrt{(16 - 9x^2)}}$

over the interval $[0, \frac{2}{3}]$

(5 marks)

Answer _____

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7 The function f is continuous and the mean value of f(x) over the interval [1, 5] is 7

a. Find the mean value of 3f(x) + 2 over the interval [1, 5].

b. Find the mean value of f(x − 2) over the interval [3, 7].

c. Find the mean value of f(2x) over the interval $[\frac{1}{2}, \frac{5}{2}]$.

d. Given also that the mean value of f(x) over the interval [5, 9] is 4, find the mean value of f(x) over the interval [1, 9].

(6 marks)

Answer _____

8 $f(x) = \dfrac{1}{\sqrt{(x^2 + 6x + 13)}}$

a. Write $x^2 + 6x + 13$ in the form $(x + p)^2 + q$, where p and q are integers.

b. Hence, using a standard integral from the formula book, find the exact mean value of f(x) over the interval $[-3, -\frac{3}{2}]$, giving your answer in the form $a \ln b$, where a and b are rational.

(6 marks)

Answer _____

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9 a. Use integration by parts to find $\int \arctan x \, dx$

b. Show that the mean value of $\arctan x$ over the interval $[0, \sqrt{3}]$ is

$\frac{\pi}{3} - \frac{\sqrt{3}}{3} \ln 2$

c. Hence write down the mean value of $\arctan\left(\frac{x}{2}\right)$ over the interval $[0, 2\sqrt{3}]$. Give a reason for your answer.

(8 marks)

Answer

10 After heavy rain, the rate of flow of water in a river, R m^3 s^{-1}, is modelled by

$R = 20 + 90te^{-\frac{t}{2}} \quad 0 \leq t \leq 10$

where t is the time in days after the rain starts.

a. Use calculus to find the mean rate of flow over the 10 days, giving your answer to 3 significant figures.

b. Hence estimate the total volume of water that flows past a point on the river during the 10 days. (1 day = 86 400 seconds.)

c. Measurements show that the actual mean rate of flow over the 10 days was 47 m^3 s^{-1}. Suggest how the model could be refined.

(8 marks)

Answer

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11 The temperature, θ °C, inside a greenhouse is modelled by

$$\theta = 14 - 5\cos\left(\frac{\pi t}{12}\right) \quad 0 \leq t \leq 24$$

where t is the number of hours after midnight.

a. Use calculus to show that, according to the model, the mean temperature over the whole day is 14 °C.

b. Find, according to the model, the mean temperature between 6 am and 6 pm. Give your answer in °C to 3 significant figures.

c. The temperature in degrees Fahrenheit is $F = 1.8\theta + 32$. Find the mean temperature between 6 am and 6 pm in degrees Fahrenheit, explaining how you used your answer to part (b).

d. Give one reason why the mean temperature recorded inside the greenhouse on a particular day might differ from 14 °C.

(9 marks)

Answer

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12 $f(x) = \dfrac{1}{x^2 + 2x + 5}$

a. Show that the mean value of $f(x)$ over the interval $[-1, 1]$ is $\dfrac{\pi}{16}$

b. Find the value of x in the interval $[-1, 1]$ for which $f(x)$ is equal to its mean value over this interval. Give your answer to 3 decimal places.

c. Show that $\int_{-1}^{\infty} f(x)\,dx$ converges and find its exact value.

d. Hence explain what happens to the mean value of $f(x)$ over the interval $[-1, k]$ as $k \to \infty$

(10 marks)

Answer _____

End of questions | Total 73 marks

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