Method of differences

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** $1/(2(2r - 1)) - 1/(2(2r + 1))$ **b)** Shown: the sum telescopes to $(1/2)(1 - 1/(2n + 1)) = n/(2n + 1)$	5
2	**a)** Shown: $r^2[(r + 1)^2 - (r - 1)^2] = r^2 \times 4r = 4r^3$ **b)** Shown: $\Sigma 4r^3 = f(n) - f(0) = n^2(n + 1)^2$	5
3	**a)** Shown: multiply top and bottom by $\sqrt{(r + 1)} - \sqrt{r}$; the denominator becomes $(r + 1) - r = 1$ **b)** Shown: the terms telescope to $\sqrt{(n + 1)} - \sqrt{1}$ **c)** $n = 440$ **d)** $\sqrt{(n + 1)} - 1$ increases without limit as $n \to \infty$, so the partial sums do not converge	7
4	**a)** Shown: $(r + 1)! - r! = (r + 1)r! - r! = r \times r!$ **b)** Shown: the sum telescopes to $(n + 1)! - 1!$ **c)** $n = 9$	6
5	$a = 5, b = 13$	6
6	**a)** Shown: $((r + 2) - r)/(r(r + 1)(r + 2)) = 2/(r(r + 1)(r + 2))$ **b)** Shown: $(1/2)(1/2 - 1/((n + 1)(n + 2)))$ **c)** $1/4$ **d)** $1/220$	8
7	**a)** Shown: $1 - 1/r^2 = (r - 1)(r + 1)/r^2$ **b)** Shown: the sum telescopes to $g(2) - g(n + 1)$ with $g(r) = \ln(r - 1) - \ln r$ **c)** $\ln(41/44)$ **d)** $-\ln 2$	7
8	**a)** Shown: terms $2/(r - 1) - 2/(r + 1)$ leave $3 - 2/n - 2/(n + 1)$ **b)** Shown: $S(2n) - S(n) = 1/n + 2/(n + 1) - 2/(2n + 1)$	8
9	**a)** $2/r - 1/(r + 1) - 1/(r + 2)$ **b)** Shown: the sum is $5/2 - 2/(n + 1) - 1/(n + 2)$ **c)** $5/2$	9
10	**a)** Shown: $((r + 1)^2 - r^2)/(r^2(r + 1)^2) = (2r + 1)/(r^2(r + 1)^2)$ **b)** $1 - 1/(n + 1)^2$ **c)** $1/100$ **d)** $n = 100$	8
11	**a)** Each $f(k)$ added in row $k - 1$ is subtracted in row $k + 1$; only $f(n + 1)$ and $f(n)$ are never subtracted and $f(0)$ and $f(1)$ are never added **b)** Shown: $(r + 1)^3 - (r - 1)^3 = 6r^2 + 2$ **c)** Shown: $6\Sigma r^2 + 2n = (n + 1)^3 + n^3 - 1$, so $6\Sigma r^2 = n(n + 1)(2n + 1)$ **d)** 242792	10
12	**a)** Shown: $6000(1/2 - 1/(n + 2)) = 3000n/(n + 2)$ **b)** $3000n/(n + 2) = 3000 - 6000/(n + 2)$, which is always less than 3000 **c)** 59 **d)** £129.87	9

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Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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