Method of differences

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 88 marks

Further Maths topic practice

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1

a. Express $\frac{1}{(2r-1)(2r+1)}$ in partial fractions.

b. Hence use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$$

(5 marks)

Answer _____

2

$f(r) = r^2(r+1)^2$

a. Show that $f(r) - f(r-1) = 4r^3$

b. Hence use the method of differences to prove that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5 marks)

Answer _____

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3

a. Show that $\frac{1}{\sqrt{(r+1)}+\sqrt{r}} = \sqrt{(r+1)} - \sqrt{r}$

b. Hence show that $\sum_{r=1}^{n} \frac{1}{\sqrt{(r+1)}+\sqrt{r}} = \sqrt{(n+1)} - 1$

c. Find the value of n for which the sum in part (b) is equal to 20

d. Explain why the infinite series $\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots$ does not have a finite sum.

(7 marks)

Answer _____

4

a. Show that $(r+1)! - r! = r \times r!$

b. Hence show that $\sum_{r=1}^{n} r \times r! = (n+1)! - 1$

c. Find the smallest value of n for which $\sum_{r=1}^{n} r \times r! > 1\,000\,000$

(6 marks)

Answer _____

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5 Use the method of differences to show that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{4}{(r+1)(r+3)} = \frac{n(an+b)}{3(n+2)(n+3)}$$

where a and b are integers to be found.

(6 marks)

Answer

6 a. Show that $\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{2}{r(r+1)(r+2)}$

b. Hence show that $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

c. Write down the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$

d. Find the exact value of $\sum_{r=10}^{\infty} \frac{1}{r(r+1)(r+2)}$

(8 marks)

Answer

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7

a. Show that $\ln(1 - \frac{1}{r^2}) = \ln(r - 1) + \ln(r + 1) - 2\ln r$

b. Hence use the method of differences to show that, for $n \geq 2$,

$$\sum_{r=2}^{n} \ln(1 - \frac{1}{r^2}) = \ln(\frac{n+1}{2n})$$

c. Find the exact value of $\sum_{r=11}^{40} \ln(1 - \frac{1}{r^2})$, giving your answer in the form $\ln(\frac{p}{q})$ where p and q are integers.

d. State the value of $\sum_{r=2}^{\infty} \ln(1 - \frac{1}{r^2})$

(7 marks)

Answer

8

a. Use the method of differences to show that, for $n \geq 2$,

$$\sum_{r=2}^{n} \frac{4}{r^2 - 1} = \frac{(n - 1)(3n + 2)}{n(n + 1)}$$

b. Hence show that $\sum_{r=n+1}^{2n} \frac{4}{r^2 - 1} = \frac{4n^2 + 3n + 1}{n(n + 1)(2n + 1)}$

(8 marks)

Answer

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9

a. Express $\frac{3r + 4}{r(r + 1)(r + 2)}$ in partial fractions.

b. Hence use the method of differences to show that

$$\sum_{r = 1}^{n} \frac{3r + 4}{r(r + 1)(r + 2)} = \frac{n(5n + 9)}{2(n + 1)(n + 2)}$$

c. Find the sum to infinity of the series.

(9 marks)

Answer _____

10

a. Show that $\frac{1}{r^2} - \frac{1}{(r + 1)^2} = \frac{2r + 1}{r^2(r + 1)^2}$

b. Hence find $\sum_{r = 1}^{n} \frac{2r + 1}{r^2(r + 1)^2}$, giving your answer in terms of n.

c. Find the exact value of $\sum_{r = 10}^{\infty} \frac{2r + 1}{r^2(r + 1)^2}$

d. Find the smallest value of n for which $\sum_{r = 1}^{n} \frac{2r + 1}{r^2(r + 1)^2} > 0.9999$

(8 marks)

Answer _____

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11 a. Explain why, for any function f,

$$\sum_{r=1}^{n} [f(r+1) - f(r-1)] = f(n+1) + f(n) - f(1) - f(0)$$

b. Given that $f(r) = r^3$, show that $f(r+1) - f(r-1) = 6r^2 + 2$

c. Hence prove that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

d. Without using the formula in part (c), evaluate $\sum_{r=20}^{50} (6r^2 + 2)$

(10 marks)

Answer

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12 A games company runs an online puzzle. The rth player to solve the puzzle wins a prize of £$\dfrac{6000}{(r+1)(r+2)}$

a. Use the method of differences to show that the total amount paid to the first n players to solve the puzzle is £ $\dfrac{3000n}{n+2}$

b. Explain why the company will never pay out £3000 or more in total, however many players solve the puzzle.

c. Find the number of players who must solve the puzzle for the total paid out to first exceed £2900

d. Find the total amount won by the 21st to the 40th players inclusive, giving your answer to the nearest penny.

(9 marks)

Answer _____

End of questions | Total 88 marks

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