Nth roots and roots of unity

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** ±1, ±i, ±√2/2 ± (√2/2)i **b)** Eight points equally spaced on the circle $\|z\| = 1$, π/4 apart, starting at 1: a regular octagon	4
2	√3 + i, -√3 + i, -2i	4
3	**a)** Shown: $z^3 - 1 = (z - 1)(z^2 + z + 1)$ and $\omega \neq 1$, so $\omega^2 + \omega + 1 = 0$ **b)** Shown: $4 + 2(\omega + \omega^2) + \omega^3 = 4 - 2 + 1 = 3$ **c)** 1	6
4	**a)** $2e^{\pi i/6}$, $2e^{2\pi i/3}$, $2e^{-\pi i/3}$, $2e^{-5\pi i/6}$ **b)** Four points on the circle $\|z\| = 2$ at arguments π/6, 2π/3, -π/3 and -5π/6: the vertices of a square	7
5	**a)** 1, $e^{\pi i/3}$, $e^{2\pi i/3}$, $e^{\pi i}$, $e^{4\pi i/3}$, $e^{5\pi i/3}$ **b)** Shown: $3^6 e^{\pi i} = 729 \times (-1) = -729$ **c)** $3e^{\pi i/6}$, $3e^{\pi i/2}$, $3e^{5\pi i/6}$, $3e^{7\pi i/6}$, $3e^{3\pi i/2}$, $3e^{11\pi i/6}$	7
6	**a)** $2e^{\pi i/4}$, $2e^{11\pi i/12}$, $2e^{-5\pi i/12}$ **b)** √2 + √2i **c)** 3√3	9
7	**a)** $6e^{4\pi i/15}$, $6e^{-2\pi i/15}$, $6e^{-8\pi i/15}$, $6e^{-14\pi i/15}$ **b)** 85.6	7
8	**a)** 1, -1/2 + (√3/2)i, -1/2 - (√3/2)i **b)** p = 4i, q = 2 + i **c)** (2 - 2√3) - i and (2 + 2√3) - i **d)** 12√3	9
9	**a)** 1, $e^{2\pi i/5}$, $e^{-2\pi i/5}$, $e^{4\pi i/5}$, $e^{-4\pi i/5}$ **b)** The roots satisfy $z^5 - 1 = 0$, which has no z^4 term, so the sum of the roots is -0/1 = 0 **c)** Shown: $1 + 2\cos(2\pi/5) + 2\cos(4\pi/5) = 0$ **d)** Shown: $4c^2 + 2c - 1 = 0$ with $c > 0$ gives $c = (\sqrt{5} - 1)/4$	9
10	**a)** 1 + i, -1 + i, -1 - i, 1 - i **b)** 1 + 3i, -1 + 3i, -1 + i, 1 + i **c)** The roots of $w^4 = -4$ translated by 2i: equally spaced on a circle with centre 2i, so a square of side 2 with area 4	8
11	**a)** Relative to C the bolts are 4 units away, π/4 apart, starting at 4: they are $4e^{k\pi i/4}$, the roots of $w^8 = 4^8 = 65536$ with w = z - 3 - 2i **b)** (3 + 2√2) + (2 + 2√2)i, 3 + 6i, (3 - 2√2) + (2 + 2√2)i, -1 + 2i, (3 - 2√2) + (2 - 2√2)i, 3 - 2i, (3 + 2√2) + (2 - 2√2)i **c)** 245 cm	9

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12	**a)** Expanding, the z^5 terms cancel to leave $5z^4 + 10z^3 + 10z^2 + 5z + 1 = 0$, a quartic, which has exactly four roots **b)** Shown: $((z + 1)/z)^5 = 1$, so $w = (z + 1)/z$, $wz = z + 1$, $z = 1/(w - 1)$ **c)** Shown **d)** $z = -1/2 - (1/2)i \cot(k\pi/5)$, $k = 1, 2, 3, 4$; they lie on $\mathrm{Re}(z) = -1/2$	10

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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