Nth roots and roots of unity

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 89 marks

Further Maths topic practice

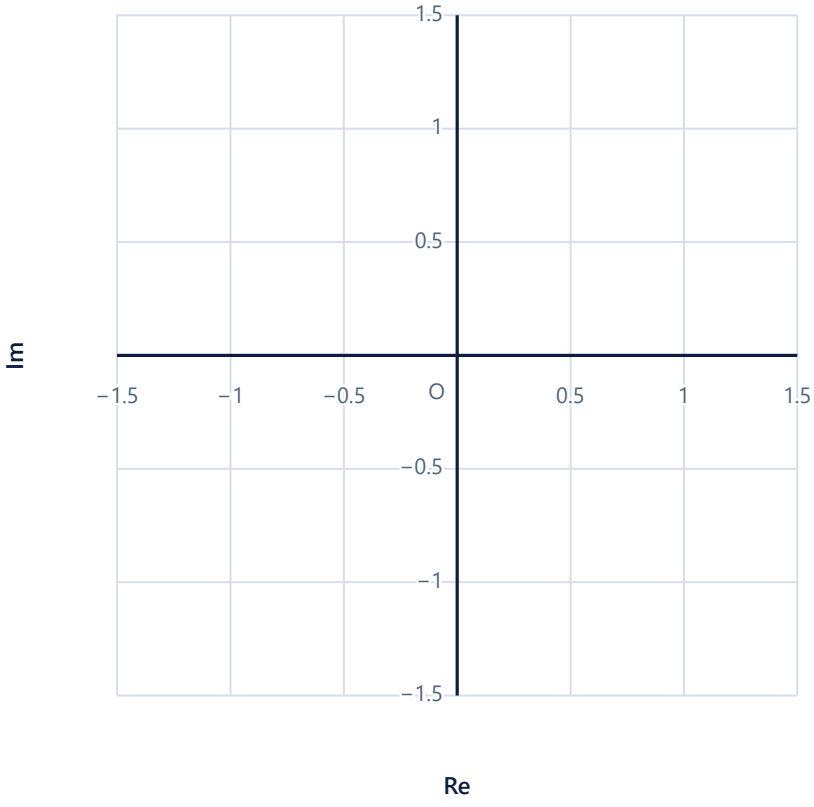
Scan for the solutions

Name: _____ **Date:** _____ **Score:** _____ **/ 89**

1 The equation $z^8 = 1$ has eight roots.

a. Solve the equation, giving each root in the form $a + bi$, where a and b are exact real numbers.

b. Show the eight roots on the Argand diagram below.



(4 marks)

Answer _____

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2 Solve the equation

$z^3 = 8i$

giving your answers in the form $a + bi$, where a and b are exact real numbers.

(4 marks)

Answer _____

3 The complex number ω is a root of the equation $z^3 = 1$, where $\omega \neq 1$

a. Show that $1 + \omega + \omega^2 = 0$

b. Show that $(2 + \omega)(2 + \omega^2) = 3$

c. Find the value of $(1 + \omega^2)^6$

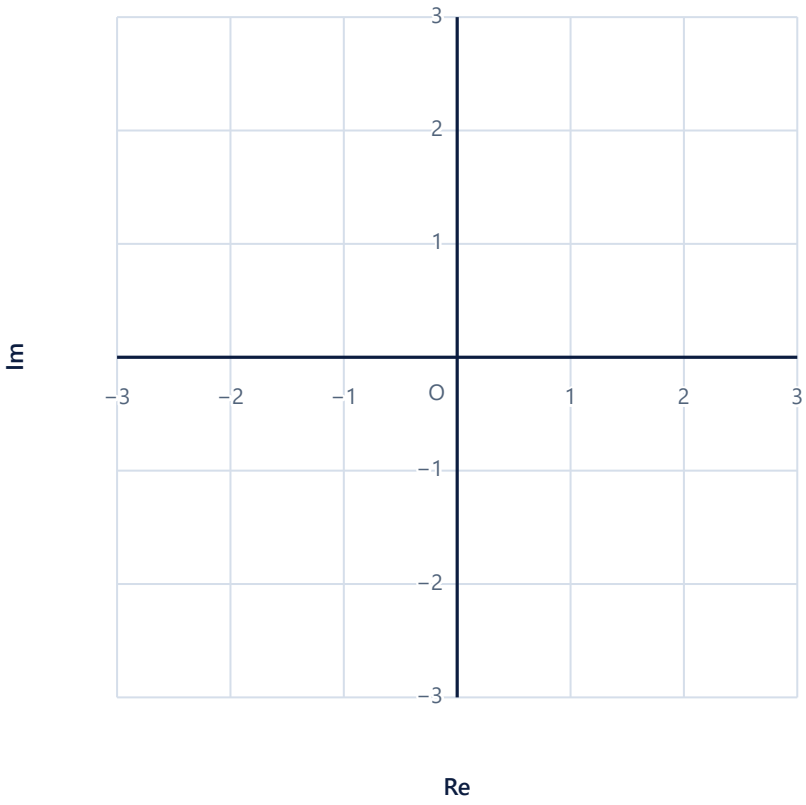
(6 marks)

Answer _____

4

a. Solve the equation $z^4 = -8 + 8\sqrt{3}i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$

b. Show the roots on the Argand diagram below and describe the shape they form.



(7 marks)

Answer

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5 a. Write down the roots of the equation $w^6 = 1$, giving your answers in the form $e^{i\theta}$, where $0 \leq \theta < 2\pi$

b. Show that $(3e^{\pi i/6})^6 = -729$

c. Hence, or otherwise, solve the equation $z^6 + 729 = 0$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$

(7 marks)

Answer

6 The complex number $w = -4\sqrt{2} + 4\sqrt{2}i$

a. Solve the equation $z^3 = w$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$

b. Write the root that lies in the first quadrant in the form $a + bi$, where a and b are exact.

The points representing the three roots form the vertices of a triangle T in an Argand diagram.

c. Find the exact area of T.

(9 marks)

Answer

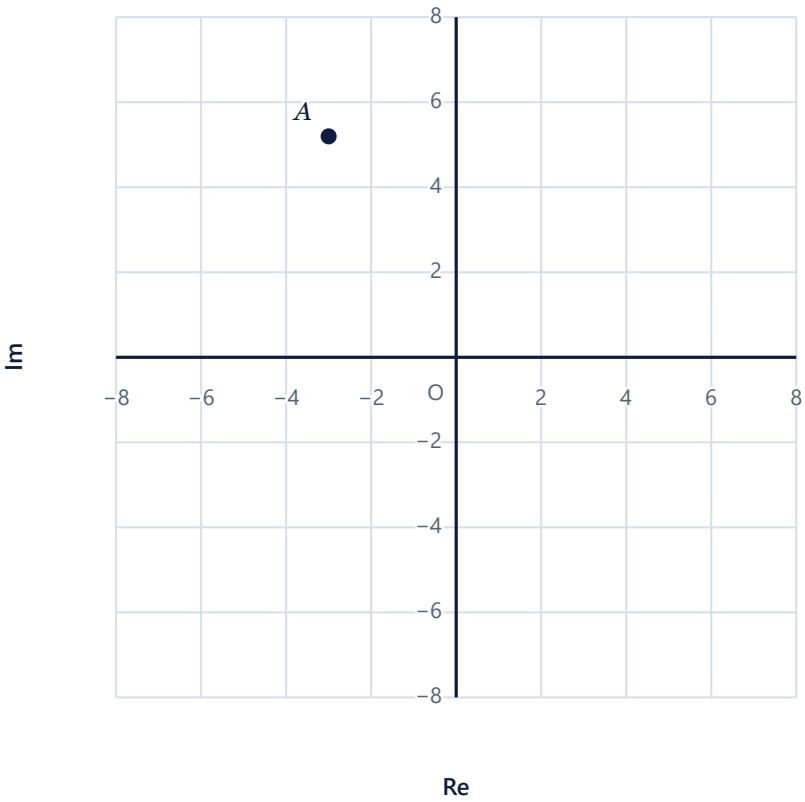
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7 The point A represents the complex number $-3 + 3\sqrt{3}i$ in the Argand diagram below.

A is one vertex of a regular pentagon whose centre is the origin O.

a. Find the complex numbers represented by the other four vertices of the pentagon, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$

b. Find the area of the pentagon, giving your answer to 3 significant figures.



(7 marks)

Answer

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8 The points A, B and C are the vertices of an equilateral triangle in an Argand diagram. The centre of the triangle is the point G, which represents $2 + i$, and A represents $2 + 5i$.

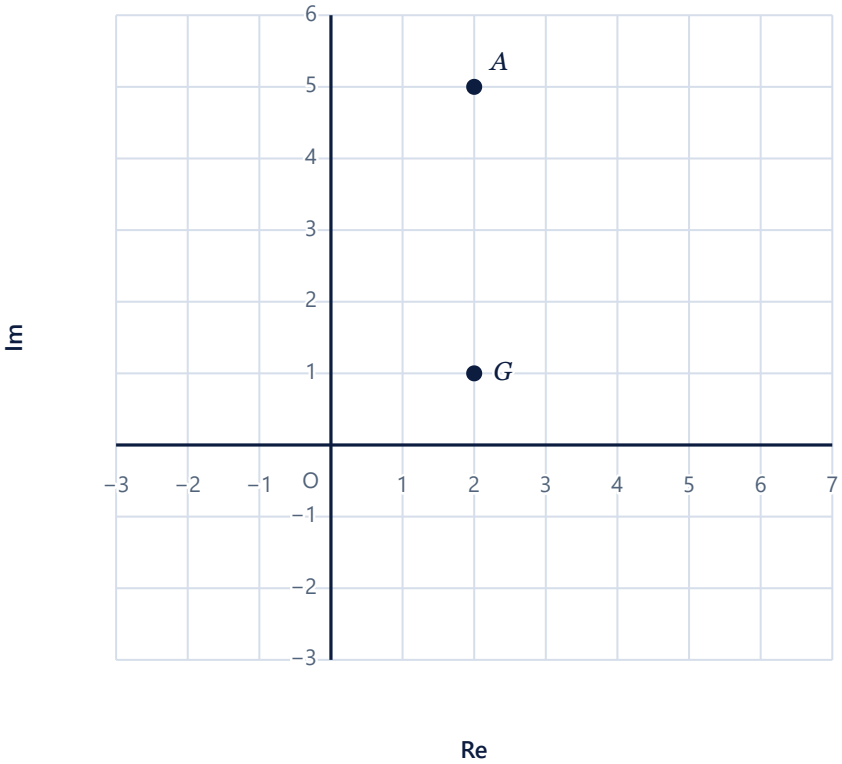
a. Write down the three roots of $z^3 = 1$ in the form $a + bi$, where a and b are exact.

The transformation $f(z) = pz + q$, where p and q are complex constants, maps the three roots of $z^3 = 1$ onto A, B and C, with the root 1 mapped to A.

b. Find p and q.

c. Hence find the complex numbers represented by B and C, giving your answers in exact form $a + bi$.

d. Find the exact area of triangle ABC.



(9 marks)

Answer

9

a. Solve the equation $z^5 = 1$, giving your answers in the form $e^{i\theta}$, where $-\pi < \theta \leq \pi$

b. Explain why the sum of the five roots is 0

c. Hence show that $\cos \frac{2\pi}{5} + \cos \frac{4\pi}{5} = -\frac{1}{2}$

d. Using the identity $\cos 2A \equiv 2\cos^2 A - 1$, show that $\cos \frac{2\pi}{5} = \frac{\sqrt{5} - 1}{4}$

(9 marks)

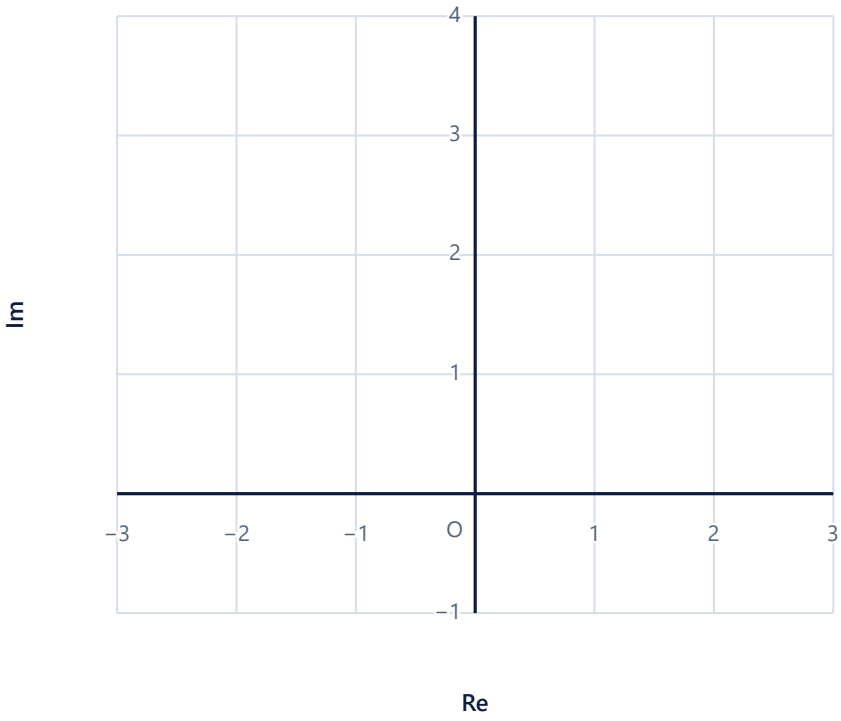
Answer

10

a. Solve the equation $w^4 = -4$, giving your answers in the form $a + bi$, where a and b are real.

b. Hence solve the equation $(z - 2i)^4 = -4$

c. Show the roots from part (b) on an Argand diagram, show that they are the vertices of a square and find the area of the square.



(8 marks)

Answer

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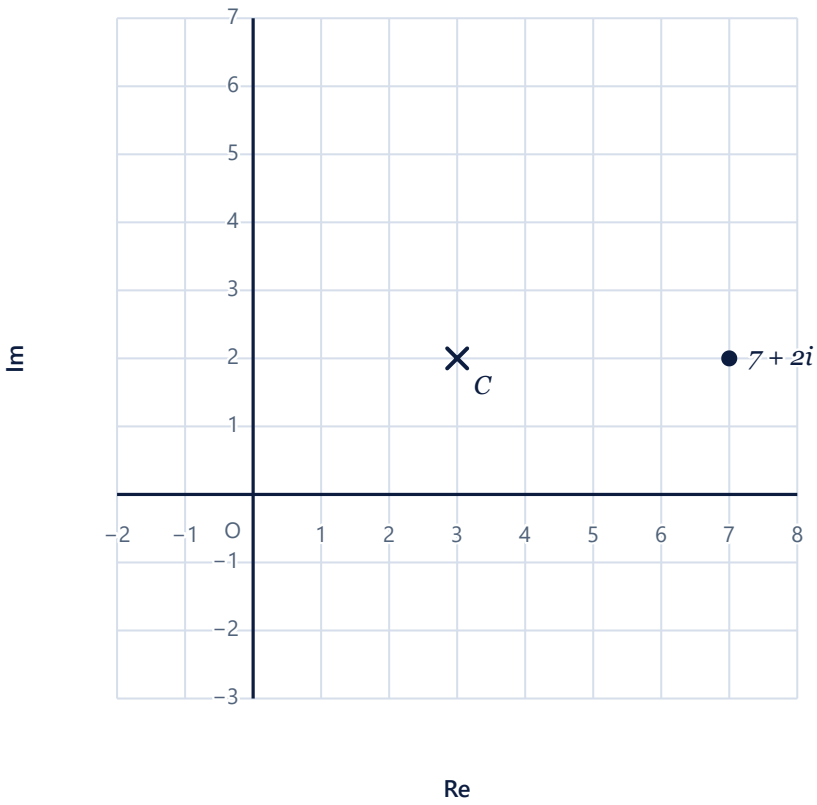
11 An engineer designs a decorative steel panel. On an Argand diagram, where 1 unit represents 10 cm, eight bolts are placed at the vertices of a regular octagon whose centre C represents the complex number $3 + 2i$. One of the bolts is at the point representing $7 + 2i$.

a. Explain why the eight bolts are at the points representing the roots of the equation $(z - 3 - 2i)^8 = 65\,536$

b. Find the complex numbers representing the other seven bolts, giving your answers in exact form $a + bi$.

A strip of steel is fixed in straight lines between adjacent bolts, all the way round the octagon.

c. Find the total length of the strip, giving your answer in cm to 3 significant figures.



(9 marks)

Answer

12 This question is about the equation $(z + 1)^5 = z^5$

a. Explain why the equation has exactly four roots.

b. Show that each root can be written as $z = \frac{1}{w - 1}$, where w is a fifth root of unity and $w \neq 1$

c. Show that, for $0 < \theta < 2\pi$, $\frac{1}{e^{i\theta} - 1} = -\frac{1}{2} - \frac{1}{2}i \cot \frac{\theta}{2}$

d. Hence write down the four roots of the equation, and state the equation of the line in the Argand diagram on which they all lie.

(10 marks)

Answer

End of questions | Total 89 marks

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