Second order homogeneous differential equations

A-level Core Pure Mathematics | Edexcel 9FM0 | 12 questions, 83 marks

Further Maths topic practice

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1 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

(3 marks)

Answer _____

2 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$$

(3 marks)

Answer _____

3 Find the general solution of each differential equation.

a. $\frac{d^2y}{dx^2} + 16y = 0$

b. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$

(4 marks)

Answer _____

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4 The differential equation

$$2\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

has a particular solution for which $y = 3$ and $\frac{dy}{dx} = 1$ when $x = 0$

a. Find this particular solution.

b. Find the exact value of y when $x = \ln 4$

(6 marks)

Answer

5 The curve C has equation $y = f(x)$, where y satisfies the differential equation

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$$

The curve passes through the point (0, 2), where the gradient of C is 0

a. Find $f(x)$.

b. Find the coordinates of the point where C crosses the x-axis.

c. Show that (0, 2) is the only stationary point of C and that it is a maximum.

(7 marks)

Answer

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6 The differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$$

has a particular solution for which $y = 1$ and $\frac{dy}{dx} = 3$ when $x = 0$

a. Find this particular solution.

b. Find the smallest positive value of x for which $y = 0$, giving your answer to 3 significant figures.

(7 marks)

Answer

7 The differential equation

$$\frac{d^2y}{dx^2} + 9y = 0$$

has a particular solution for which $y = 2$ when $x = 0$ and $y = 5$ when $x = \frac{\pi}{6}$

a. Find this particular solution.

b. Write your answer to part (a) in the form $y = R\cos(3x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving the exact value of R and the value of α to 3 significant figures.

c. Hence find the maximum value of y and the smallest positive value of x at which it occurs.

(7 marks)

Answer

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8 The differential equation

$$\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + 9y = 0$$

where k is a positive constant, is to be solved.

a. Find the value of k for which the auxiliary equation has a repeated root, and write down the general solution of the differential equation for this value of k.

b. Find the general solution of the differential equation when $k = 5$

c. Find the general solution of the differential equation when $k = 2$

d. State the set of values of k for which the general solution involves sines and cosines.

(8 marks)

Answer

9 a. The general solution of a differential equation of the form

$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$

is $y = Ae^{-½x} + Be^{3x}$

Find possible integer values of a, b and c.

b. Find a differential equation of the form $\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = 0$ whose general solution is

$y = e^{-x}(A \cos 3x + B \sin 3x)$

c. Show, by substituting directly into the differential equation, that $y = xe^{4x}$ is a solution of

$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0$

(7 marks)

Answer

10 A metal rod of length 8 cm sticks out from a hot wall. At a point x cm from the wall, the temperature of the rod is θ °C above the temperature of the surrounding air. The temperature is modelled by the differential equation

$$\frac{d^2\theta}{dx^2} = 0.25\theta, \quad 0 \le x \le 8$$

At the wall, θ = 60. The free end of the rod is insulated, so that $\frac{d\theta}{dx} = 0$ when $x = 8$

a. Find the general solution of the differential equation.

b. Show that the particular solution can be written as

$$\theta = \frac{60\cosh(4 - \frac{1}{2}x)}{\cosh 4}$$

c. Find, according to the model, the temperature of the free end of the rod above the temperature of the air.

d. Find the distance from the wall of the point on the rod where the temperature is 10 °C above the temperature of the air.

(10 marks)

Answer

11 The differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

has a particular solution for which $y = 0$ and $\frac{dy}{dx} = 4$ when $x = 0$

a. Show that this particular solution is $y = 4e^{-x} \sin x$

b. Show that the stationary points of the curve $y = 4e^{-x} \sin x$, $x \geq 0$, occur when $x = \frac{\pi}{4} + n\pi$, where n is a non-negative integer.

c. Find the exact value of y at the first maximum point, for which $x = \frac{\pi}{4}$

d. The next maximum point is at $x = \frac{9\pi}{4}$. Show that the ratio of the value of y at the first maximum to the value of y at this maximum is $e^{2\pi}$

(10 marks)

Answer

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12 A straight smooth wire of length 3 m rotates in a horizontal plane about one end, O. A small bead is threaded on the wire. At time t seconds the bead is r metres from O, where

$$\frac{d^2r}{dt^2} = 4r$$

Initially the bead is 0.5 m from O and at rest relative to the wire.

a. Show that $r = 0.5 \cosh 2t$

b. Find the time taken for the bead to reach the end of the wire.

c. Find the exact speed of the bead along the wire as it reaches the end of the wire.

In a second experiment the bead again starts 0.5 m from O, but is projected towards O with speed 1 m s^{-1} along the wire.

d. Find r in terms of t for the second experiment.

e. Explain what the model predicts for the motion of the bead in the second experiment. (11 marks)

Answer

End of questions | Total 83 marks

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