Second order non-homogeneous differential equations

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	$y = Ae^{x} + Be^{2x} + 2x + 3$	4
2	$y = A\cos 2x + B\sin 2x + 5\sin x$	4
3	$y = Ae^{x} + Be^{-3x} + 2e^{2x}$	5
4	**a)** $y = Ae^{3x} + Be^{-2x}$; λe^{3x} is part of the complementary function, so substituting it gives 0, not $10e^{3x}$ **b)** $y = Ae^{3x} + Be^{-2x} + 2xe^{3x}$	6
5	**a)** $y = (A + Bx)e^{2x} + 2x^2 + 4x + 3$ **b)** $y = (2x - 2)e^{2x} + 2x^2 + 4x + 3$	8
6	**a)** $y = 2\sin x - \cos x$ **b)** $y = e^{-x}(\cos 2x - (1/2)\sin 2x) + 2\sin x - \cos x$ **c)** The e^{-x} terms die away, so $y \approx 2\sin x - \cos x$: an oscillation of amplitude $\sqrt{5}$ and period 2π	9
7	**a)** shown **b)** $y = A\cos 4x + B\sin 4x + 3x\sin 4x$ **c)** $y = 2\cos 4x + 2\sin 4x + 3x\sin 4x$ **d)** $2 + 3\pi/8$	9
8	**a)** The CF is $(A + Bx)e^{3x}$, which contains both e^{3x} and xe^{3x}; $\lambda = 3$ **b)** $y = (1 + 2x + 3x^2)e^{3x}$ **c)** shown: $3x^2 + 2x + 1$ has discriminant -8 0	8
9	**a)** $y = Ae^{2x} + Be^{-3x} + x - 2 - 2e^{-x}$ **b)** $y = 4e^{-3x} - 2e^{-x} + x - 2$ **c)** -9	9
10	**a)** $\theta = 180 + e^{-t}(A\cos 3t + B\sin 3t)$ **b)** shown **c)** shown **d)** 236 °C **e)** 180 °C	11
11	**a)** $x = A + Be^{-4t}$; a constant is part of the CF, so substituting $x = k$ gives 0, not 8 **b)** $x = 2t + (1/2)e^{-4t} - 1/2$ **c)** 5.50 m **d)** 1.73 m s^{-1}; 2 m s^{-1}	10
12	**a)** $x = Ae^{-t} + Be^{-2t} + t + 2$ **b)** $x = e^{-t} + 2e^{-2t} + t + 2$ **c)** shown **d)** 3.64 m; $d^2x/dt^2 = e^{-t} + 8e^{-2t} > 0$ so it is a minimum **e)** $x \approx t + 2$: P moves away from O with speed approaching 1 m s^{-1}	12

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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