Simple harmonic motion and damped oscillations

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** $x = A\cos 4t + B\sin 4t$ **b)** $x = 0.3\cos 4t$ **c)** period $\pi/2$ s; maximum speed 1.2 m s^{-1}	5
2	**a)** $d^2x/dt^2 = -25x$, giving $x = 0.6\sin 5t$ (shown) **b)** 0.6 m **c)** $t = \pi/30$ s, speed $3\sqrt{3}/2$ m s^{-1} (2.60 m s^{-1})	6
3	**a)** critical damping; $x = (A + Bt)e^{-3t}$ **b)** heavy damping; $x = Ae^{-t} + Be^{-5t}$ **c)** light damping; $x = e^{-3t}(A\cos 2t + B\sin 2t)$	6
4	**a)** light damping for $0 < k < 5$; critical damping when $k = 5$ **b)** $x = (0.2 + t)e^{-5t}$ **c)** $0.2 + t > 0$ and $e^{-5t} > 0$ for $t \geq 0$, so $x > 0$ and P never reaches O	7
5	**a)** the auxiliary equation has two distinct real (negative) roots, -1 and -4 **b)** $x = 20e^{-t} - 20e^{-4t}$ **c)** 9.45 mm (at $t = (1/3)\ln 4$) **d)** $x = 20e^{-4t}(e^{3t} - 1) > 0$ for $t > 0$	8
6	**a)** $h = -e^{-t}(12\cos 3t + 2\sin 3t)$ **b)** 1.72 cm below the equilibrium position ($h = -1.72$) **c)** $t = (\pi - \arctan 6)/3 = 0.579$ s **d)** it keeps oscillating with period $2\pi/3$ s but the amplitude decays like e^{-t}, so the axle settles at its equilibrium position	9
7	**a)** $x = e^{-t}(A\cos 2t + B\sin 2t)$ **b)** $x = 2\cos t + \sin t$ **c)** $x = e^{-t}(A\cos 2t + B\sin 2t) + 2\cos t + \sin t$ **d)** the e^{-t} terms die away, leaving $x \approx 2\cos t + \sin t$: oscillations of period 2π s and amplitude $\sqrt{5}$ cm, whatever the initial conditions	9
8	**a)** $k = 29$ **b)** $x = -e^{-2t}(6\cos 5t + 2.4\sin 5t)$ **c)** $x = 0.141$ cm (above equilibrium); velocity 14.2 cm s^{-1} upwards **d)** $e^{-2t} \to 0$, so the oscillations die away and the weight comes to rest at its equilibrium position	10
9	**a)** $\lambda = -3/4$ **b)** $x = A\cos 2t + B\sin 2t - (3/4)t\cos 2t$ **c)** $x = (3/8)\sin 2t - (3/4)t\cos 2t$ **d)** $x = -2.72$ cm **e)** the $t\cos 2t$ term makes the amplitude grow without limit, which is unrealistic: in practice damping would limit the sway (or the bridge would fail), so the model is not suitable for large t	11
10	**a)** $2\,d^2x/dt^2 = -50x - \lambda\,dx/dt$ (shown) **b)** $\lambda = 20$ **c)** $x = (0.5 - 1.5t)e^{-5t}$ **d)** $x = 0$ at $t = 1/3$; greatest distance $0.3e^{-8/3} = 0.0208$ m	12

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	e) the auxiliary equation then has two distinct real negative roots, so the motion is heavily damped: P returns towards O more slowly, with no oscillation	
11	**a)** $x = 0.5e^{-0.2t} \cos 2t$ (shown) **b)** stationary points are $\pi/2$ apart and alternate, so maxima are π s apart; ratio $e^{-0.2\pi} = 0.533$ **c)** $T = 5 \ln 50 = 19.6$ s **d)** e.g. it ignores further waves from wind or other boats, which would keep the buoy moving	11
12	**a)** $x = e^{-2t}(A \cos 4t + B \sin 4t) + 2$ **b)** $x = 2 - e^{-2t}(2 \cos 4t + \sin 4t)$ **c)** $dx/dt = 10e^{-2t} \sin 4t$ (shown) **d)** greatest distance $2 + 2e^{-\pi/2}$ cm (2.42 cm); settles 2 cm below the unloaded position **e)** $c = 4\sqrt{5}$; the platform then settles at 2 cm without overshooting or oscillating, so the reading is steady sooner	12

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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