Volumes of revolution (Year 2)

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	$27\pi/8$	4
2	$V = \pi\int(1 + \sin 2x)\,dx$ from 0 to $\pi/2 = \pi(\pi/2 + 1) = \pi/2(\pi + 2)$	4
3	$3\pi/5$	4
4	$\pi/2(e^4 - 1)$	5
5	**a)** $\operatorname{artanh}(1/2) = \frac{1}{2}\ln((1 + \frac{1}{2})/(1 - \frac{1}{2})) = \frac{1}{2}\ln 3$, so $x = 1/2$ **b)** $\pi/2(\ln 3 - 1)$	8
6	**a)** $dx/dt = 2 + 2\cos 2t = 2 + 2(2\cos^2 t - 1) = 4\cos^2 t$ **b)** $3\pi^2/4$	7
7	**a)** $\sec^2 t = 1 + \tan^2 t = 1 + x^2$, so $y = \ln\sec t = \frac{1}{2}\ln\sec^2 t = \frac{1}{2}\ln(1 + x^2)$ **b)** $V = \pi\int \tan^3 t\,dt$ from 0 to $\pi/4 = \pi/2(1 - \ln 2)$	7
8	**a)** $(\pi/3, \sqrt{3})$ **b)** $\pi^2 - (3/2)\pi\sqrt{3}$	7
9	**a)** $e^x + e^{-x} = 10/3$ gives $3e^{2x} - 10e^x + 3 = 0$, so $e^x = 3$ or $1/3$ and $x = \pm\ln 3$ **b)** $V = \pi\int(25/9 - \cosh^2 x)\,dx$ from $-\ln 3$ to $\ln 3 = \pi/9(41\ln 3 - 20)$	8
10	**a)** $(z - 1/z)^4 = (2i\sin\theta)^4 = 16\sin^4\theta$ and $= 2\cos 4\theta - 8\cos 2\theta + 6$, so $8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$ **b)** $k = 1.5$ **c)** $27\pi^2/32 = 8.33\ \text{cm}^3$ **d)** Overestimates: the hole means some of the modelled solid is not wood	11
11	**a)** $V = \pi\int e^y\,dy$ from 0 to $h = \pi(e^h - 1)$ **b)** 10.9 litres **c)** $0.0715\ \text{dm s}^{-1}$ **d)** 1.01 dm	10
12	**a)** $a\sin(\pi/6) = 4$ gives $a = 8$; height $b\sqrt{3} = 15$ gives $b = 5\sqrt{3}$ **b)** $V = \pi\int x^2\,(dy/dt)\,dt = 320\sqrt{3}\pi\int \sin^3 t\,dt = 720\pi$ **c)** 57 seconds **d)** For example: the glass has a thickness so the inside is smaller; the real side may not follow this curve exactly	10

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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