Critical path analysis

Worked solutions

Further Maths topic practice

This topic online

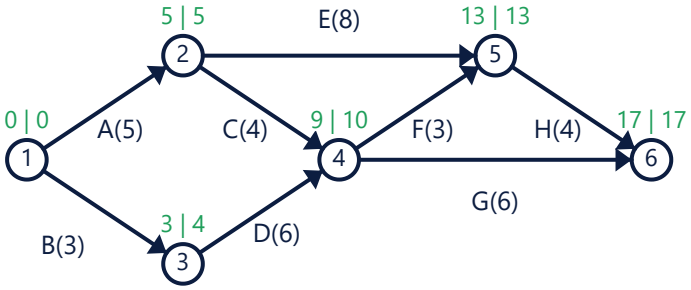
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1 The activity network below shows the activities, A to H, needed to complete a project. Each activity is represented by an arc and its duration, in days, is shown in brackets. The events are numbered 1 to 6.

a. Calculate the early event time and the late event time for each event.

b. State the minimum completion time of the project.

c. State the critical activities.



Event times: early | late

Diagram NOT accurately drawn

(5 marks)

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 5 = 5

Early 3 = 0 + 3 = 3

Early 4 = max(5 + 4, 3 + 6) = 9

Early 5 = max(5 + 8, 9 + 3) = 13

Early 6 = max(9 + 6, 13 + 4) = 17

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 6 = early 6 = 17

Late 5 = 17 - 4 = 13

Late 4 = min(13 - 3, 17 - 6) = 10

Late 3 = 10 - 6 = 4

Late 2 = min(10 - 4, 13 - 8) = 5

Late 1 = min(5 - 5, 4 - 3) = 0

1: (0, 0); 2: (5, 5); 3: (3, 4); 4: (9, 10); 5: (13, 13); 6: (17, 17)

b)

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17 days

c)

Total float = late time at end - duration - early time at start

Float A = 5 - 5 - 0 = 0

Float B = 4 - 3 - 0 = 1

Float C = 10 - 4 - 5 = 1

Float D = 10 - 6 - 3 = 1

Float E = 13 - 8 - 5 = 0

Float F = 13 - 3 - 9 = 1

Float G = 17 - 6 - 9 = 2

Float H = 17 - 4 - 13 = 0

Critical activities have total float 0

A, E, H

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2 The precedence table shows the activities needed to refurbish a classroom.

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A
E	B, C
F	D
G	E, F

Draw the activity network, using activity on arc. Number the events.

Draw your activity network in this space

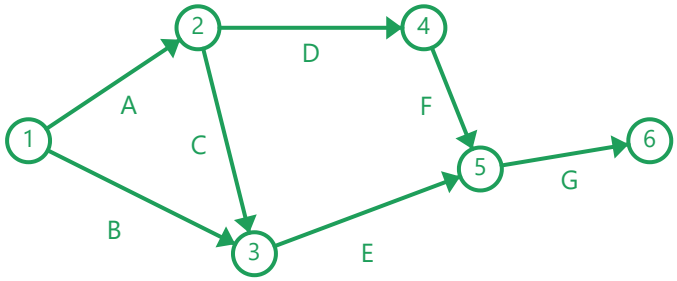


Diagram NOT accurately drawn

(4 marks)

Start with activities that have no predecessors

A and B leave event 1

A ends at event 2: C and D start there

E needs B and C: end B and C at the same event

B and C end at event 3, E starts at 3

D ends at event 4, F starts at 4

G needs E and F: end them at the same event

E and F end at event 5, G from 5 to 6

Check each activity against the table

Network drawn; no dummies needed

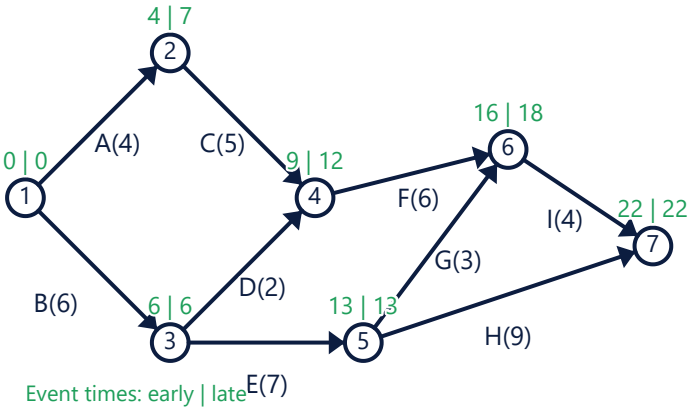
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3 The activity network below represents a project to build a garden office in Cheshire. The duration of each activity is given in days.

a. Find the early and late event times.

b. State the critical path and the minimum completion time.

c. Find the total float of each non-critical activity.



Event times: early | late

Diagram NOT accurately drawn

(6 marks)

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 4 = 4

Early 3 = 0 + 6 = 6

Early 4 = max(4 + 5, 6 + 2) = 9

Early 5 = 6 + 7 = 13

Early 6 = max(9 + 6, 13 + 3) = 16

Early 7 = max(13 + 9, 16 + 4) = 22

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 7 = early 7 = 22

Late 6 = 22 - 4 = 18

Late 5 = min(18 - 3, 22 - 9) = 13

Late 4 = 18 - 6 = 12

Late 3 = min(12 - 2, 13 - 7) = 6

Late 2 = 12 - 5 = 7

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Late 1 = min(7 - 4, 6 - 6) = 0

1: (0, 0); 2: (4, 7); 3: (6, 6); 4: (9, 12); 5: (13, 13); 6: (16, 18); 7: (22, 22)

b)

Critical events have early time = late time

Follow the arcs with zero float from 1 to 7

BEH; 22 days

c)

Total float = late time at end - duration - early time at start

Float A = 7 - 4 - 0 = 3

Float C = 12 - 5 - 4 = 3

Float D = 12 - 2 - 6 = 4

Float F = 18 - 6 - 9 = 3

Float G = 18 - 3 - 13 = 2

Float I = 22 - 4 - 16 = 2

A 3, C 3, D 4, F 3, G 2, I 2

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

4 The precedence table shows the activities in a project to launch a new school website.

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A, B
E	C
F	D
G	E, F

a. Draw the activity network, using activity on arc and the minimum number of dummies.

b. Explain why a dummy is needed.

Draw your activity network in this space

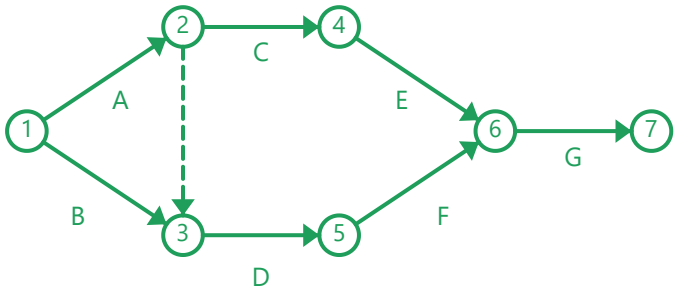


Diagram NOT accurately drawn

(5 marks)

a)

A and B leave event 1

C needs only A: C starts at event 2 (end of A)

D needs A and B: D starts at event 3 (end of B)

Dummy from event 2 to event 3 carries A to D

E after C, F after D

E and F end at event 6, G from 6 to 7

Network with one dummy, 2 to 3

b)

C depends on A only, D depends on A and B

Without the dummy, either C would need B or D would not need A

Dummy shows D depends on A as well as B

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

5 The activity network below represents a project. The dashed arcs are dummies.

a. Write down the immediately preceding activities for each of the activities A to H.

b. Explain why the dummy from event 5 to event 6 is needed.

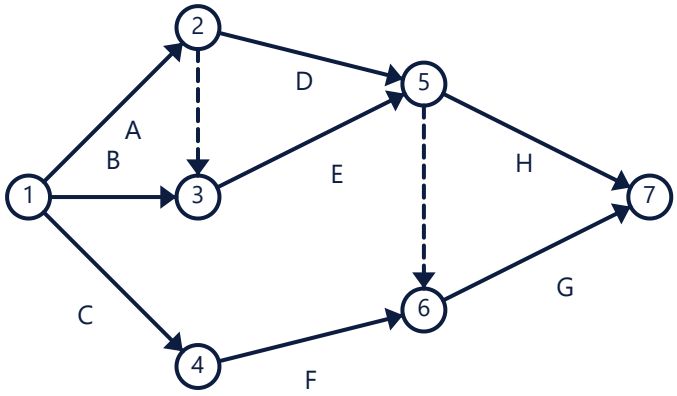


Diagram NOT accurately drawn

(4 marks)

a)

Look at the event where each activity starts; follow dummies back

A, B, C start at event 1: no predecessors

D starts at event 2: A ends there

E starts at event 3: B ends there, and A arrives by the dummy 2 to 3

F starts at event 4: C ends there

H starts at event 5: D and E end there

G starts at event 6: F ends there, D and E arrive by the dummy 5 to 6

A: none; B: none; C: none; D: A; E: A, B; F: C; G: D, E, F; H: D, E

b)

G needs D, E and F; H needs only D and E

Dummy keeps H independent of F

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6 The activity network below represents the activities needed to stage a sixth-form drama production in Exeter. Each duration is in days and the dashed arc is a dummy.

a. Find the early and late event times.

b. State the critical activities and the minimum completion time.

c. Find the total float of each non-critical activity.

d. Explain what the total float of an activity means in this context.

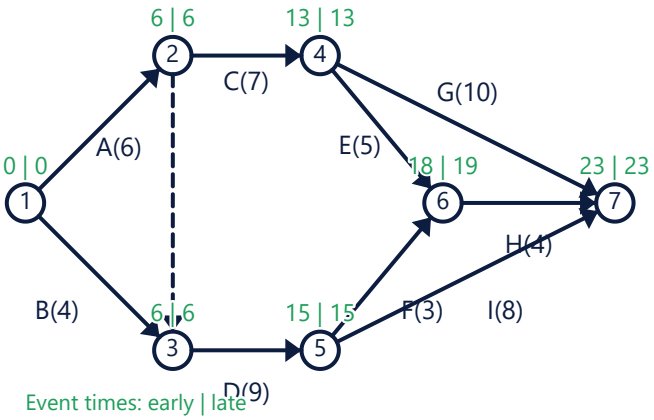


Diagram NOT accurately drawn

(7 marks)

a)

The dummy has duration 0

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 6 = 6

Early 3 = max(0 + 4, 6 + 0) = 6

Early 4 = 6 + 7 = 13

Early 5 = 6 + 9 = 15

Early 6 = max(13 + 5, 15 + 3) = 18

Early 7 = max(13 + 10, 18 + 4, 15 + 8) = 23

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 7 = early 7 = 23

Late 6 = 23 - 4 = 19

Late 5 = min(19 - 3, 23 - 8) = 15

Late 4 = min(19 - 5, 23 - 10) = 13

Late 3 = 15 - 9 = 6

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

Late 2 = min(6 − 0, 13 − 7) = 6

Late 1 = min(6 − 6, 6 − 4) = 0

1: (0, 0); 2: (6, 6); 3: (6, 6); 4: (13, 13); 5: (15, 15); 6: (18, 19); 7: (23, 23)

b)

Zero float, from event 1 to event 7

A, C, D, G, I; 23 days

c)

Total float = late time at end − duration − early time at start

Float B = 6 − 4 − 0 = 2

Float E = 19 − 5 − 13 = 1

Float F = 19 − 3 − 15 = 1

Float H = 23 − 4 − 18 = 1

B 2, E 1, F 1, H 1

d)

Days an activity can be delayed without delaying the project

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

7 The activity network below shows the tasks, with durations in hours, involved in setting up a village fete in Rutland. Each task needs one worker.

a. Find the minimum completion time for the project.

b. Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

c. Explain why the project may need more workers than your lower bound.

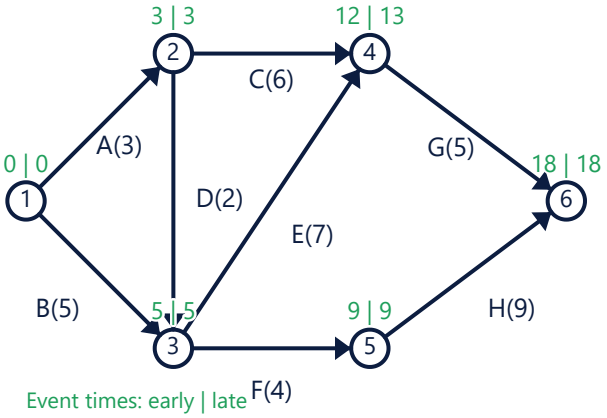


Diagram NOT accurately drawn

(6 marks)

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 3 = 3

Early 3 = max(0 + 5, 3 + 2) = 5

Early 4 = max(3 + 6, 5 + 7) = 12

Early 5 = 5 + 4 = 9

Early 6 = max(12 + 5, 9 + 9) = 18

18 hours

b)

Lower bound = total of all durations ÷ critical time, rounded up

Total = 3 + 5 + 6 + 2 + 7 + 4 + 5 + 9 = 41

$\frac{41}{18} = 2.278...$

Round up: a fraction of a worker is not possible

3 workers

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c)

The bound assumes every worker is busy for the whole time

Tasks must wait for others, so some workers may be idle

More may be needed because of idle time

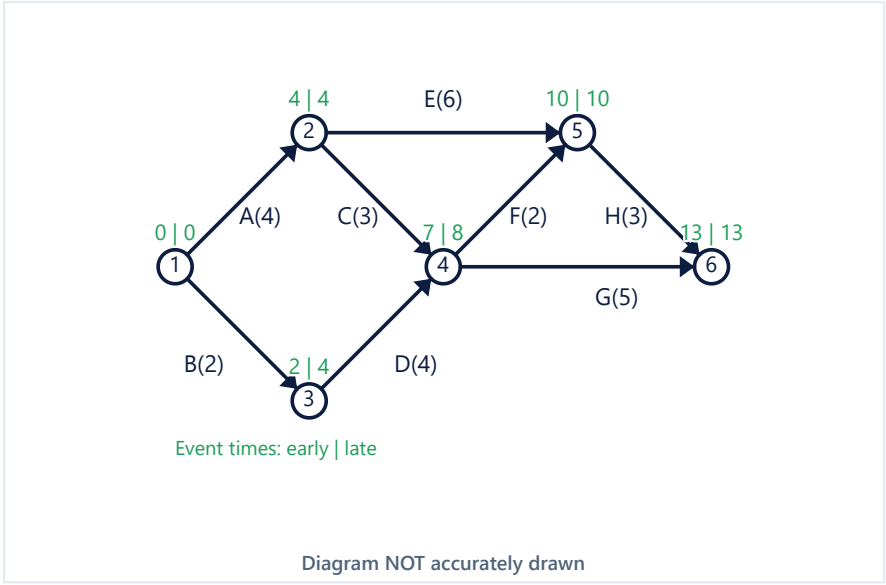
Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

8 The activity network below represents a kitchen refit in Nottingham. Durations are in days.

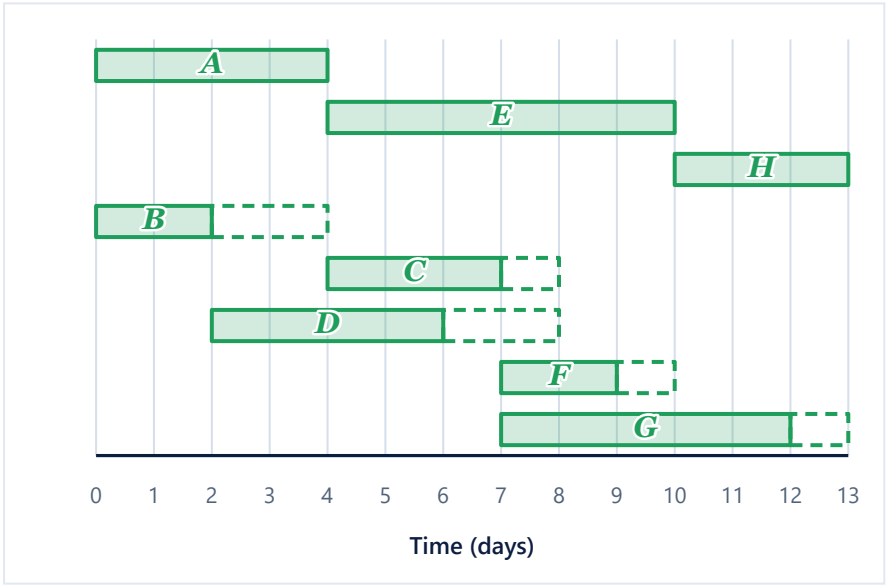
a. Find the early and late event times and state the critical path.

b. Find the total float of each non-critical activity.

c. On the grid below, draw a Gantt (cascade) chart for the project, showing the critical activities on the top rows and each float as a dashed box.



Activity network



Grid for the Gantt chart

(7 marks)

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 4 = 4

Early 3 = 0 + 2 = 2

Early 4 = max(4 + 3, 2 + 4) = 7

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Early 5 = max(4 + 6, 7 + 2) = 10

Early 6 = max(7 + 5, 10 + 3) = 13

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 6 = early 6 = 13

Late 5 = 13 - 3 = 10

Late 4 = min(10 - 2, 13 - 5) = 8

Late 3 = 8 - 4 = 4

Late 2 = min(8 - 3, 10 - 6) = 4

Late 1 = min(4 - 4, 4 - 2) = 0

1: (0, 0); 2: (4, 4); 3: (2, 4); 4: (7, 8); 5: (10, 10); 6: (13, 13);
AEH

b)

Total float = late time at end - duration - early time at start

Float B = 4 - 2 - 0 = 2

Float C = 8 - 3 - 4 = 1

Float D = 8 - 4 - 2 = 2

Float F = 10 - 2 - 7 = 1

Float G = 13 - 5 - 7 = 1

B 2, C 1, D 2, F 1, G 1

c)

Each bar starts at the early time of its start event

A: 0 to 4, critical

E: 4 to 10, critical

H: 10 to 13, critical

B: 0 to 2, float to 4

C: 4 to 7, float to 8

D: 2 to 6, float to 8

F: 7 to 9, float to 10

G: 7 to 12, float to 13

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Gantt chart drawn (second diagram)

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

9 The table shows the activities, with durations in weeks, for a project to convert a barn in Shropshire into holiday cottages.

Activity	Immediately preceding activities	Duration (weeks)
A	–	5
B	–	3
C	A	4
D	A, B	6
E	B	9
F	C	7
G	D, E	2

a. Draw the activity network, using activity on arc and the minimum number of dummies.

b. Explain why two dummies are needed.

c. Find the minimum completion time and state the critical activities.

Draw your activity network in this space

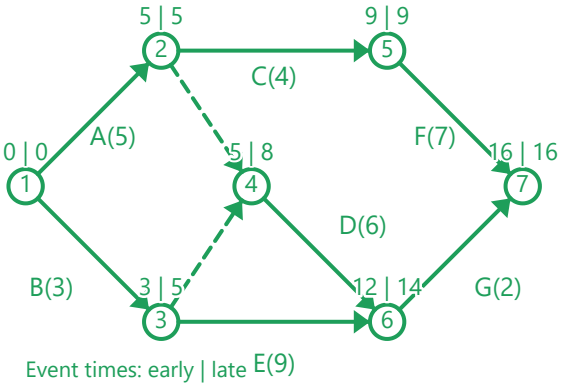


Diagram NOT accurately drawn

(8 marks)

a)

A and B leave event 1

A ends at event 2, B ends at event 3

C (after A only) starts at 2

E (after B only) starts at 3

D needs A and B: new event 4, dummies 2 to 4 and 3 to 4

D and E both end at event 6, G starts at 6

F starts at the end of C

F and G end at event 7

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Network with two dummies

b)

One dummy would make C depend on B or E depend on A

Two dummies needed

c)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 5 = 5

Early 3 = 0 + 3 = 3

Early 4 = max(5 + 0, 3 + 0) = 5

Early 5 = 5 + 4 = 9

Early 6 = max(5 + 6, 3 + 9) = 12

Early 7 = max(9 + 7, 12 + 2) = 16

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 7 = early 7 = 16

Late 6 = 16 - 2 = 14

Late 5 = 16 - 7 = 9

Late 4 = 14 - 6 = 8

Late 3 = min(8 - 0, 14 - 9) = 5

Late 2 = min(8 - 0, 9 - 4) = 5

Late 1 = min(5 - 5, 5 - 3) = 0

Total float = late time at end - duration - early time at start

Float A = 5 - 5 - 0 = 0

Float B = 5 - 3 - 0 = 2

Float C = 9 - 4 - 5 = 0

Float D = 14 - 6 - 5 = 3

Float E = 14 - 9 - 3 = 2

Float F = 16 - 7 - 9 = 0

Float G = 16 - 2 - 12 = 2

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

16 weeks; A, C, F

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

10 The activity network below represents a project to renovate a café in Brighton. The durations are in days, and activity F takes x days.

a. Find, in terms of x where necessary, the early event times.

b. Find the range of values of x for which activity F is critical.

c. Given that $x = 2$, find the minimum completion time and the total float of F.

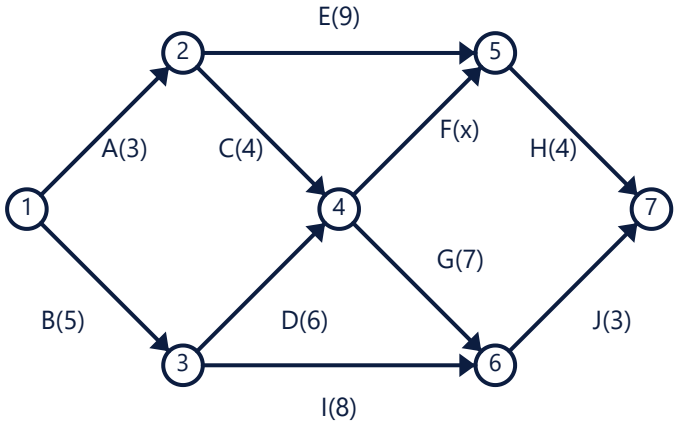


Diagram NOT accurately drawn

(7 marks)

a)

Early 1 = 0

Early 2 = 0 + 3 = 3

Early 3 = 0 + 5 = 5

Early 4 = max(3 + 4, 5 + 6) = 11

Early 5 = max(3 + 9, 11 + x) = max(12, 11 + x)

Early 6 = max(11 + 7, 5 + 8) = 18

Early 7 = max(early 5 + 4, 18 + 3)

1: 0, 2: 3, 3: 5, 4: 11, 5: max(12, 11 + x), 6: 18, 7: max(16, 15 + x, 21)

b)

Longest path not using F: B, D, G, J

5 + 6 + 7 + 3 = 21

Path through F: B, D, F, H

5 + 6 + x + 4 = 15 + x

F is critical when this path is a longest path

15 + x ⩾ 21

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$x \geqslant 6$

Check: then $11 + x \geqslant 17 > 12$, so F also beats E into event 5

$\boxed{x \geqslant 6}$

c)

Early 5 = $\max(12, 13) = 13$

Early 7 = $\max(13 + 4, 21) = 21$

Late 5 = $21 - 4 = 17$

Float F = $17 - 2 - 11 = 4$

$\boxed{\text{21 days; float of F} = 4}$

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11 The activity network below shows a project to install solar panels on a primary school in Leicester. Durations are in days.

a. Find the early and late event times and the minimum completion time.

b. Find the total floats of activities B, G and I.

Explain the effect on the completion time of the project of each of the following separate delays:

c. activity I takes 5 days longer than planned,

d. activity G takes 3 days longer than planned,

e. activity B takes 2 days longer than planned.

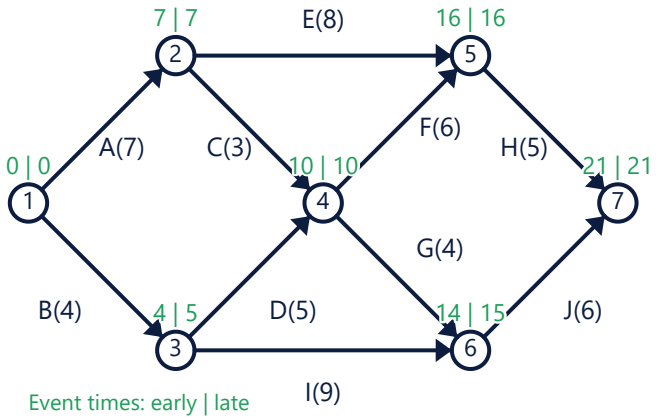


Diagram NOT accurately drawn

(8 marks)

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 7 = 7

Early 3 = 0 + 4 = 4

Early 4 = max(7 + 3, 4 + 5) = 10

Early 5 = max(7 + 8, 10 + 6) = 16

Early 6 = max(10 + 4, 4 + 9) = 14

Early 7 = max(16 + 5, 14 + 6) = 21

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 7 = early 7 = 21

Late 6 = 21 - 6 = 15

Late 5 = 21 - 5 = 16

Late 4 = min(16 - 6, 15 - 4) = 10

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

Late 3 = min(10 − 5, 15 − 9) = 5

Late 2 = min(10 − 3, 16 − 8) = 7

Late 1 = min(7 − 7, 5 − 4) = 0

1: (0, 0); 2: (7, 7); 3: (4, 5); 4: (10, 10); 5: (16, 16); 6: (14, 15); 7: (21, 21); 21 days

b)

Total float = late time at end − duration − early time at start

Float B = 5 − 4 − 0 = 1

Float G = 15 − 4 − 10 = 1

Float I = 15 − 9 − 4 = 2

B 1, G 1, I 2

c)

Delay beyond the float is added to the project

Extra = 5 − 2 = 3

24 days

d)

Extra = 3 − 1 = 2

23 days

e)

Extra = 2 − 1 = 1

22 days

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

12 The table shows the activities, with durations in days, needed to set up a new community library in Sheffield.

Activity	Immediately preceding activities	Duration (days)
A	–	4
B	–	6
C	–	3
D	A	5
E	A, B	4
F	C	7
G	D	6
H	D, E	3
I	F	5
J	G, H, I	2

a. Draw the activity network, using activity on arc and the minimum number of dummies.

b. Find the early and late event times and state the critical activities.

c. Find the total float of each non-critical activity.

d. Each activity needs one worker. Calculate a lower bound for the number of workers needed to finish the project in the minimum time.

Draw your activity network in this space

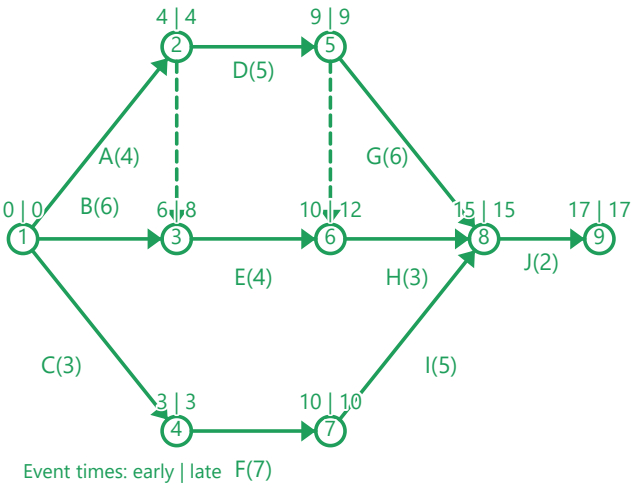


Diagram NOT accurately drawn

(10 marks)

a)

A, B, C leave event 1

D after A only: starts at 2 (end of A)

E after A and B: dummy 2 to 3, E starts at 3

G after D only: starts at 5 (end of D)

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

H after D and E: dummy 5 to 6, H starts at 6

F after C; I after F

G, H and I end at event 8

J from event 8 to event 9

Network with two dummies

b)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 4 = 4

Early 3 = max(0 + 6, 4 + 0) = 6

Early 4 = 0 + 3 = 3

Early 5 = 4 + 5 = 9

Early 6 = max(6 + 4, 9 + 0) = 10

Early 7 = 3 + 7 = 10

Early 8 = max(9 + 6, 10 + 3, 10 + 5) = 15

Early 9 = 15 + 2 = 17

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 9 = early 9 = 17

Late 8 = 17 - 2 = 15

Late 7 = 15 - 5 = 10

Late 6 = 15 - 3 = 12

Late 5 = min(12 - 0, 15 - 6) = 9

Late 4 = 10 - 7 = 3

Late 3 = 12 - 4 = 8

Late 2 = min(8 - 0, 9 - 5) = 4

Late 1 = min(4 - 4, 8 - 6, 3 - 3) = 0

1: (0, 0); 2: (4, 4); 3: (6, 8); 4: (3, 3); 5: (9, 9); 6: (10, 12); 7: (10, 10); 8: (15, 15); 9: (17, 17); critical A, C, D, F, G, I, J

c)

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

Total float = late time at end - duration - early time at start

Float B = 8 - 6 - 0 = 2

Float E = 12 - 4 - 6 = 2

Float H = 15 - 3 - 10 = 2

B 2, E 2, H 2

d)

Total = 4 + 6 + 3 + 5 + 4 + 7 + 6 + 3 + 5 + 2 = 45

$$\frac{45}{17} = 2.647...$$

3 workers

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.