Minimum spanning trees

AS Decision Mathematics 1 | Edexcel 8FM0 | 12 questions, 71 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 71**

1 The network below has six vertices, A to F. The number on each arc is its weight.

a. Use Kruskal's algorithm to find a minimum spanning tree for the network. List the arcs in the order that you consider them and state whether each arc is added to the tree.

b. State the weight of the minimum spanning tree.

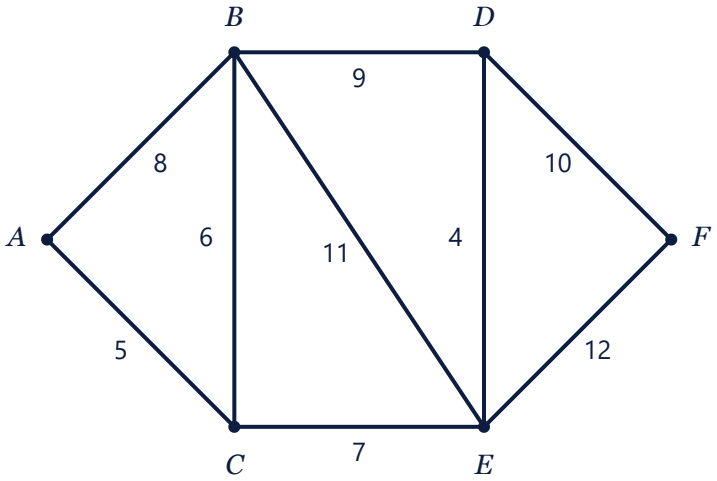


Diagram NOT accurately drawn

(4 marks)

Answer _____

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2 The network below shows the lengths, in metres, of paths between seven huts, A to G, at a scout camp in the Lake District.

a. Use Prim's algorithm, starting at G, to find a minimum spanning tree for the network. State the order in which you select the arcs.

b. State the total length of the minimum spanning tree.

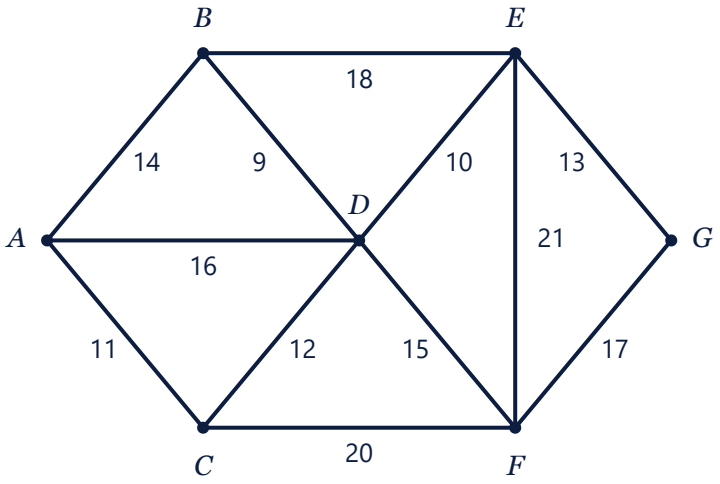


Diagram NOT accurately drawn

(4 marks)

Answer

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3 The table shows the distances, in miles, between six villages in Shropshire, A to F.

	A	B	C	D	E	F
A	–	23	31	18	27	35
B	23	–	16	29	21	30
C	31	16	–	25	14	22
D	18	29	25	–	26	33
E	27	21	14	26	–	19
F	35	30	22	33	19	–

a. Use Prim's algorithm, starting at A, to find a minimum spanning tree for the six villages. You must state the order in which you select the arcs.

b. State the weight of the minimum spanning tree.

(4 marks)

Answer _____

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4 The table shows the lengths, in km, of the roads between seven towns, A to G, in Lincolnshire. A dash means there is no direct road.

	A	B	C	D	E	F	G
A	–	32	–	28	25	–	–
B	32	–	30	–	22	–	–
C	–	30	–	–	27	34	–
D	28	–	–	–	24	–	36
E	25	22	27	24	–	29	31
F	–	–	34	–	29	–	26
G	–	–	–	36	31	26	–

a. The seven towns are shown in the diagram. Draw the network of roads, labelling each arc with its length.

b. Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order that you consider them and state whether each is added.

c. State the total length of your minimum spanning tree.

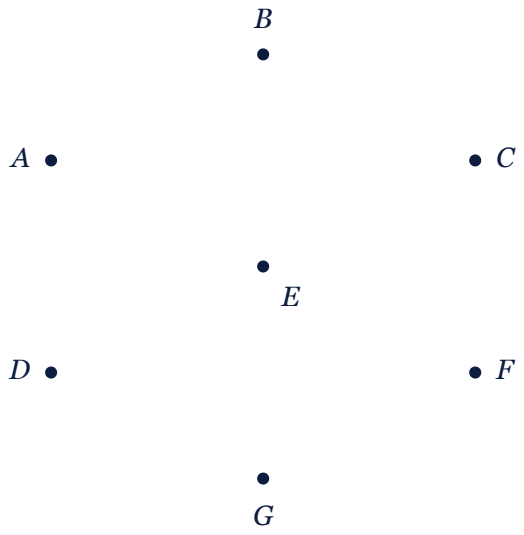


Diagram NOT accurately drawn

(6 marks)

Answer

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5 The table shows the times, in minutes, to drive between seven depots, A to G, of a delivery firm in the West Midlands.

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	–	42	57	36	61	48	70
B	42	–	33	51	45	39	64
C	57	33	–	47	29	55	41
D	36	51	47	–	53	31	66
E	61	45	29	53	–	50	38
F	48	39	55	31	50	–	58
G	70	64	41	66	38	58	–

a. Use Prim's algorithm, starting at D, to find a minimum spanning tree. State the order in which you add the vertices to the tree.

b. Find the weight of the tree.

c. Kruskal's algorithm is also applied to the same table. Explain whether Kruskal's algorithm could give a different minimum spanning tree.

(6 marks)

Answer

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6 The network below shows the costs, in thousands of pounds, of laying water pipes between six farms, A to F, in Devon.

a. Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order that you consider them and state whether each is added.

b. State the minimum cost.

c. Find the number of different minimum spanning trees for this network. Give a reason for your answer.

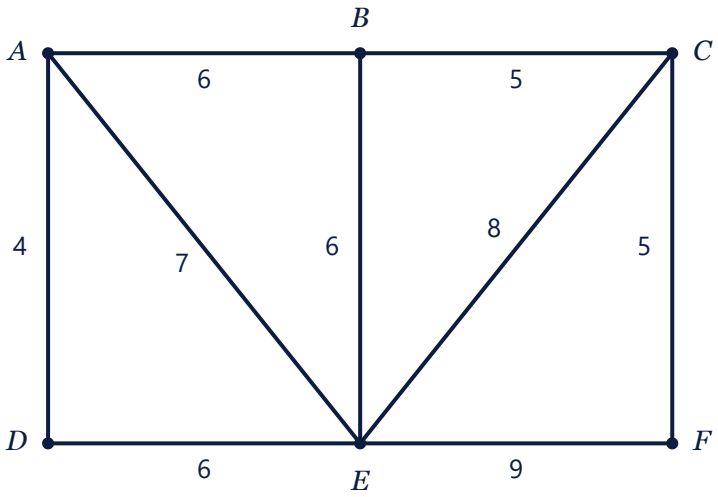


Diagram NOT accurately drawn

(5 marks)

Answer

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7 The network below shows the lengths, in metres, of cable needed to link eight buildings, A to H, on a university campus in Leeds.

a. Use Prim's algorithm, starting at C, to find a minimum spanning tree. State the order in which you select the arcs.

b. Kruskal's algorithm is applied to the same network. State the order in which the arcs are added to the tree.

c. State the total length of cable needed.

d. Explain why Kruskal's algorithm needs a check for cycles at every step but Prim's algorithm does not.

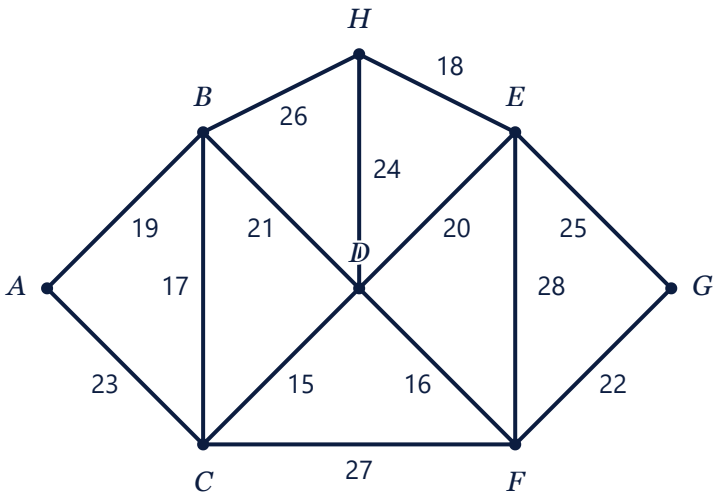


Diagram NOT accurately drawn

(7 marks)

Answer

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8 The network below has six vertices. The weight of arc BD is x, where $x > 0$

a. Given that $x = 15$, use Kruskal's algorithm to find a minimum spanning tree and state its weight.

b. Find the range of values of x for which arc BD is in every minimum spanning tree of the network.

c. For these values, write down the weight of the minimum spanning tree in terms of x

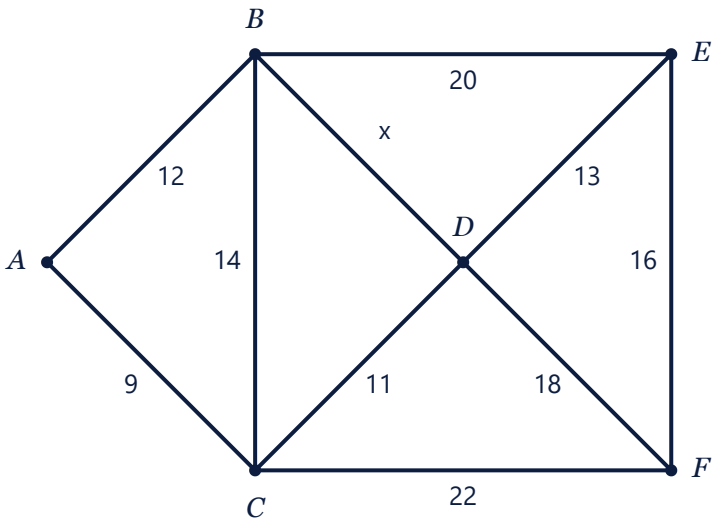


Diagram NOT accurately drawn

(6 marks)

Answer

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9 A cycle club in Cornwall wants to resurface tracks so that seven checkpoints, P to V, are all linked. The network shows the lengths, in km, of the possible tracks.

a. Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order that you consider them and state whether each is added.

b. State the total length of track in the tree.

The landowner insists that the track QS must be resurfaced.

c. Find the minimum total length of track that links all seven checkpoints and includes QS, stating the arcs used.

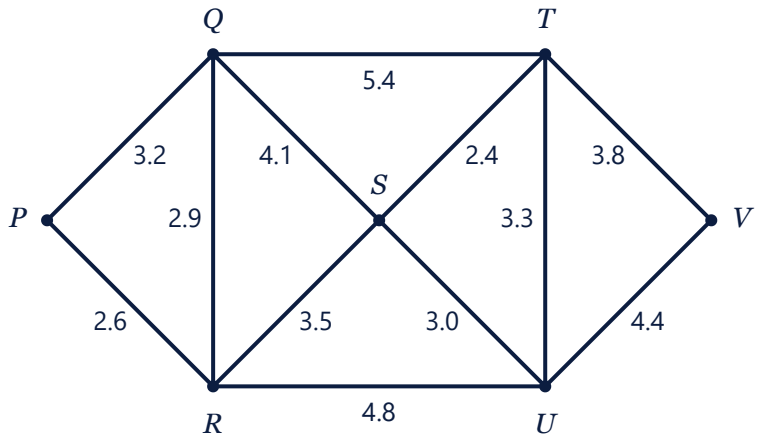


Diagram NOT accurately drawn

(7 marks)

Answer _____

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10 A broadband company plans to link six villages in Leicestershire with fibre cable laid along roads: Ashby (A), Brill (B), Cotes (C), Dunton (D), Elm (E) and Foxley (F). The table shows the road distances, in km, between the villages.

	A	**B**	**C**	**D**	**E**	**F**
A	–	6.4	8.1	5.2	9.7	7.3
B	6.4	–	4.6	7.8	5.9	8.8
C	8.1	4.6	–	6.9	3.8	9.1
D	5.2	7.8	6.9	–	8.4	4.9
E	9.7	5.9	3.8	8.4	–	7.6
F	7.3	8.8	9.1	4.9	7.6	–

a. Use Prim's algorithm, starting at Ashby, to find the minimum length of cable needed to link all six villages. You must state the order in which you select the arcs.

b. Laying the cable costs £18 500 per km. Find the minimum cost of the network.

c. Give one practical reason why the company might not use the network found in part (a). (7 marks)

Answer

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11 The table shows the distances, in metres, between seven classroom blocks, A to G, in a large secondary school in Bristol. The school wants to link every block with a covered walkway of minimum total length.

	A	B	C	D	E	F	G
A	–	18	25	31	22	37	29
B	18	–	14	27	30	33	24
C	25	14	–	16	28	26	35
D	31	27	16	–	19	21	34
E	22	30	28	19	–	32	17
F	37	33	26	21	32	–	23
G	29	24	35	34	17	23	–

a. Use Prim's algorithm, starting at A, to find a minimum spanning tree. State the order in which you select the arcs.

b. State the length of the minimum spanning tree.

A new sports hall, H, is built. Its distances, in metres, to the other blocks are

	A	B	C	D	E	F	G
H	20	26	15	12	27	13	30

c. Find a minimum spanning tree for all eight buildings, explaining your method, and state its length.

(8 marks)

Answer _____

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12 The table shows the distances, in km, between six petrol stations, A to F, on a motorway network. The distance from C to D is k km.

	A	**B**	**C**	**D**	**E**	**F**
A	–	30	41	26	38	45
B	30	–	24	33	29	40
C	41	24	–	k	35	27
D	26	33	k	–	36	39
E	38	29	35	36	–	31
F	45	40	27	39	31	–

Prim's algorithm, starting at A, is applied to the table. The vertices are added to the tree in the order A, D, C, B, F, E.

a. Find the range of possible values of k

b. Write down, in terms of k, the weight of the minimum spanning tree.

c. Given instead that $k = 34$, find the order in which the vertices are added and the weight of the minimum spanning tree.

(7 marks)

Answer _____

End of questions | Total 71 marks

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