Route inspection

Worked solutions

Further Maths topic practice

This topic online

1 The network below shows the lengths, in km, of the roads in a small housing estate in Swindon. The total length of the roads is 57 km.

A council officer must travel along every road at least once, starting and finishing at A. Use an appropriate algorithm to find the length of the shortest possible route, stating the roads that must be travelled twice.

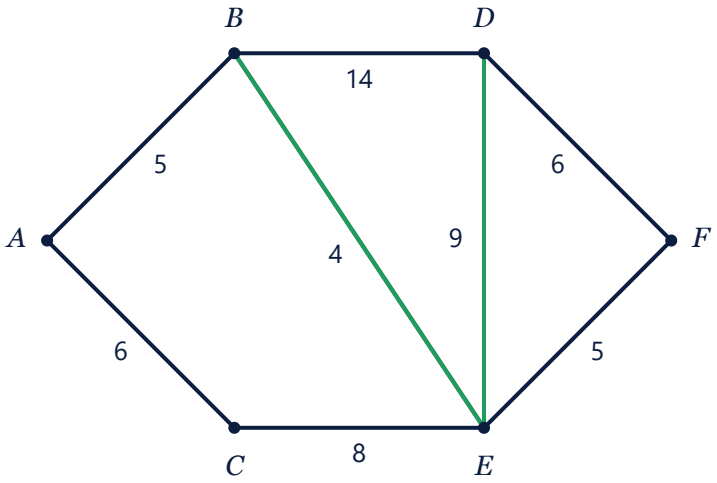


Diagram NOT accurately drawn

(4 marks)

Route inspection (Chinese postman)

Find the degree of each vertex

A 2, B 3, C 2, D 3, E 4, F 2

Odd vertices: B, D

Shortest distance between each pair of odd vertices

BD: BED = 4 + 9 = 13

Repeat BE and DE

Length = 57 + 13 = 70

70 km, repeating BE and DE

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2 The network below shows the lengths, in hundreds of metres, of the paths in a park in Glasgow. The total length of the paths is 76 hundred metres.

A park keeper must walk along every path at least once to check for damage, starting and finishing at A.

a. Use the route inspection algorithm to find which paths must be walked twice. You must make your method clear, considering every possible pairing of the odd vertices.

b. Find the length of the shortest possible route.

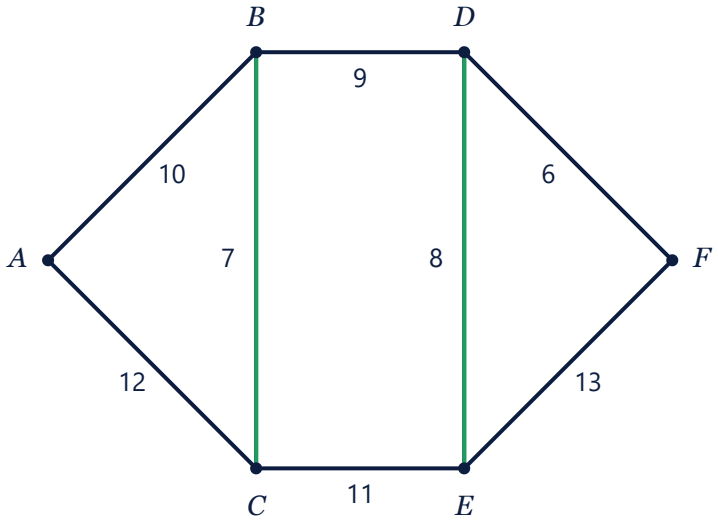


Diagram NOT accurately drawn

(5 marks)

a)

Find the degree of each vertex

A 2, B 3, C 3, D 3, E 3, F 2

Odd vertices: B, C, D, E

Shortest distance between each pair of odd vertices

BC = 7 (direct)

BD = 9 (direct)

BE: BDE = 9 + 8 = 17

CD: CBD = 7 + 9 = 16

CE = 11 (direct)

DE = 8 (direct)

Pair the odd vertices: three possible pairings

BC + DE = 7 + 8 = 15

BD + CE = 9 + 11 = 20

BE + CD = 17 + 16 = 33

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Least pairing: BC + DE = 15

Repeat BC and DE

b)

Length = 76 + 15 = 91

91 hundred metres

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3 The network below shows the lengths, in km, of the cycle lanes in a town in Suffolk. The total length is 75 km.

a. State, with a reason, whether the network is Eulerian, semi-Eulerian or neither.

b. Ella wants to cycle along every lane exactly once. State where she must start and finish, and the length of her route.

c. Instead, Ella must start and finish at P. Find the length of the shortest route that covers every lane at least once.

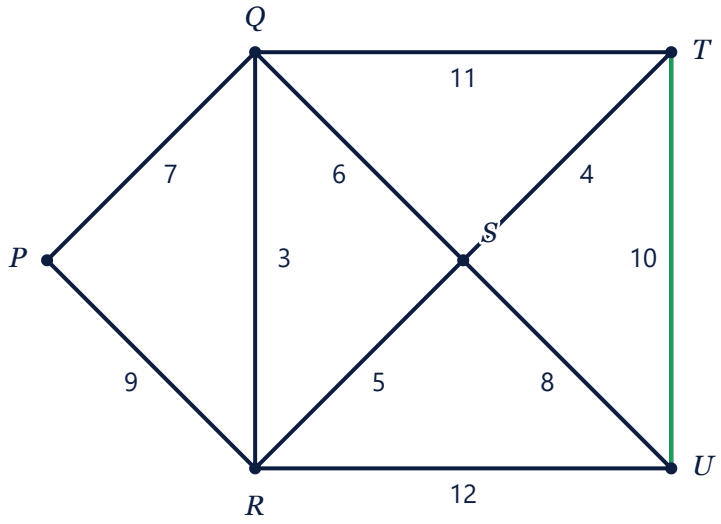


Diagram NOT accurately drawn

(5 marks)

a)

Find the degree of each vertex

P 2, Q 4, R 4, S 4, T 3, U 3

Odd vertices: T, U

Exactly two odd vertices

Semi-Eulerian

b)

A route using every arc once must start and finish at the odd vertices

T to U, 75 km

c)

Now T and U must be made even: repeat the shortest path from T to U

TU = 10 (direct)

Length = 75 + 10 = 85

85 km

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4 The network below shows the lengths, in metres, of the corridors in a hospital wing. A cleaner must travel along every corridor at least once, starting and finishing at A.

a. Find the total length of the corridors.

b. Use the route inspection algorithm to find the corridors that will be travelled twice.

c. Find the length of the cleaner's shortest route.

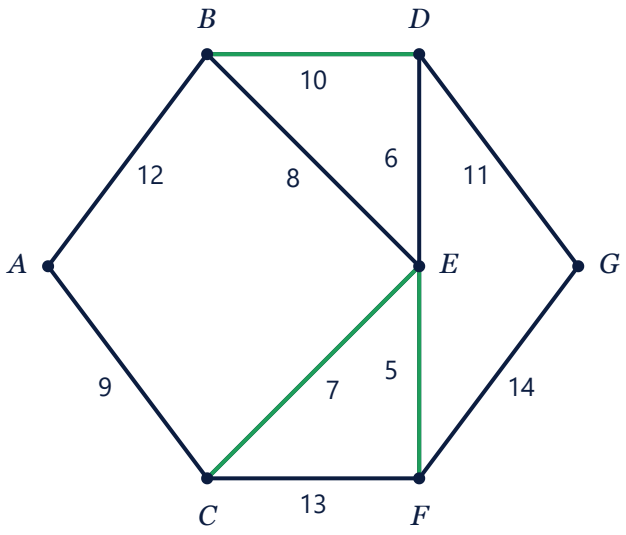


Diagram NOT accurately drawn

(6 marks)

a)

Total = 12 + 9 + 10 + 8 + 7 + 13 + 6 + 11 + 5 + 14

Total = 95

95 m

b)

Find the degree of each vertex

A 2, B 3, C 3, D 3, E 4, F 3, G 2

Odd vertices: B, C, D, F

Shortest distance between each pair of odd vertices

BC: BEC = 8 + 7 = 15

BD = 10 (direct)

BF: BEF = 8 + 5 = 13

CD: CED = 7 + 6 = 13

CF: CEF = 7 + 5 = 12

DF: DEF = 6 + 5 = 11

Pair the odd vertices: three possible pairings

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$BC + OF = 15 + 11 = 26$

$BO + CF = 10 + 12 = 22$

$BF + CO = 13 + 13 = 26$

Least pairing: $BO + CF = 22$

BO uses BO; CF uses CEF

Repeat BO, CE, EF

c)

Length $= 95 + 22 = 117$

117 m

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5 A road-sweeping lorry in Coventry must travel along every road in the network below at least once. The number on each arc is the length of the road in hundreds of metres, and the total length of the roads is 162 hundred metres.

a. Find the length of the shortest route that starts and finishes at the depot, A.

The lorry may instead start and finish at any two vertices.

b. State where the lorry should start and finish so that its route is as short as possible, and find the length of this route.

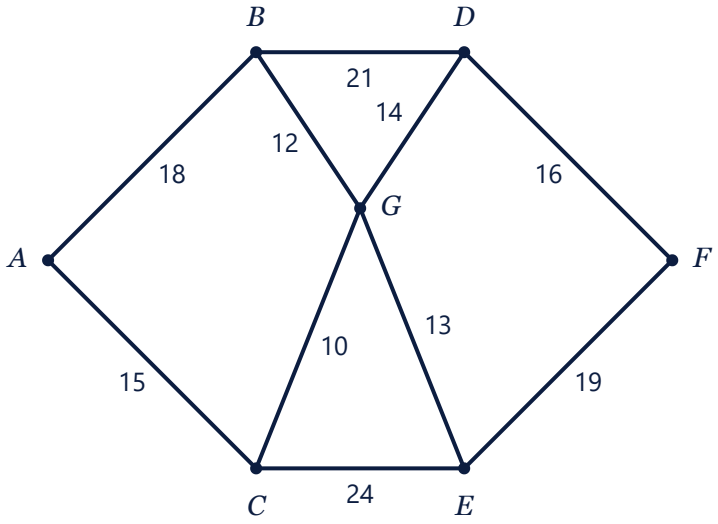


Diagram NOT accurately drawn

(7 marks)

a)

Find the degree of each vertex

A 2, B 3, C 3, D 3, E 3, F 2, G 4

Odd vertices: B, C, D, E

Shortest distance between each pair of odd vertices

BC: BGC = 12 + 10 = 22

BD = 21 (direct)

BE: BGE = 12 + 13 = 25

CD: CGD = 10 + 14 = 24

CE: CGE = 10 + 13 = 23

DE: DGE = 14 + 13 = 27

Pair the odd vertices: three possible pairings

BC + DE = 22 + 27 = 49

BD + CE = 21 + 23 = 44

BE + CD = 25 + 24 = 49

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Length = 162 + 44 = 206

206 hundred metres

b)

Start and finish at two odd vertices; only the other two need a repeated path

Choose the pair with the shortest distance to repeat

Repeat BD (21): start and finish at C and E

Repeat BC (22): start and finish at D and E

Repeat CE (23): start and finish at B and D

Repeat CD (24): start and finish at B and E

Repeat BE (25): start and finish at C and D

Repeat DE (27): start and finish at B and C

Shortest is BD = 21

Length = 162 + 21 = 183

C and E; 183 hundred metres

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6 The network below shows the lengths, in metres, of the pipes in a water treatment works. The total length is 3180 m. An engineer must inspect every pipe, starting and finishing at A.

The table shows the shortest distances, in metres, between some of the vertices.

	B	C	D	G
B	–	150	410	810
C	150	–	550	700
D	410	550	–	400
G	810	700	400	–

a. Explain why the vertices in the table were chosen.

b. Use the route inspection algorithm to find the length of the shortest inspection route. State the pipes that are inspected twice.

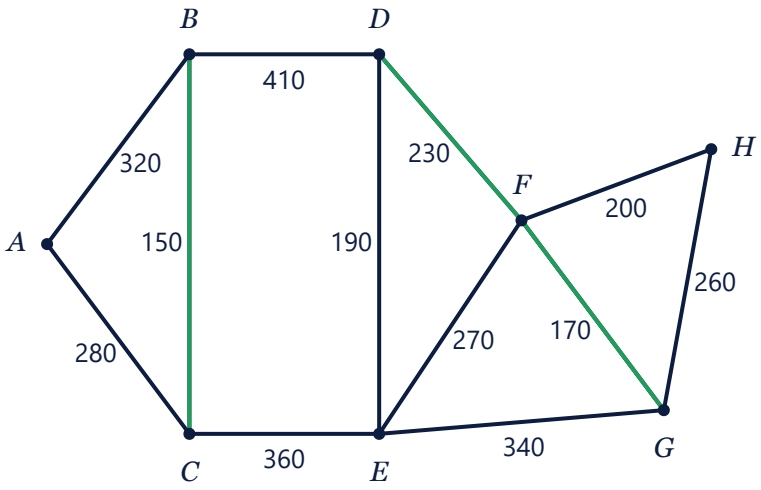


Diagram NOT accurately drawn

(6 marks)

a)

Find the degree of each vertex

A 2, B 3, C 3, D 3, E 4, F 4, G 3, H 2

Odd vertices: B, C, D, G

They are the odd vertices

b)

Use the table for each pairing

Pair the odd vertices: three possible pairings

BC + DG = 150 + 400 = 550

BD + CG = 410 + 700 = 1110

BG + CD = 810 + 550 = 1360

Least: BC + DG = 550

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BC uses BC; OG uses OFG

Length = 3180 + 550 = 3730

3730 m; repeat BC, OF, FG

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7 The network below shows the lengths, in km, of the footpaths that a ranger in the New Forest must check, starting and finishing at A. The total length is 89 km.

a. Explain why the ranger can check every footpath without walking any path twice, and state the length of her route.

b. State how many times her route passes through D.

Footpath CD is closed and no longer needs checking, and it cannot be used.

c. Find the length of the shortest route that checks every remaining footpath, starting and finishing at A.

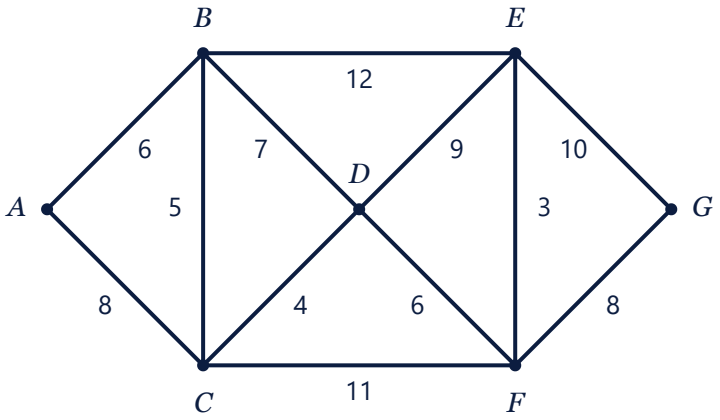


Diagram NOT accurately drawn

(7 marks)

a)

Find the degree of each vertex

A 2, B 4, C 4, D 4, E 4, F 4, G 2

Every vertex has even degree

So the network is Eulerian

No path twice; 89 km

b)

D has degree 4

Each pass through D uses one arc in and one arc out

Passes = 4 ÷ 2 = 2

2 times

c)

New total = 89 - 4 = 85

C and D now have odd degree (3)

Shortest C to D without CD: CBD = 5 + 7 = 12

CFD = 11 + 6 = 17 is longer

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Length = 85 + 12 = 97

97 km

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8 The network below shows the lengths, in km, of the streets that a postal worker in Belfast must walk along, starting and finishing at the sorting office, A.

a. Use the route inspection algorithm to find the length of the shortest route.

b. State a possible route.

c. State the number of times your route visits D, not counting the start or finish.

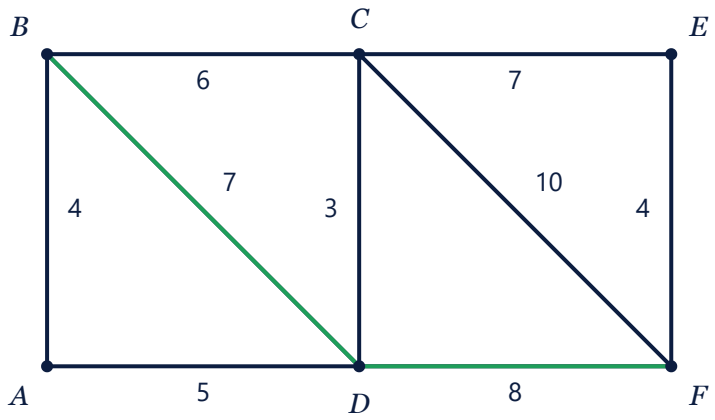


Diagram NOT accurately drawn

(7 marks)

a)

Find the degree of each vertex

A 2, B 3, C 4, D 4, E 2, F 3

Odd vertices: B, F

Shortest distance between each pair of odd vertices

BF: BDF = 7 + 8 = 15

Total = 4 + 6 + 3 + 5 + 7 + 4 + 8 + 7 + 10 = 54

Length = 54 + 15 = 69

69 km

b)

Double BD and DF: every vertex now even

Check: 11 arcs used = 9 + 2

ABCDBDFCEFDA

c)

D has degree 4, plus 2 for the repeated BD and DF

Visits = 6 ÷ 2 = 3

3 times

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9 The network below shows the lengths, in metres, of the aisles in a supermarket warehouse in Reading. The total length is 194 m. A security guard must patrol every aisle at least once, starting and finishing at A.

a. Find the length of the shortest patrol route, stating the aisles that are patrolled twice.

The manager plans to open a new aisle of length 35 m directly between C and D. The new aisle must also be patrolled.

b. Find the length of the shortest patrol route with the new aisle, and state whether the new aisle makes the patrol shorter or longer, and by how much.

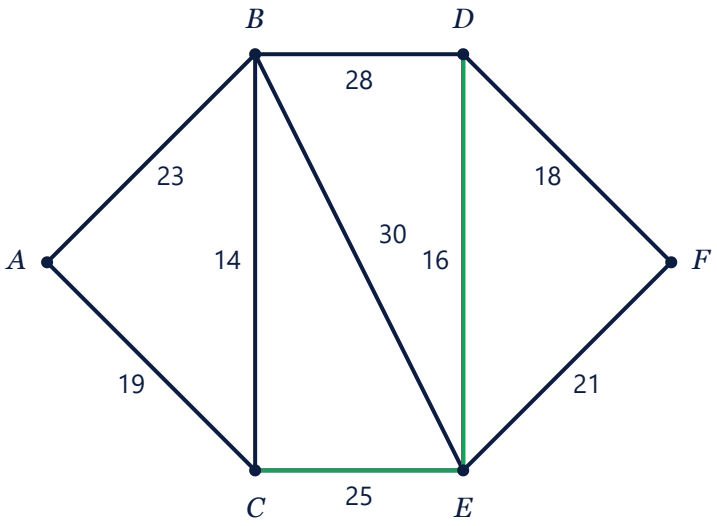


Diagram NOT accurately drawn

(7 marks)

a)

Find the degree of each vertex

A 2, B 4, C 3, D 3, E 4, F 2

Odd vertices: C, D

Shortest distance between each pair of odd vertices

CD: CED = 25 + 16 = 41

CBD = 14 + 28 = 42 is longer

Length = 194 + 41 = 235

235 m; repeat CE and DE

b)

Adding CD raises the degrees of C and D by 1

C and D become even, so every vertex is even

No aisle needs repeating

Length = 194 + 35 = 229

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Saving = 235 - 229 = 6

229 m: shorter by 6 m

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10 The network below shows the lengths, in km, of the roads between six villages in Herefordshire. The length of road BE is x km, where $8 < x < 19$

A gritting lorry must travel along every road at least once, starting and finishing at A. The shortest possible route has length 144 km.

a. Write down, in terms of x, the total length of the roads.

b. Find the value of x. You must consider every pairing of the odd vertices.

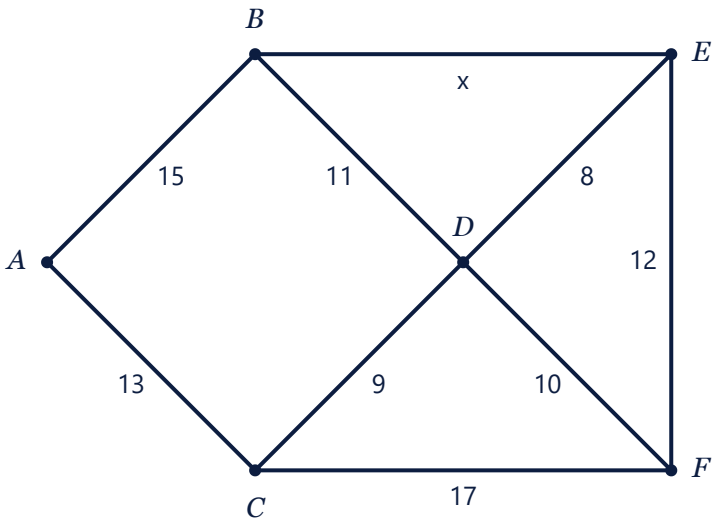


Diagram NOT accurately drawn

(8 marks)

a)

Total = 15 + 13 + 11 + 9 + 8 + 10 + 17 + 12 + x

Total = 95 + x

$\boxed{95 + x}$

b)

Find the degree of each vertex

A 2, B 3, C 3, D 4, E 3, F 3

Odd vertices: B, C, E, F

Shortest distances between the odd vertices

BC: BDC = 11 + 9 = 20

BE = x (direct, since x < 19 = BDE)

BF: BDF = 11 + 10 = 21

CE: CDE = 9 + 8 = 17

CF = 17 (direct)

EF = 12 (direct)

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Pairings

$BC + EF = 20 + 12 = 32$

$BE + CF = x + 17$

$BF + CE = 21 + 17 = 38$

Case 1: $BE + CF$ is least

$144 = 95 + x + x + 17$

$2x = 32$

$x = 16$

Then $BE + CF = 33 > 32$: not least, so reject

Case 2: $BC + EF = 32$ is least

$144 = 95 + x + 32$

$x = 17$

Check: $BE + CF = 34 > 32$ and $8 < 17 < 19$

$\boxed{x = 17}$

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11 The network below shows the lengths, in km, of the paths in a university campus in Durham that must be gritted on an icy morning. The total length is 9.9 km. The gritter starts and finishes at its store, A, and must travel along every path at least once.

a. Use the route inspection algorithm to find the length of the shortest route. State the paths that are travelled twice.

b. The gritter travels at an average speed of 20 km/h. Find the time taken for the route, to the nearest minute.

The store moves, so that the gritter may now start and finish at any vertices.

c. State the start and finish that give the shortest route, and the length of that route.

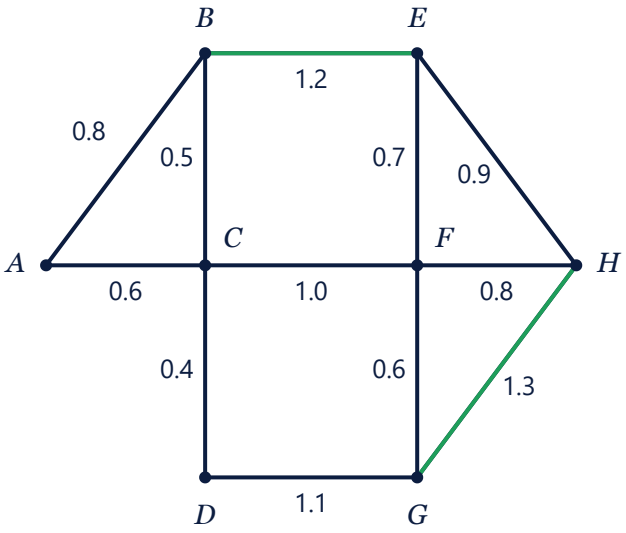


Diagram NOT accurately drawn

(9 marks)

a)

Find the degree of each vertex

A 2, B 3, C 4, D 2, E 3, F 4, G 3, H 3

Odd vertices: B, E, G, H

Shortest distance between each pair of odd vertices

BE = 1.2 (direct)

BG: BCDG = 0.5 + 0.4 + 1.1 = 2

BH: BEH = 1.2 + 0.9 = 2.1

EG: EFG = 0.7 + 0.6 = 1.3

EH = 0.9 (direct)

GH = 1.3 (direct)

Pair the odd vertices: three possible pairings

BE + GH = 1.2 + 1.3 = 2.5

BG + EH = 2 + 0.9 = 2.9

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$BH + EG = 2.1 + 1.3 = 3.4$

$\text{Length} = 9.9 + 2.5 = 12.4$

12.4 km; repeat BE and GH

b)

$$\text{Time} = \frac{12.4}{20} \text{ hours}$$

$\text{Time} = 0.62$ hours

$\text{Time} = 0.62 \times 60 = 37.2$ minutes

37 minutes

c)

Repeat the shortest single pair; start and finish at the other two odd vertices

Repeat EH (0.9): start and finish at B and G

Repeat BE (1.2): start and finish at G and H

Repeat EG (1.3): start and finish at B and H

$\text{Length} = 9.9 + 0.9 = 10.8$

B and G; 10.8 km

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12 The network below shows the lengths, in miles, of the roads that a highways inspector in Somerset must drive along. The total length of the roads is 135 miles.

a. Find the length of the shortest route that covers every road and starts and finishes at D.

The inspector now lives at B. She will start at B but may finish at any vertex.

b. Find where she should finish to make her route as short as possible, and the length of this route.

c. State which roads she drives along twice in part (b).

d. Explain why she would not finish at an even vertex.

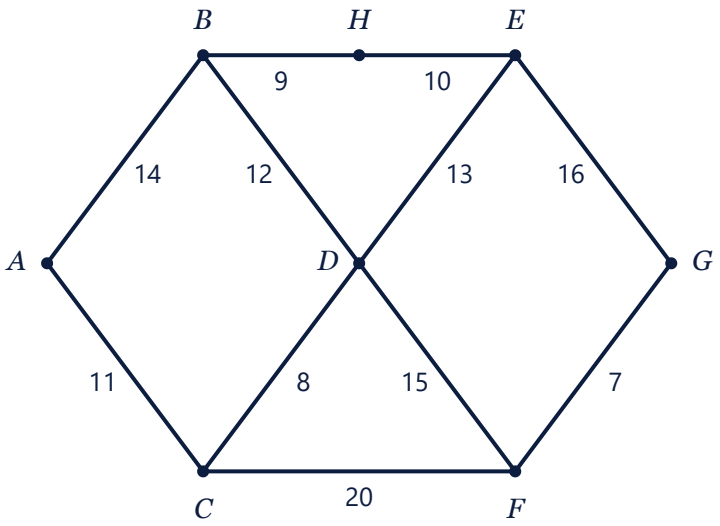


Diagram NOT accurately drawn

(10 marks)

a)

Find the degree of each vertex

A 2, B 3, C 3, D 4, E 3, F 3, G 2, H 2

Odd vertices: B, C, E, F

Shortest distance between each pair of odd vertices

BC: BDC = 12 + 8 = 20

BE: BHE = 9 + 10 = 19

BF: BDF = 12 + 15 = 27

CE: CDE = 8 + 13 = 21

CF = 20 (direct)

EF: EGF = 16 + 7 = 23

Pair the odd vertices: three possible pairings

BC + EF = 20 + 23 = 43

BE + CF = 19 + 20 = 39

BF + CE = 27 + 21 = 48

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Least: BE + CF = 39

Length = 135 + 39 = 174

174 miles

b)

B is odd; finish at another odd vertex and repeat a path between the other two

Finish C: repeat EF = 23

Finish E: repeat CF = 20

Finish F: repeat CE = 21

Least is 20: finish at E

Length = 135 + 20 = 155

Finish at E; 155 miles

c)

CF uses CF

CF

d)

An even finish leaves all four odd vertices to be paired

least pairing 39 > 20

Longer route, so finish at an odd vertex

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