Floyd's algorithm

A-level Decision Mathematics 1 | Edexcel 9FM0 | 12 questions, 78 marks

Further Maths topic practice

Scan for the solutions

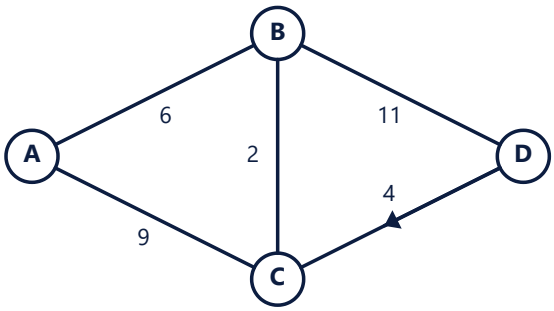
Name: _____ **Date:** _____ **Score:** _____ **/ 78**

1 The network below shows the roads between four farms, A, B, C and D. The number on each arc is the length of the road in km. The road from D to C is one-way, as shown by the arrow.

Floyd's algorithm is to be used to find the shortest distances between every pair of farms.

a. Complete the initial distance table and the initial route table.

b. Explain why the first iteration of Floyd's algorithm makes no changes to either table.



Initial tables

Distance table

	A	B	C	D
A				
B				
C				
D				

Route table

	A	B	C	D
A				
B				
C				
D				

Diagram NOT accurately drawn

(4 marks)

Answer _____

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2 The initial distance table and initial route table for a directed network with vertices A, B, C and D are shown below.

Perform the first iteration of Floyd's algorithm. You should show the distance table and the route table after this iteration.

Initial tables

Distance table

	A	**B**	**C**	**D**
A	-	5	∞	3
B	5	-	4	∞
C	2	∞	-	8
D	3	∞	8	-

Route table

	A	**B**	**C**	**D**
A	A	B	C	D
B	A	B	C	D
C	A	B	C	D
D	A	B	C	D

After the first iteration

Distance table

	A	**B**	**C**	**D**
A				
B				
C				
D				

Route table

	A	**B**	**C**	**D**
A				
B				
C				
D				

(3 marks)

Answer

3 Floyd's algorithm has been applied to a network of roads between five towns, A, B, C, D and E. Some of the roads are one-way. The final distance table, in miles, and the final route table are shown below.

a. Use the final route table to find the shortest route from A to E, explaining how you use the table. State the length of this route.

b. Find the shortest route from E to D and its length.

Final tables

Distance table

	A	**B**	**C**	**D**	**E**
A	-	8	11	5	15
B	8	-	3	9	7
C	11	3	-	6	4
D	5	9	6	-	10
E	7	7	4	10	-

Route table

	A	**B**	**C**	**D**	**E**
A	A	B	B	D	C
B	A	B	C	C	C
C	B	B	C	D	E
D	A	C	C	D	C
E	A	C	C	C	E

(4 marks)

Answer

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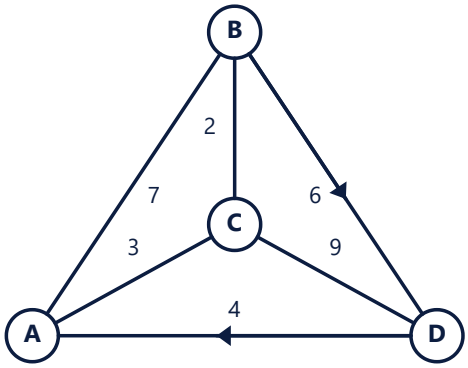
4 The network below shows the paths between four huts, A, B, C and D, on a nature reserve. The number on each arc is the walking time in minutes. The paths from B to D and from D to A are one-way, as shown by the arrows.

The initial distance and route tables are shown below the network.

a. Use Floyd's algorithm to find the shortest walking times between every pair of huts. You should show the distance table and the route table after each iteration.

b. Use your final route table to find the quickest route from D to B, and state its time.

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Initial tables

Distance table

	A	B	C	D
A	-	7	3	∞
B	7	-	2	6
C	3	2	-	9
D	4	∞	9	-

Route table

	A	B	C	D
A	A	B	C	D
B	A	B	C	D
C	A	B	C	D
D	A	B	C	D

(9 marks)

First iteration

Distance table

	A	B	C	D
A				
B				
C				
D				

Route table

	A	B	C	D
A				
B				
C				
D				

Second iteration

Distance table

	A	B	C	D
A				
B				
C				
D				

Route table

	A	B	C	D
A				
B				
C				
D				

Third iteration

Distance table

	A	B	C	D
A				
B				
C				
D				

Route table

	A	B	C	D
A				
B				
C				
D				

Fourth iteration

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Distance table

	A	B	C	D
A				
B				
C				
D				

Route table

	A	B	C	D
A				
B				
C				
D				

Diagram NOT accurately drawn

Answer _____

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5 Floyd's algorithm is being used to find the shortest times, in minutes, between five stations, A, B, C, D and E, on a tram network in which some links run in one direction only. The distance table and route table after the second iteration are shown below.

a. Perform the third, fourth and fifth iterations of Floyd's algorithm. You should show the distance table and the route table after each iteration.

b. Use your final tables to find the quickest journey from C to D and its time.

After the second iteration

Distance table

	A	B	C	D	E
A	-	4	7	11	14
B	4	-	3	7	18
C	10	14	-	21	5
D	9	7	10	-	2
E	14	18	5	2	-

Route table

	A	B	C	D	E
A	A	B	B	B	E
B	A	B	C	D	A
C	A	A	C	B	E
D	A	B	B	D	E
E	A	A	C	D	E

Third iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Fourth iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Fifth iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

(7 marks)

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Answer

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6 The initial distance table below gives the lengths, in metres, of the walkways between five buildings, A, B, C, D and E, on a hospital site. Some walkways are one-way.

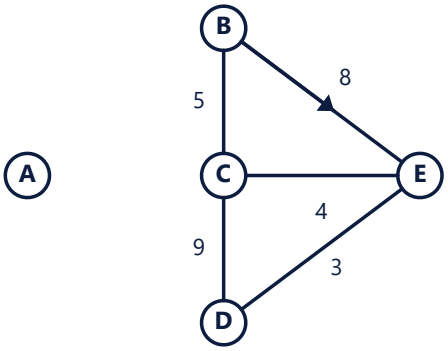
	A	**B**	**C**	**D**	**E**
A	–	12	6	∞	∞
B	∞	–	5	∞	8
C	6	5	–	9	4
D	7	∞	9	–	3
E	∞	∞	4	3	–

The drawing of the network below is incomplete.

a. By adding the arcs that join vertex A to the other vertices, with their weights and any arrows needed, complete the drawing of the network.

b. Perform the first two iterations of Floyd's algorithm, starting from the initial tables given below the network. You should show the distance table and the route table after each iteration.

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(6 marks)

Initial tables

Distance table

	A	B	C	D	E
A	-	12	6	∞	∞
B	∞	-	5	∞	8
C	6	5	-	9	4
D	7	∞	9	-	3
E	∞	∞	4	3	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

First iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Second iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Diagram NOT accurately drawn

Answer _____

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7

a. Explain the difference between the information given by Dijkstra's algorithm and the information given by Floyd's algorithm.

b. Explain why, in the kth iteration of Floyd's algorithm, the entries in the kth row and the kth column of the distance table never change.

c. Floyd's algorithm has cubic order. A computer takes 2.4 seconds to apply Floyd's algorithm to a network with 40 vertices. Estimate the time it would take to apply the algorithm to a network with 100 vertices.

(5 marks)

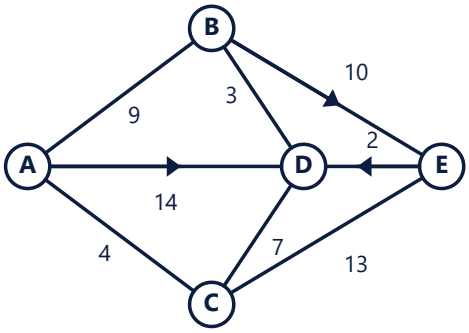
Answer

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8 The network below shows the cycle routes between five villages, A, B, C, D and E. The number on each arc is the length of the route in km. The routes from A to D, from B to E and from E to D are one-way, as shown by the arrows.

a. Complete the initial distance table and the initial route table.

b. Perform the first two iterations of Floyd's algorithm. You should show the distance table and the route table after each iteration.



Initial tables

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

First iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Second iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Diagram NOT accurately drawn

(6 marks)

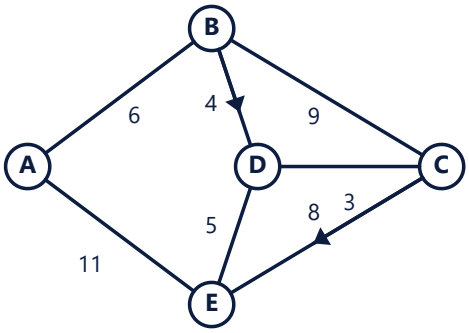
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Answer

9 The network below shows the times, in minutes, to drive along the roads between five depots, A, B, C, D and E. The roads from B to D and from C to E are one-way. Floyd's algorithm has been applied to the network, with the vertices taken in the order A, B, C, D, E. The final distance table and a partially completed final route table are shown below.

a. Complete the final route table, explaining how you found each missing entry.

b. Use the final route table to find the quickest route from E to C and its time.



Final tables

Distance table

	A	B	C	D	E
A	-	6	13	10	11
B	6	-	7	4	9
C	15	9	-	3	8
D	16	12	3	-	5
E	11	17	8	5	-

Route table

	A	B	C	D	E
A	A	B		B	E
B	A	B	D	D	D
C		B	C	D	E
D		C	C	D	E
E	A		D	D	E

Diagram NOT accurately drawn

(6 marks)

Answer

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10 A council runs a gritting service for six villages, A, B, C, D, E and F. Some of the roads between the villages are one-way. Floyd's algorithm has been used to find the shortest driving times, in minutes, between the villages. The final distance table and final route table are shown below.

The council will keep its gritting lorry at one of the villages. Each call-out is a return journey from that village to one other village and back again.

a. Explain why the time from A to C is not the same as the time from C to A.

b. Find the village at which the lorry should be kept so that the longest return journey is as short as possible. You must show your working.

c. For the village chosen in (b), find the route of its longest return journey.

Final tables

Distance table

	A	**B**	**C**	**D**	**E**	**F**
A	-	14	9	17	20	20
B	14	-	7	15	18	24
C	21	7	-	8	11	17
D	29	15	8	-	6	9
E	30	21	14	6	-	10
F	20	12	19	16	10	-

Route table

	A	**B**	**C**	**D**	**E**	**F**
A	A	B	C	C	C	F
B	A	B	C	C	C	D
C	B	B	C	D	E	D
D	C	C	C	D	E	F
E	F	D	D	D	E	F
F	A	B	B	E	E	F

(8 marks)

Answer _____

11 Floyd's algorithm has been applied to a network of roads between six towns, A, B, C, D, E and F. Some of the roads are one-way. The final table of shortest times, in minutes, and the final route table are shown below.

Leila is a courier based in town A. She must deliver parcels to D and to E, in either order, and then return to A.

a. Find the order in which Leila should make her deliveries so that her total time is as short as possible. State the total time.

b. Use the final route table to write down the complete route Leila should take, listing every town she passes through.

c. Explain why her route passes through town F twice.

Final tables

Distance table

	A	**B**	**C**	**D**	**E**	**F**
A	-	12	7	15	22	11
B	12	-	6	9	17	22
C	7	6	-	10	15	18
D	17	16	10	-	8	13
E	16	21	15	8	-	5
F	11	20	14	4	12	-

Route table

	A	**B**	**C**	**D**	**E**	**F**
A	A	B	C	F	C	F
B	A	B	C	D	D	E
C	A	B	C	D	E	A
D	C	C	C	D	E	E
E	F	C	C	D	E	F
F	A	D	D	D	D	F

(8 marks)

Answer

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12 The network below shows the footpaths between five viewpoints, A, B, C, D and E, in a country park. The number on each arc is the walking time in minutes. The paths from A to C, from C to D and from E to A are one-way, as shown by the arrows.

a. Complete the initial distance table and the initial route table.

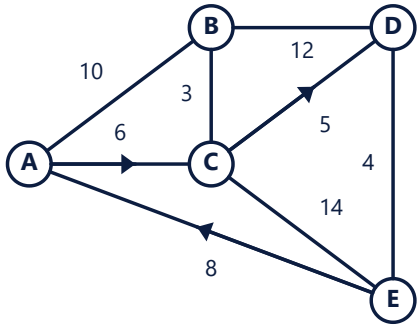
b. Perform the first three iterations of Floyd's algorithm. You should show the distance table and the route table after each iteration.

The final tables, after all five iterations, are shown below the tables for the iterations.

c. Use the final route table to find the quickest route from A to E and from E to B. State the time of each.

d. A new two-way path is to be built directly between B and E. Find the range of walking times for this path that would reduce the quickest time from B to E.

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Initial tables

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

(12 marks)

First iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Second iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Third iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Final tables

Distance table

	A	B	C	D	E
A	-	9	6	11	15
B	10		3	8	12

Route table

	A	B	C	D	E
A	A	C	C	C	D
B	A	B	C	C	D

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B	10	-	3	8	12
C	13	3	-	5	9
D	12	12	15	-	4
E	8	16	14	4	-

B	A	B	C	C	D
C	B	B	C	D	D
D	E	B	B	D	E
E	A	D	C	D	E

Diagram NOT accurately drawn

Answer _____

End of questions | Total 78 marks

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