Planarity algorithm

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** A graph that can be drawn in a plane so that no two arcs meet (cross) except at a vertex **b)** A cycle (closed path) that passes through every vertex exactly once and returns to the start vertex **c)** e.g. A B C F E D A	4
2	AC inside, BD outside: planar	3
3	Planar: AC, CE, AE inside; BD, BF outside	4
4	**a)** e.g. A B C D E F G A **b)** Planar: AC, AE, CE inside; BG, DF, DG outside	5
5	**a)** e.g. A X B Y C Z A **b)** Not planar: AY inside, BZ and CX outside, but BZ and CX cross; the pipes cannot be laid without a crossing	5
6	**a)** 12 **b)** Not planar: AC I, BD O, BF O, CE I, AD I, but AD and CE both inside and they cross	5
7	**a)** C X A V D Y E Z B W C **b)** Not planar	5
8	**a)** Planar: BD, BE, AE inside; CF outside **b)** Not planar: AE I, CF O, BD I, BE I, AD I, then AD crosses BE with both inside	7
9	**a)** n(n - 1)/2 **b)** 3: AD, BD and CE **c)** AC I, BD O, BE O, CE I, AD I but AD crosses CE, both inside: not planar	6
10	**a)** AB, BC, CD, DE, EF, FG and GA are all arcs and each vertex is visited once **b)** Not planar, so the tracks cannot all be printed on one side **c)** e.g. AD: the algorithm then gives a valid labelling (other answers: BF, CG)	8
11	**a)** Planar: BE, BF, CE inside; AD, DG outside **b)** AF only: it crosses only an outside arc so can go inside, while AE and CF each cross an inside arc and an outside arc	7
12	**a)** Semi-Eulerian: exactly two vertices (B and D) have odd degree **b)** 5 **c)** A B C D E F A **d)** Planar: AC, AD inside; BE, CE outside **e)** Eulerian, since every vertex now has even degree; not planar	10

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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