Planarity algorithm

A-level Decision Mathematics 1 | Edexcel 9FM0 | 12 questions, 69 marks

Further Maths topic practice

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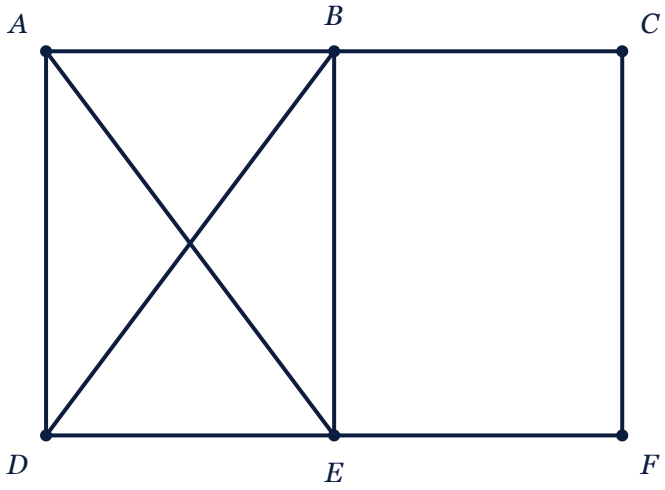
Name: _____ **Date:** _____ **Score:** _____ **/ 69**

1 The graph G is shown below.

a. Explain what is meant by a planar graph.

b. Explain what is meant by a Hamiltonian cycle.

c. Write down a Hamiltonian cycle for G, starting and finishing at A.



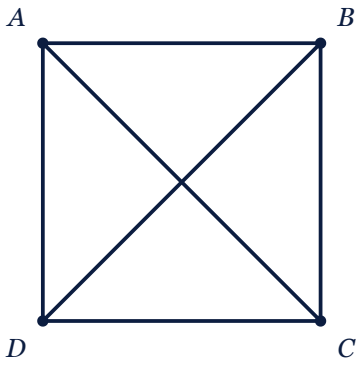
(4 marks)

Answer _____

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2 The graph below has four vertices and six arcs. In the drawing, arcs AC and BD cross.

Taking ABCDA as the Hamiltonian cycle, use the planarity algorithm to show that the graph is planar. Draw the graph without any arcs crossing.

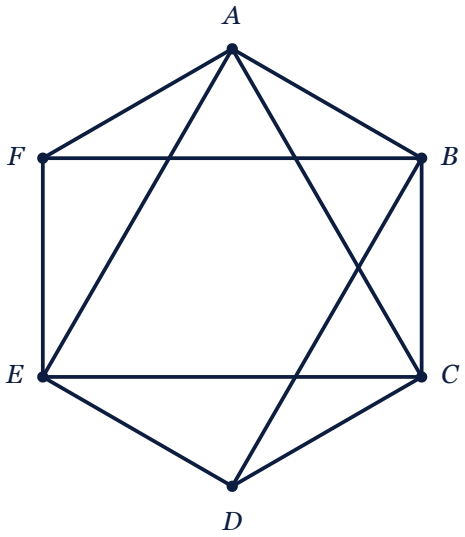


(3 marks)

Answer

3 The graph below has six vertices, A to F.

Taking ABCDEFA as the Hamiltonian cycle, use the planarity algorithm to determine whether the graph is planar. You must make your working clear and justify your answer.



(4 marks)

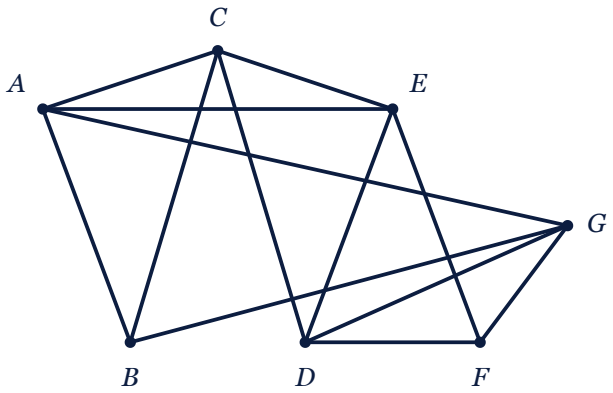
Answer

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4 The graph below has seven vertices, A to G, and thirteen arcs.

a. Starting at A, find a Hamiltonian cycle for the graph.

b. Use your Hamiltonian cycle and the planarity algorithm to determine whether the graph is planar. You must make your working clear and justify your answer.



(5 marks)

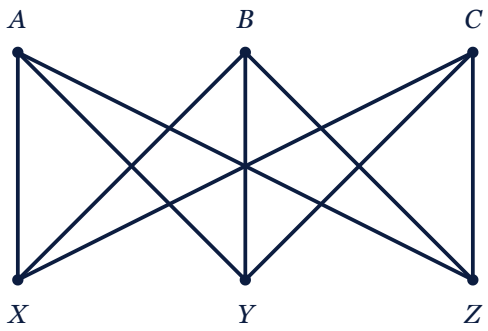
Answer

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5 Three new houses, A, B and C, are each to be connected by an underground pipe to each of three supply points: water X, gas Y and a heat pump Z. The pipes must all be laid at the same depth, so no two pipes may cross. The graph below shows the nine connections.

a. Write down a Hamiltonian cycle for the graph, starting at A.

b. Use the planarity algorithm to determine whether the pipes can be laid without crossing. You must make your working clear.



(5 marks)

Answer

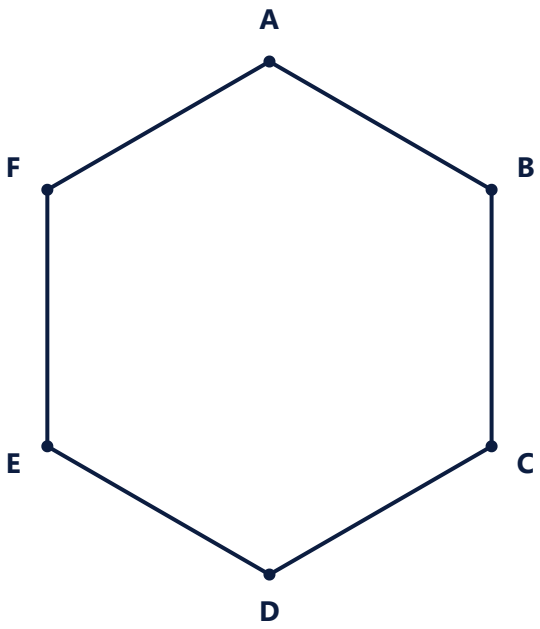
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6 A graph G has six vertices, A to F. The vertices adjacent to each vertex are listed below.

Vertex	Adjacent to
A	B, C, D, F
B	A, C, D, F
C	A, B, D, E
D	A, B, C, E, F
E	C, D, F
F	A, B, D, E

a. State the number of arcs in G.

b. Taking ABCDEFA as the Hamiltonian cycle, which is drawn below, use the planarity algorithm to determine whether G is planar. You must make your working clear and justify your answer.



(5 marks)

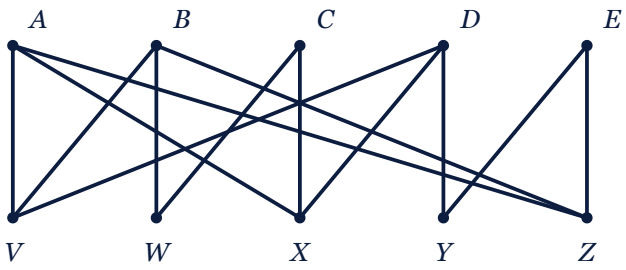
Answer

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7 The graph below has ten vertices and thirteen arcs. A Hamiltonian cycle for the graph begins C, X, A, V, ...

a. Complete the Hamiltonian cycle.

b. Hence use the planarity algorithm to determine whether the graph is planar. You must make your working clear and justify your answer.



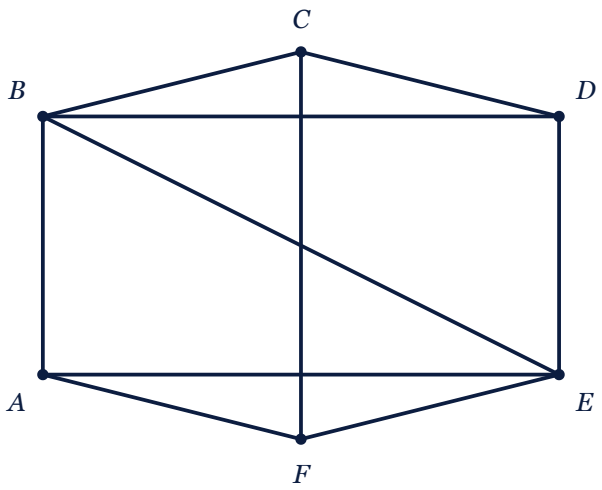
(5 marks)

Answer

8 The graph below shows the footpaths, drawn as arcs, between six picnic areas, A to F, in a park.

a. Taking ABCDEFA as the Hamiltonian cycle, use the planarity algorithm to show that the graph is planar.

b. A new footpath AD is to be added. Determine whether the graph remains planar. You must make your working clear.



(7 marks)

Answer

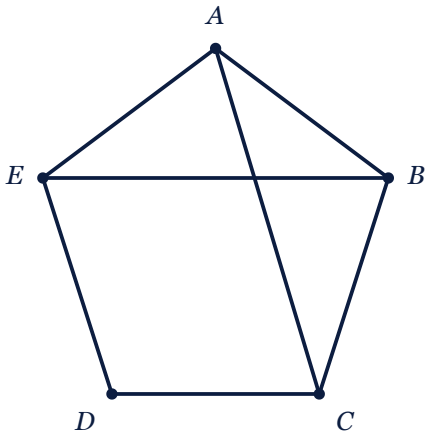
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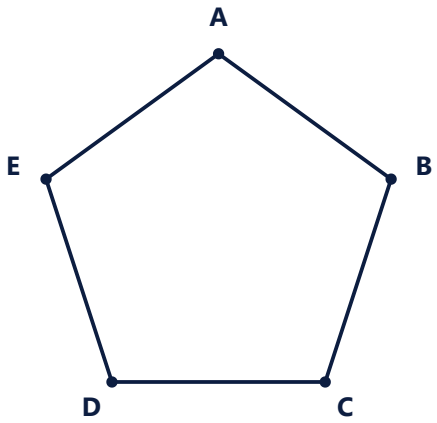
9 The graph G below has five vertices and seven arcs.

a. State the number of arcs in the complete graph K_n, in terms of n.

b. Find the number of arcs that would need to be added to G to make the complete graph K_5, and list them.

c. Taking ABCDEA as the Hamiltonian cycle, which is drawn on the right below, use the planarity algorithm to show that K_5 is not planar.





(6 marks)

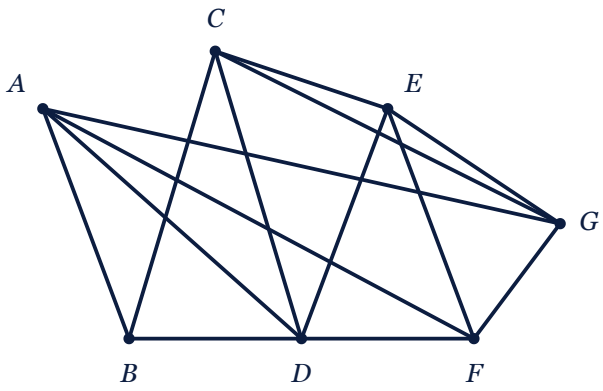
Answer

10 A circuit board has seven components, A to G. The graph below shows the thirteen copper tracks that must join pairs of components. Tracks printed on one side of the board must not cross.

a. Show that A B C D E F G A is a Hamiltonian cycle for the graph.

b. Use the planarity algorithm to determine whether all thirteen tracks can be printed on one side of the board.

c. The designer moves one track to the other side of the board. Find a track that could be moved so that the remaining twelve tracks can be printed on one side without crossing. Justify your answer.



(8 marks)

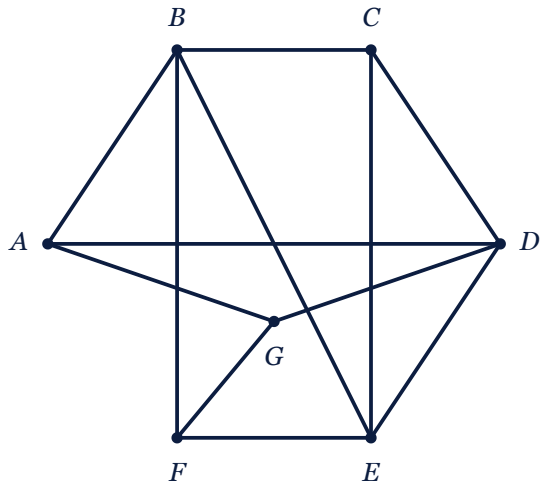
Answer

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11 The graph below has seven vertices, A to G, and twelve arcs. A B C D E F G A is a Hamiltonian cycle for the graph.

a. Use the planarity algorithm to show that the graph is planar.

b. One more arc is to be added to the graph. It will be one of AE, AF or CF. Determine which of these arcs could be added so that the graph is still planar. You must justify your answer.



(7 marks)

Answer

12 The graph G below has six vertices, A to F, and ten arcs.

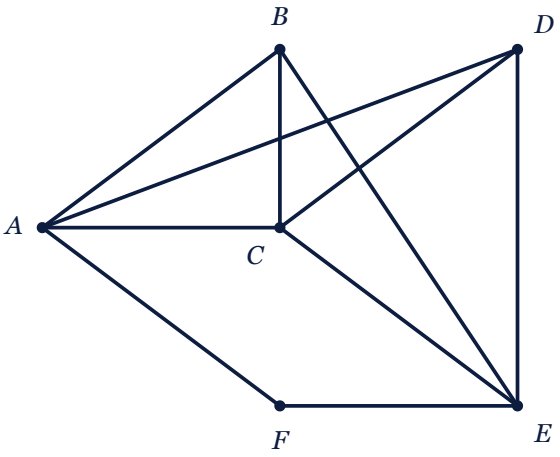
a. State whether G is Eulerian, semi-Eulerian or neither, giving a reason for your answer.

b. Find the number of arcs that would need to be added to G to make the complete graph K_6.

c. Write down a Hamiltonian cycle for G, starting at A.

d. Use your Hamiltonian cycle and the planarity algorithm to show that G is planar.

e. The arc BD is added to G. State, with a reason, whether the new graph is Eulerian, and determine whether it is planar.



(10 marks)

Answer _____

End of questions | Total 69 marks