Resource histograms and scheduling

Worked solutions

Further Maths topic practice

This topic online

1 A café in Leeds is being refurbished. The table shows the ten activities involved and the time, in hours, each one takes.

Activity	A	B	C	D	E	F	G	H	I	J
Time (hours)	4	7	5	6	3	8	5	9	6	8

The minimum completion time of the project is 23 hours. Each activity requires one worker.

a. Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must make your method clear.

b. Explain why the project cannot be completed in 23 hours by two workers.

(3 marks)

a)

Lower bound = sum of all activity times ÷ minimum completion time, rounded up

Sum = 4 + 7 + 5 + 6 + 3 + 8 + 5 + 9 + 6 + 8 = 61

61 ÷ 23 = 2.652

Round up (workers come in whole numbers)

3 workers

b)

2 × 23 = 46 hours of work

46 < 61, so two workers cannot do all the work in 23 hours

2 workers give only 46 hours, less than 61

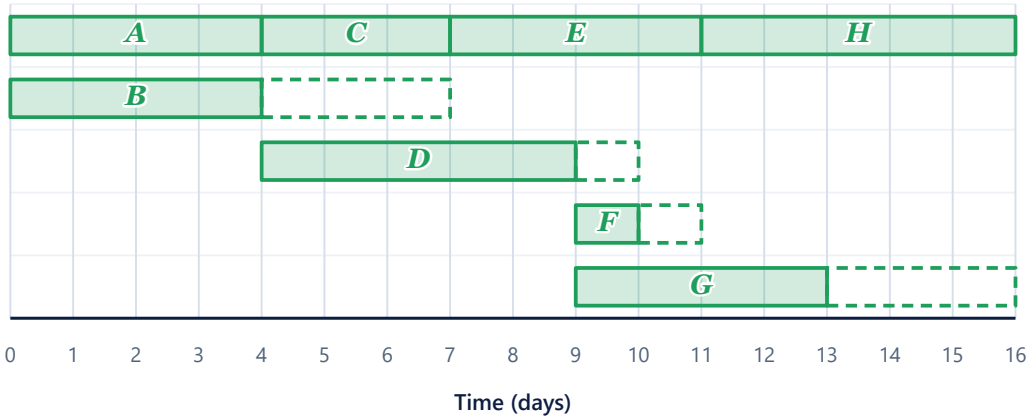
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2 Kemi is planning the landscaping of a garden. The table shows each activity, its duration in days, its earliest start time and its latest finish time. The minimum completion time is 16 days.

Activity	Duration (days)	Earliest start time	Latest finish time
A	4	0	4
B	4	0	7
C	3	4	7
D	5	4	10
E	4	7	11
F	1	9	11
G	4	9	16
H	5	11	16

a. Find the total float of each activity and hence state the critical activities.

b. On the grid below, draw a cascade (Gantt) chart for the project. Show the critical activities on the top row and the float of each other activity as a dashed box.



(5 marks)

a)

Total float = late time at end − duration − early time at start

Float A = 4 − 4 − 0 = 0

Float B = 7 − 4 − 0 = 3

Float C = 7 − 3 − 4 = 0

Float D = 10 − 5 − 4 = 1

Float E = 11 − 4 − 7 = 0

Float F = 11 − 1 − 9 = 1

Float G = 16 − 4 − 9 = 3

Float H = 16 − 5 − 11 = 0

Critical activities have total float 0

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A, C, E, H

b)

Critical activities on the top row; each other bar starts at its early start time, float dashed after it

A: 0 to 4, critical

C: 4 to 7, critical

E: 7 to 11, critical

H: 11 to 16, critical

B: 0 to 4, float to 7

D: 4 to 9, float to 10

F: 9 to 10, float to 11

G: 9 to 13, float to 16

Cascade chart drawn on the grid

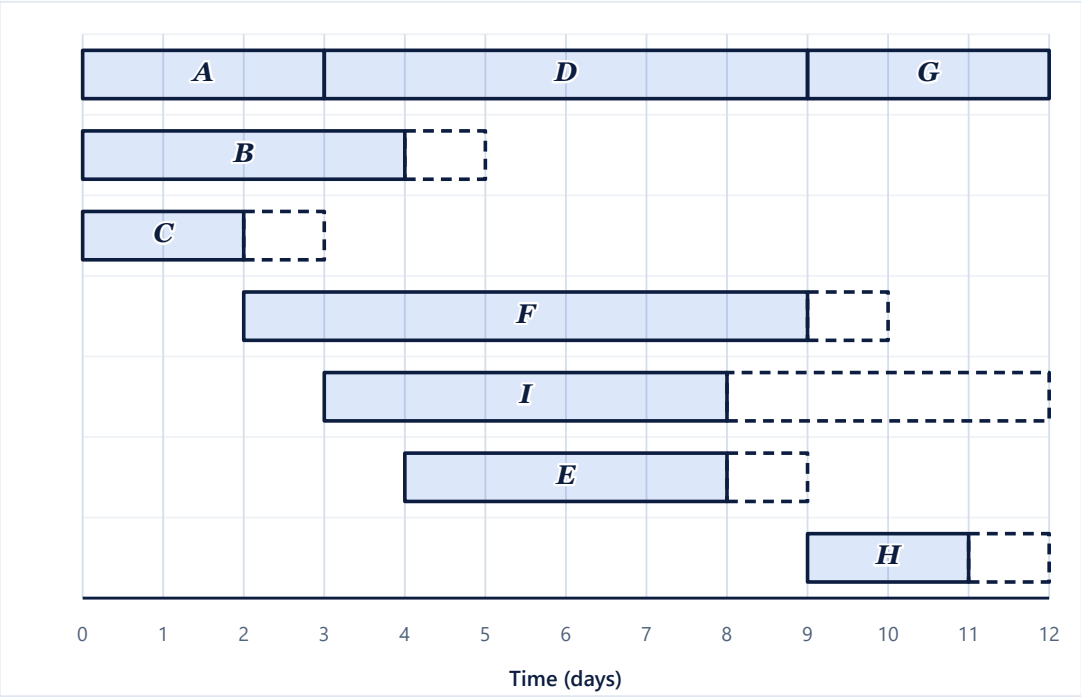
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3 A pop-up shop is being set up in Bristol. The cascade (Gantt) chart below shows the activities, A to I, with times in days. The critical activities are on the top row and the dashed boxes show float.

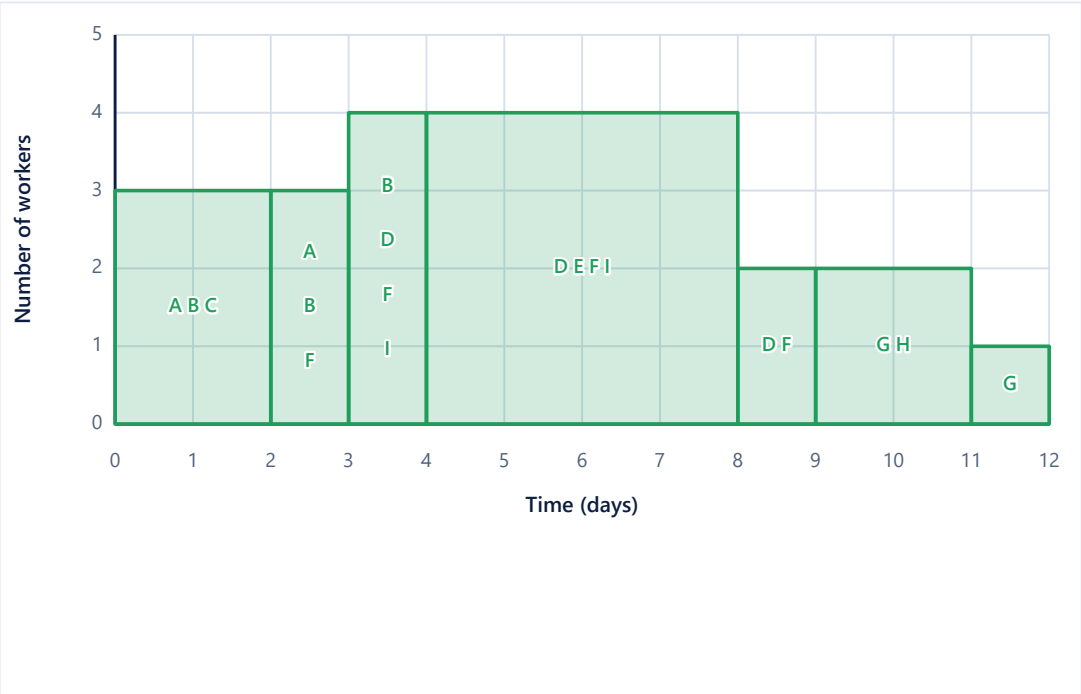
Each activity requires one worker and each activity starts at its earliest start time.

a. On Grid 1, draw a resource histogram for the project.

b. State the greatest number of workers needed at any time, and when this happens.



Cascade chart



Grid 1

(4 marks)

a)

Each bar runs from its earliest start to its earliest finish; count the bars in each day

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0 to 2: A, B, C → 3

2 to 3: A, B, F → 3

3 to 4: B, D, F, I → 4

4 to 8: D, E, F, I → 4

8 to 9: D, F → 2

9 to 11: G, H → 2

11 to 12: G → 1

Histogram drawn on Grid 1

b)

Largest count is 4, from 3 to 8

4 workers, from 3 to 8 days

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4 A charity is having a new website built. The activity network below shows the activities, A to J, with their durations in days in brackets. Each activity requires one worker and the project is to be completed in the minimum time.

a. Complete the diagram to show the early event time and the late event time at each event.

b. State the minimum completion time and the critical activities.

c. Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must make your method clear.

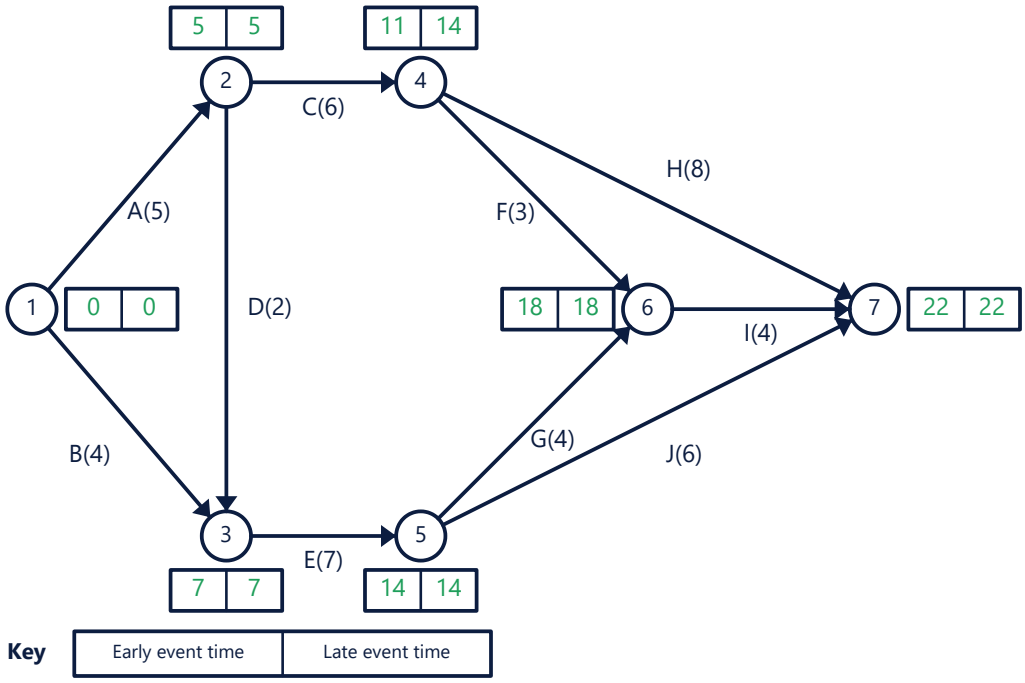


Diagram NOT accurately drawn

(6 marks)

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 5 = 5

Early 3 = max(0 + 4, 5 + 2) = 7

Early 4 = 5 + 6 = 11

Early 5 = 7 + 7 = 14

Early 6 = max(11 + 3, 14 + 4) = 18

Early 7 = max(11 + 8, 18 + 4, 14 + 6) = 22

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 7 = early 7 = 22

Late 6 = 22 - 4 = 18

Late 5 = min(18 - 4, 22 - 6) = 14

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Late 4 = min(18 - 3, 22 - 8) = 14

Late 3 = 14 - 7 = 7

Late 2 = min(14 - 6, 7 - 2) = 5

Late 1 = min(5 - 5, 7 - 4) = 0

1: (0, 0); 2: (5, 5); 3: (7, 7); 4: (11, 14); 5: (14, 14); 6: (18, 18); 7: (22, 22)

b)

Critical activities: early = late at both ends and duration = difference

22 days; A, D, E, G, I

c)

Lower bound = sum of all activity times ÷ critical path length, rounded up

Sum = 5 + 4 + 6 + 2 + 7 + 3 + 4 + 8 + 4 + 6 = 49

49 ÷ 22 = 2.227

Round up: 3 workers

3 workers

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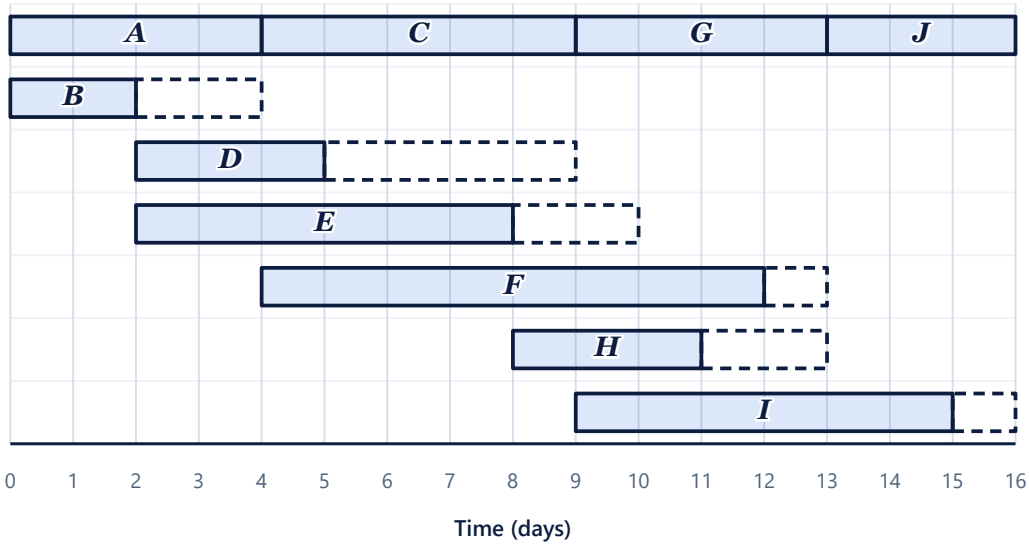
5 A community centre in Belfast is being renovated. The cascade (Gantt) chart below shows the activities, A to J, with times in days. The critical activities are on the top row and the dashed boxes show float. Each activity requires one worker.

a. State the activities that must be happening at time 5.5 days, and the activities that could be happening at that time.

b. State the activities that must be happening at time 10.5 days.

c. Calculate a lower bound for the number of workers needed to complete the project in the minimum time.

d. Use your answer to part (b) to explain why the project cannot be completed in the minimum time using the number of workers found in part (c).



(7 marks)

a)

Must be happening: latest start < t < earliest finish

C: latest start 9 - 5 = 4, earliest finish 4 + 5 = 9

E: latest start 10 - 6 = 4, earliest finish 2 + 6 = 8

F: latest start 13 - 8 = 5, earliest finish 4 + 8 = 12

All three bars cover 5.5 even when started as late as possible

Could be happening: earliest start < t < latest finish

D: earliest start 2, latest finish 9, latest start 6, earliest finish 5

Latest start 6 > 5.5 and earliest finish 5 < 5.5

So D may or may not be happening at 5.5

Must: C, E, F; could: D

b)

F: latest start 13 - 8 = 5, earliest finish 4 + 8 = 12

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G: Latest start 13 - 4 = 9, earliest finish 9 + 4 = 13

H: Latest start 13 - 3 = 10, earliest finish 8 + 3 = 11

I: Latest start 16 - 6 = 10, earliest finish 9 + 6 = 15

F, G, H, I

c)

Lower bound = sum of all activity times ÷ critical path length, rounded up

Sum = 4 + 2 + 5 + 3 + 6 + 8 + 4 + 3 + 6 + 3 = 44

44 ÷ 16 = 2.75

Round up: 3 workers

3 workers

d)

At 10.5 days F, G, H and I must all be in progress

That needs 4 workers at the same time

3 workers are not enough: at least 4 are needed to finish in 16 days

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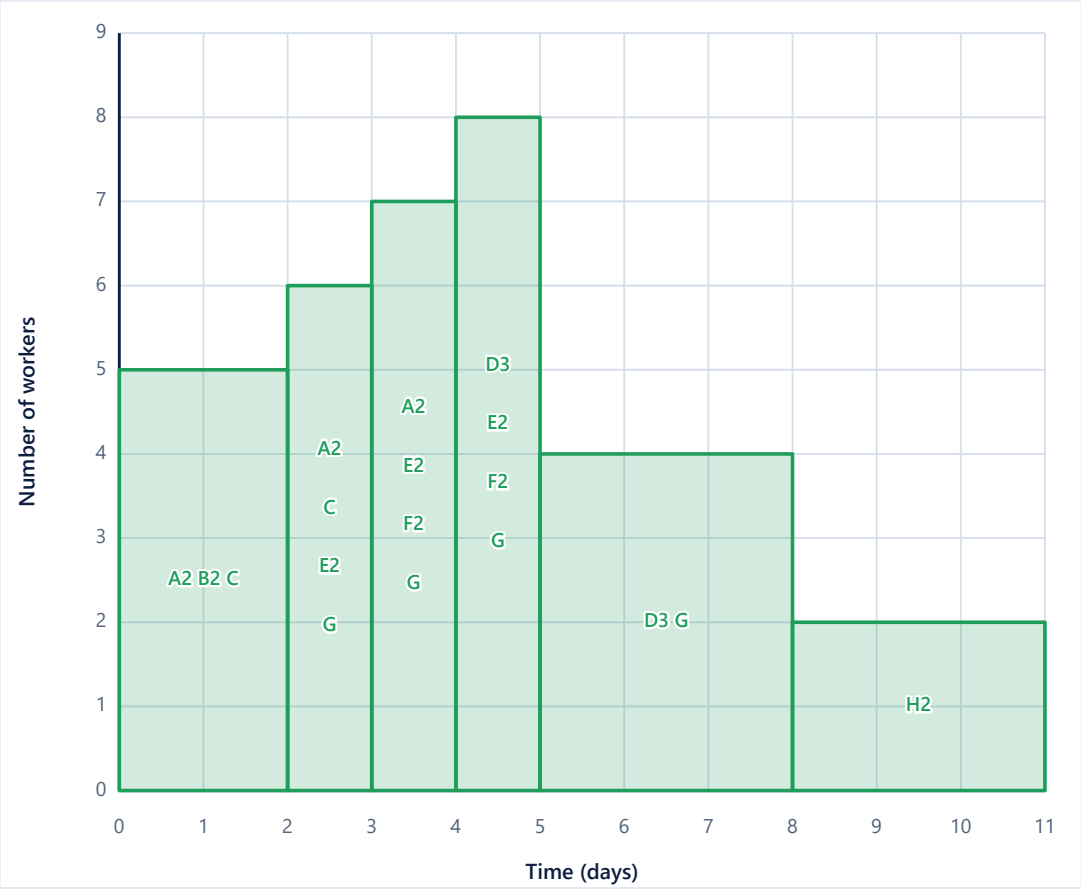
6 Solar panels are being fitted to a primary school in Norwich. The table shows each activity, its duration in days, the number of workers it needs, its earliest start time and its latest finish time. The minimum completion time is 11 days. Once an activity has started it continues until it is finished.

Activity	Duration (days)	Number of workers	Earliest start time	Latest finish time
A	4	2	0	4
B	2	2	0	5
C	3	1	0	6
D	4	3	4	8
E	3	2	2	8
F	2	2	3	8
G	6	1	2	11
H	3	2	8	11

a. On Grid 1, draw a resource histogram for the project, given that every activity starts at its earliest start time.

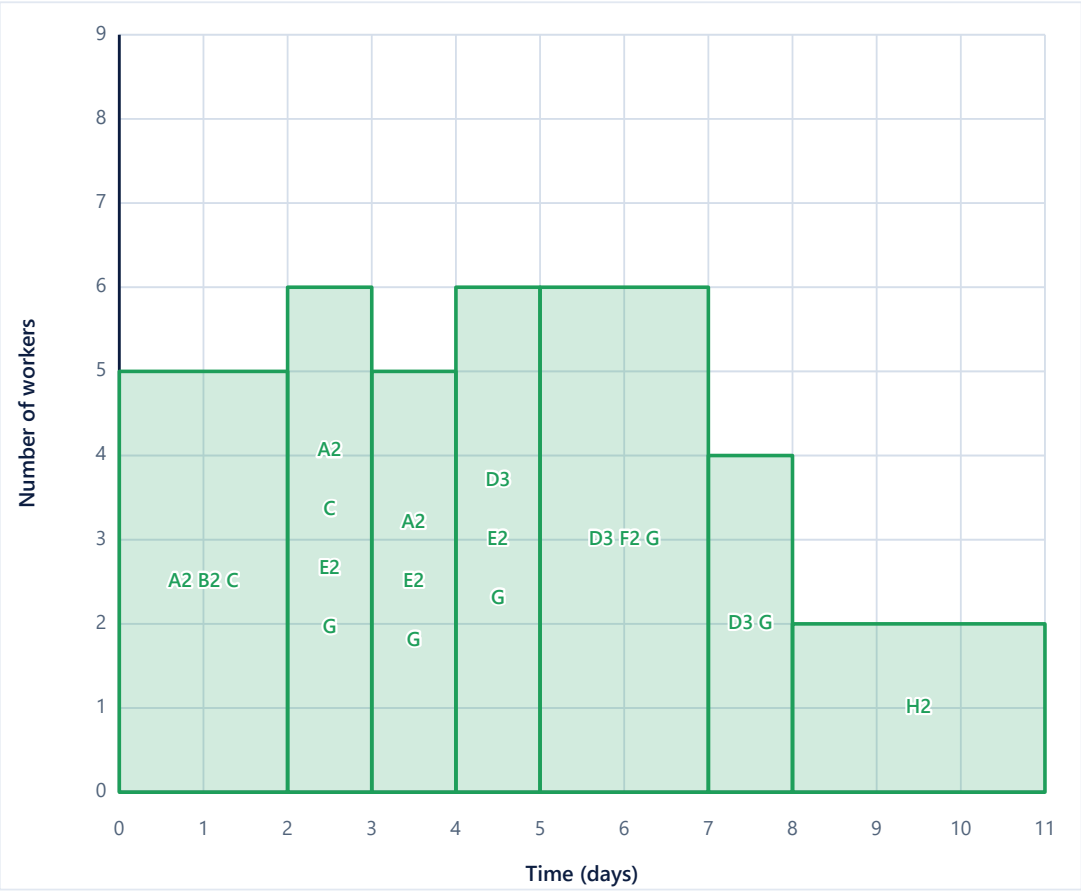
b. The contractor can provide only 6 workers on any one day. By delaying one activity within its float, draw on Grid 2 a resource histogram showing how the project can be completed in 11 days using no more than 6 workers at any time. State the new start time of the activity you delay.

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(7 marks)

Grid 1



Grid 2

a)

Workers in each interval: add the workers of every activity in progress

0 to 2: A(2) + B(2) + C(1) = 5

2 to 3: A(2) + C(1) + E(2) + G(1) = 6

3 to 4: A(2) + E(2) + F(2) + G(1) = 7

4 to 5: D(3) + E(2) + F(2) + G(1) = 8

5 to 8: D(3) + G(1) = 4

8 to 11: H(2) = 2

Histogram drawn on Grid 1 (peak 8 workers from 4 to 5)

b)

Too many workers from 3 to 4 (7) and from 4 to 5 (8)

A and D are critical; E, F and G have float

Float F = 8 - 2 - 3 = 3

Delay F (2 workers) to start at 5: it finishes at 7 ≤ 8

F still finishes by its latest finish time of 8, so no later activity is delayed and the project still takes 11 days

0 to 2: A(2) + B(2) + C(1) = 5

2 to 3: A(2) + C(1) + E(2) + G(1) = 6

3 to 4: A(2) + E(2) + G(1) = 5

4 to 5: D(3) + E(2) + G(1) = 6

5 to 7: D(3) + F(2) + G(1) = 6

7 to 8: D(3) + G(1) = 4

8 to 11: H(2) = 2

Greatest number now 6

F starts at day 5; at most 6 workers, project still 11 days

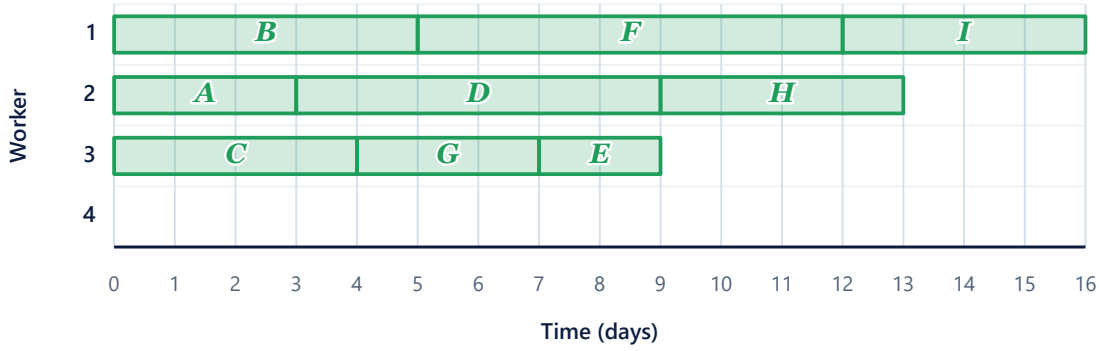
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7 The table gives the activities needed to set up a wedding fair in York, with durations in days, earliest start times and latest finish times. Each activity requires one worker. The project is to be completed in the minimum time of 16 days.

Activity	Immediately preceding activities	Duration (days)	Earliest start time	Latest finish time
A	–	3	0	6
B	–	5	0	5
C	–	4	0	9
D	A	6	3	12
E	B	2	5	12
F	B	7	5	12
G	C	3	4	12
H	D, E	4	9	16
I	F, G	4	12	16

a. Calculate a lower bound for the number of workers needed to complete the project in the minimum time.

b. Schedule the activities on the grid below, using the number of workers found in part (a), so that the project is completed in 16 days.



(7 marks)

a)

Lower bound = sum of all activity times ÷ critical path length, rounded up

Sum = 3 + 5 + 4 + 6 + 2 + 7 + 3 + 4 + 4 = 38

38 ÷ 16 = 2.375

Round up: 3 workers

3 workers

b)

When a worker becomes free, start the available activity with the smallest latest finish time (critical first)

Time 0: B to worker 1, A to worker 2, C to worker 3

Time 3: D to worker 2

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Time 4: G to worker 3

Time 5: F to worker 1 (available F, E)

Time 7: E to worker 3

Time 9: H to worker 2

Time 12: I to worker 1

Check: every activity starts after its preceding activities finish

Worker 1: B (0 to 5), F (5 to 12), I (12 to 16)

Worker 2: A (0 to 3), D (3 to 9), H (9 to 13)

Worker 3: C (0 to 4), G (4 to 7), E (7 to 9)

Schedule drawn: finishes at 16 days with 3 workers

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8 A bathroom in a flat in Swansea is being refitted. The network below shows the activities, with durations in days in brackets. Each activity requires one worker.

a. Complete the diagram to show the early event time and the late event time at each event.

b. On Grid 1, draw a resource histogram for the project, given that each activity starts at its earliest start time.

c. Show that a lower bound for the number of workers needed to complete the project in 13 days is 3, and explain why the project cannot in fact be completed in 13 days by 3 workers.

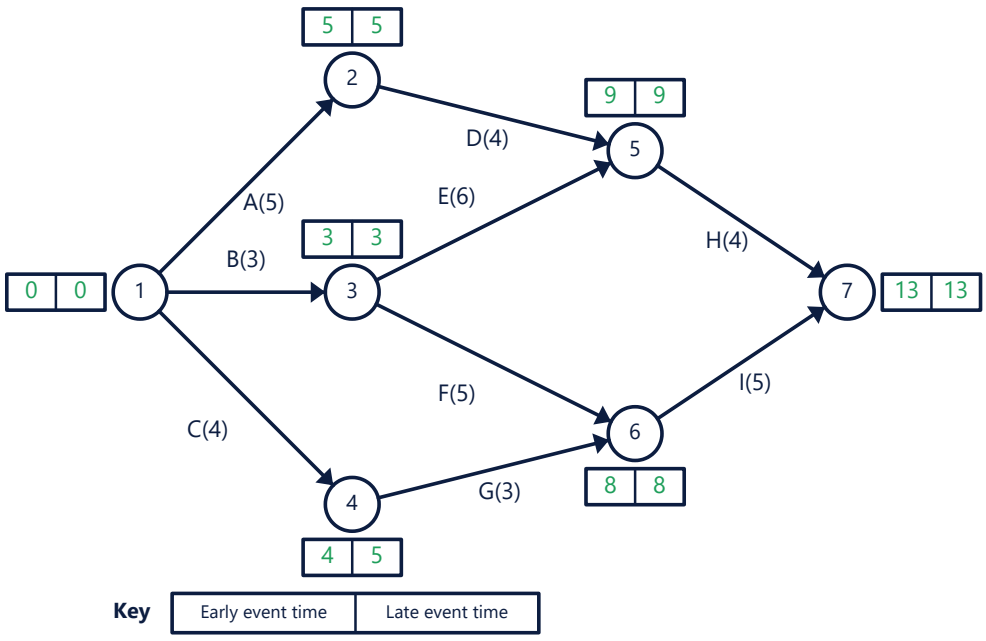
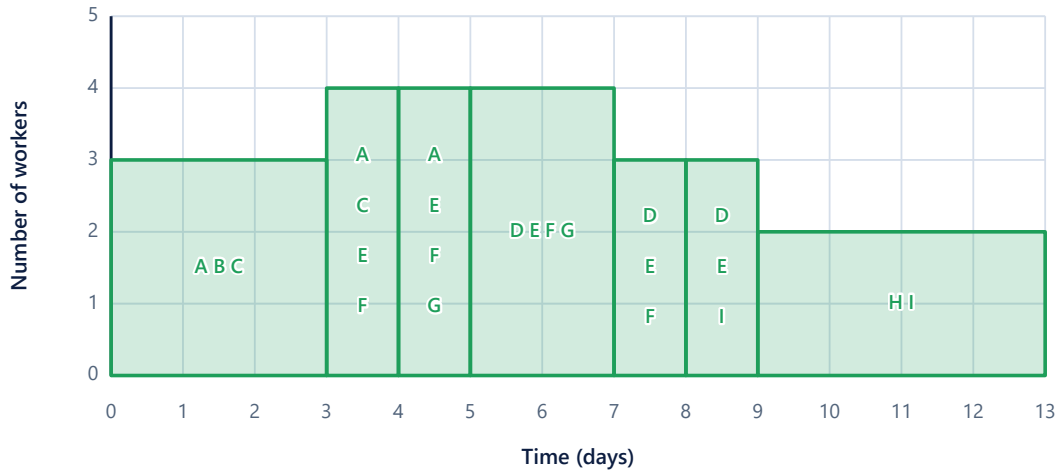


Diagram NOT accurately drawn

Activity network



Grid 1

(8 marks)

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a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 5 = 5

Early 3 = 0 + 3 = 3

Early 4 = 0 + 4 = 4

Early 5 = max(5 + 4, 3 + 6) = 9

Early 6 = max(3 + 5, 4 + 3) = 8

Early 7 = max(9 + 4, 8 + 5) = 13

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 7 = early 7 = 13

Late 6 = 13 - 5 = 8

Late 5 = 13 - 4 = 9

Late 4 = 8 - 3 = 5

Late 3 = min(9 - 6, 8 - 5) = 3

Late 2 = 9 - 4 = 5

Late 1 = min(5 - 5, 3 - 3, 5 - 4) = 0

1: (0, 0); 2: (5, 5); 3: (3, 3); 4: (4, 5); 5: (9, 9); 6: (8, 8); 7: (13, 13)

b)

Each activity from its earliest start; count the activities in each day

0 to 3: A, B, C → 3

3 to 4: A, C, E, F → 4

4 to 5: A, E, F, G → 4

5 to 7: D, E, F, G → 4

7 to 8: D, E, F → 3

8 to 9: D, E, I → 3

9 to 13: H, I → 2

Histogram drawn on Grid 1

c)

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Lower bound = sum of all activity times ÷ critical path length, rounded up

Sum = 5 + 3 + 4 + 4 + 6 + 5 + 3 + 4 + 5 = 39

39 ÷ 13 = 3

3 exactly, so 3 workers

Look for a time when 4 activities must be happening

A, E and F are critical: A from 0 to 5, E and F from 3

C: latest start 5 - 4 = 1, earliest finish 0 + 4 = 4

So C must be happening from 1 to 4

From 3 to 4: A, C, E and F all in progress

Lower bound 3, but 4 activities must overlap from 3 to 4, so 3 workers cannot finish in 13 days

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9 Volunteers are preparing a village fete in Devon. The network below shows the activities, with durations in hours in brackets. Each activity requires one worker.

a. Complete the diagram to show the early event time and the late event time at each event, and state the minimum completion time.

b. Calculate a lower bound for the number of workers needed to complete the project in the minimum time.

c. Only two volunteers, Ffion and Ravi, are available. On Grid 1, draw a schedule for the two workers that completes the project in the shortest possible time, and state this time.

d. Explain why the project cannot be completed by the two workers in less time than your answer to part (c).

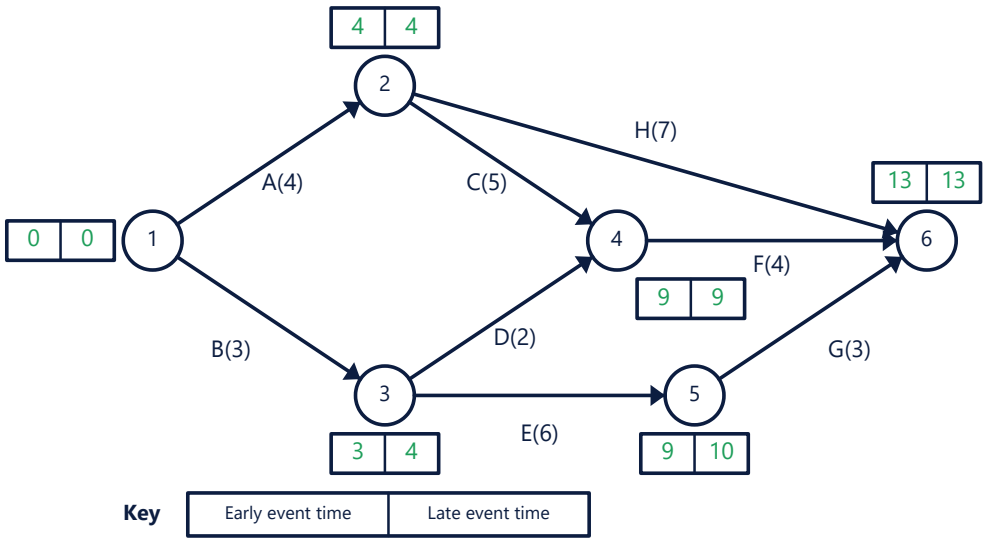
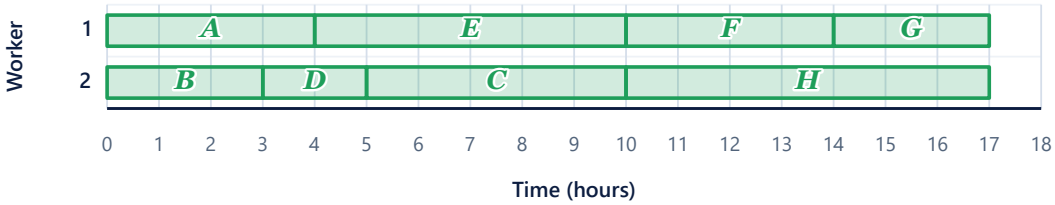


Diagram NOT accurately drawn

Activity network



Grid 1

(9 marks)

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 4 = 4

Early 3 = 0 + 3 = 3

Early 4 = max(4 + 5, 3 + 2) = 9

Early 5 = 3 + 6 = 9

Early 6 = max(9 + 4, 9 + 3, 4 + 7) = 13

Backward pass: late time = smallest (late time at end − duration) over arcs out

Late 6 = early 6 = 13

Late 5 = 13 − 3 = 10

Late 4 = 13 − 4 = 9

Late 3 = min(9 − 2, 10 − 6) = 4

Late 2 = min(9 − 5, 13 − 7) = 4

Late 1 = min(4 − 4, 4 − 3) = 0

1: (0, 0); 2: (4, 4); 3: (3, 4); 4: (9, 9); 5: (9, 10); 6: (13, 13); 13 hours

b)

Lower bound = sum of all activity times ÷ critical path length, rounded up

Sum = 4 + 3 + 5 + 2 + 6 + 4 + 3 + 7 = 34

34 ÷ 13 = 2.615

Round up: 3 workers

3 workers

c)

Two workers: aim to keep both busy with no waiting

Worker 1 takes A, the start of the critical path; worker 2 takes B

At 3 worker 2 is free: D and E (both need only B) are ready; give worker 2 D, the shorter one

At 4 A ends: worker 1 starts E, worker 2 starts C at 5 after D

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At 10 both free: F (needs C, D) to worker 1, H (needs A) to worker 2

At 14 worker 1 starts G (needs E)

Worker 1: A (0 to 4), E (4 to 10), F (10 to 14), G (14 to 17)

Worker 2: B (0 to 3), D (3 to 5), C (5 to 10), H (10 to 17)

17 hours

d)

Total work = 34 hours

$34 \div 2 = 17$ hours

Two workers cannot do 34 hours of work in under 17 hours, and the schedule has no idle time

17 hours is the least possible

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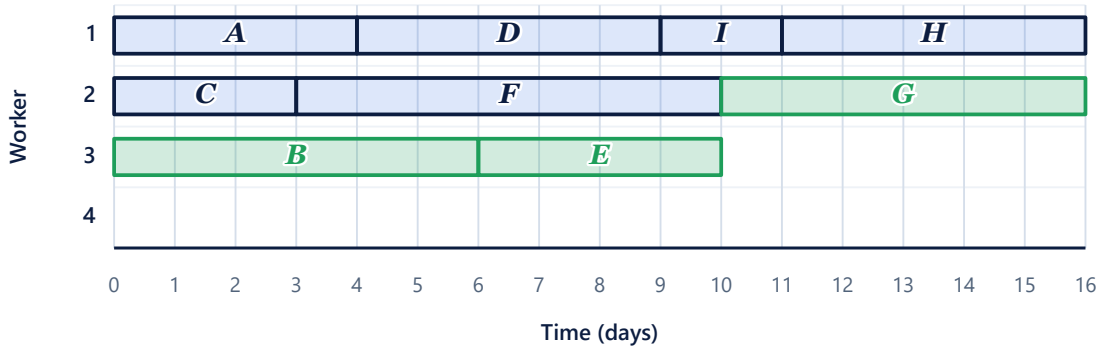
10 Mehmet is building a garden office. The precedence table shows the activities, with durations in days. Each activity requires one worker.

Activity	Immediately preceding activities	Duration (days)
A	–	4
B	–	6
C	–	3
D	A	5
E	A, B	4
F	C	7
G	D	6
H	E, F, I	5
I	D	2

a. Find the earliest start time and the latest finish time of each activity.

b. State the minimum completion time of the project and the critical activities.

c. Three workers are available. The grid below shows a partly completed schedule, with six of the activities already placed. Complete the schedule so that the project is finished in the minimum time.



(9 marks)

a)

Forward pass: ES = largest (ES + duration) over the immediately preceding activities

$ES(A) = 0$

$ES(B) = 0$

$ES(C) = 0$

$ES(D) = 0 + 4 = 4$

$ES(E) = \max(0 + 4, 0 + 6) = 6$

$ES(F) = 0 + 3 = 3$

$ES(G) = 4 + 5 = 9$

$ES(H) = \max(6 + 4, 3 + 7, 9 + 2) = 11$

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$ES(I) = 4 + 5 = 9$

Backward pass: LF = smallest (LF - duration) over the activities that follow; activities with none follow LF = end time

End time = max(ES + duration) = 16

$LF(I) = 16 - 5 = 11$

$LF(H) = 16$

$LF(G) = 16$

$LF(F) = 16 - 5 = 11$

$LF(E) = 16 - 5 = 11$

$LF(D) = \min(16 - 6, 11 - 2) = 9$

$LF(C) = 11 - 7 = 4$

$LF(B) = 11 - 4 = 7$

$LF(A) = \min(9 - 5, 11 - 4) = 4$

A: 0, 4; B: 0, 7; C: 0, 4; D: 4, 9; E: 6, 11; F: 3, 11; G: 9, 16; H: 11, 16; I: 9, 11

b)

Total float = late time at end - duration - early time at start

Float A = $4 - 4 - 0 = 0$

Float B = $7 - 6 - 0 = 1$

Float C = $4 - 3 - 0 = 1$

Float D = $9 - 5 - 4 = 0$

Float E = $11 - 4 - 6 = 1$

Float F = $11 - 7 - 3 = 1$

Float G = $16 - 6 - 9 = 1$

Float H = $16 - 5 - 11 = 0$

Float I = $11 - 2 - 9 = 0$

16 days; A, D, H, I

c)

Left to place: B, E and G

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B: no predecessors, LF 7: worker 3 from 0 to 6

E: needs A (ends 4) and B (ends 6): worker 3 from 6 to 10, LF 11

G: needs D (ends 9), duration 6, LF 16: worker 2 is free at 10, so 10 to 16

H still starts at 11, after E (10), F (10) and I (11)

Worker 3: B 0 to 6, E 6 to 10; worker 2: G 10 to 16; finished in 16 days

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11 A crew is building a stage for a music festival in Glasgow. The network below shows the activities, with durations in hours in brackets. The table gives the number of workers each activity needs. Once an activity has started it must continue until it is finished.

Activity	A	B	C	D	E	F	G	H
Number of workers	3	2	2	3	1	2	2	1

a. Complete the diagram to show the early event time and the late event time at each event, and state the minimum completion time.

b. On Grid 1, draw a resource histogram for the project, given that each activity starts at its earliest start time.

c. Only five workers are available at any time. Given that the project cannot now be completed in less than 14 hours, draw on Grid 2 a resource histogram showing how the project can be completed in 14 hours using no more than five workers at any time. State the start time of each activity.

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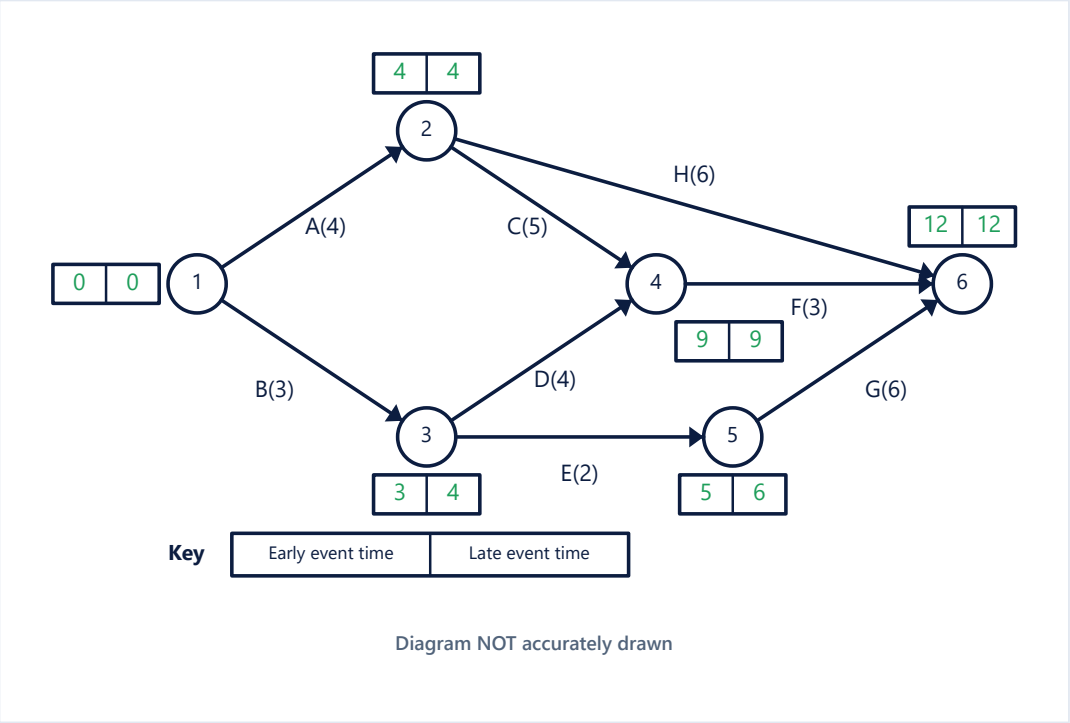
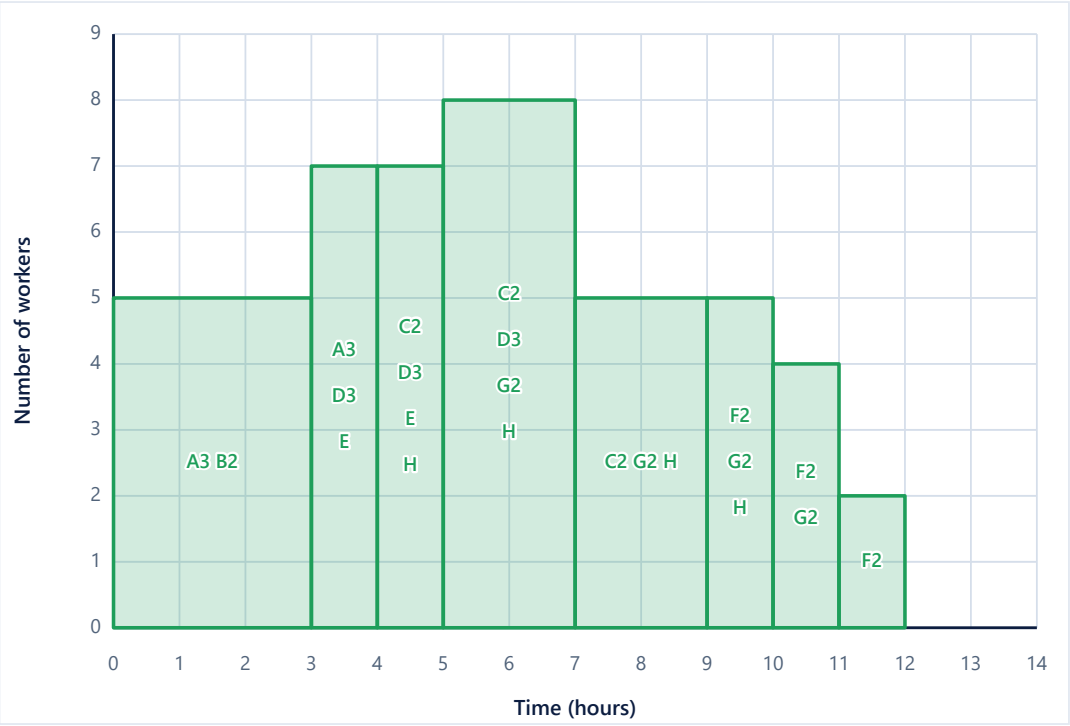


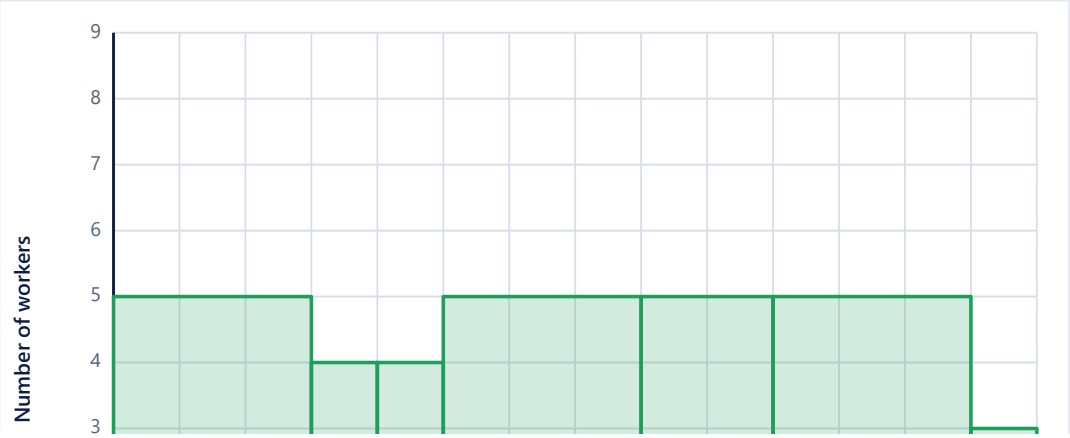
Diagram NOT accurately drawn

Activity network

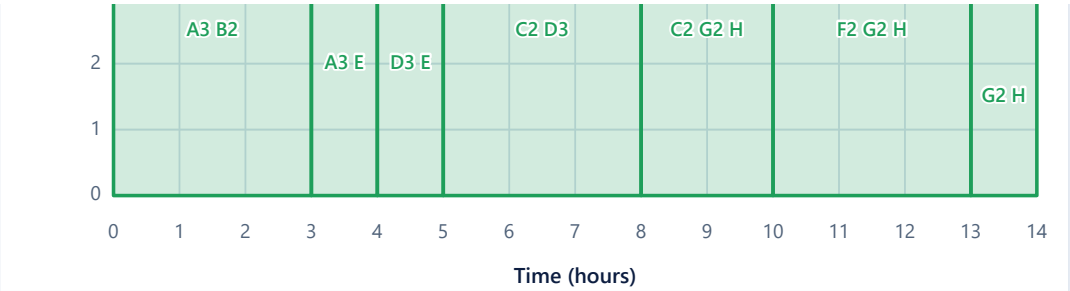
(10 marks)



Grid 1



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Grid 2

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 4 = 4

Early 3 = 0 + 3 = 3

Early 4 = max(4 + 5, 3 + 4) = 9

Early 5 = 3 + 2 = 5

Early 6 = max(9 + 3, 5 + 6, 4 + 6) = 12

Backward pass: late time = smallest (late time at end − duration) over arcs out

Late 6 = early 6 = 12

Late 5 = 12 − 6 = 6

Late 4 = 12 − 3 = 9

Late 3 = min(9 − 4, 6 − 2) = 4

Late 2 = min(9 − 5, 12 − 6) = 4

Late 1 = min(4 − 4, 4 − 3) = 0

1: (0, 0); 2: (4, 4); 3: (3, 4); 4: (9, 9); 5: (5, 6); 6: (12, 12); 12 hours

b)

Workers in each interval: add the workers of every activity in progress

0 to 3: A(3) + B(2) = 5

3 to 4: A(3) + D(3) + E(1) = 7

4 to 5: C(2) + D(3) + E(1) + H(1) = 7

5 to 7: C(2) + D(3) + G(2) + H(1) = 8

7 to 9: C(2) + G(2) + H(1) = 5

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9 to 10: F(2) + G(2) + H(1) = 5

10 to 11: F(2) + G(2) = 4

11 to 12: F(2) = 2

Histogram drawn on Grid 1 (up to 8 workers)

c)

At most 5 workers: delay activities, keeping precedence

0 to 3: A(3) + B(2) = 5

E (1 worker) after B: start at 3, alongside A

D (3 workers) after B: cannot join A(3), so start at 4 when A ends

C (2) after A: D(3) + C(2) = 5, start C at 5 after E ends

G (2) after E and H (1) after A: wait until D ends at 8

F (2) after C and D: C ends at 10, start F at 10

0 to 3: A(3) + B(2) = 5

3 to 4: A(3) + E(1) = 4

4 to 5: D(3) + E(1) = 4

5 to 8: C(2) + D(3) = 5

8 to 10: C(2) + G(2) + H(1) = 5

10 to 13: F(2) + G(2) + H(1) = 5

13 to 14: G(2) + H(1) = 3

Last activities G, H and F finish at 14

Start times: A 0, B 0, C 5, D 4, E 3, F 10, G 8, H 8; finished in 14 hours

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12 A house extension in Birmingham is being built. The activity network below shows the activities, with durations in days in brackets. Each activity requires one worker and the project is to be completed in the shortest possible time.

a. Complete the diagram to show the early event time and the late event time at each event.

b. On Grid 1, draw a cascade (Gantt) chart for the project, with the critical activities on the top row.

c. Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time.

d. On Grid 2, schedule the activities using the number of workers found in part (c) so that the project is completed in the shortest possible time.

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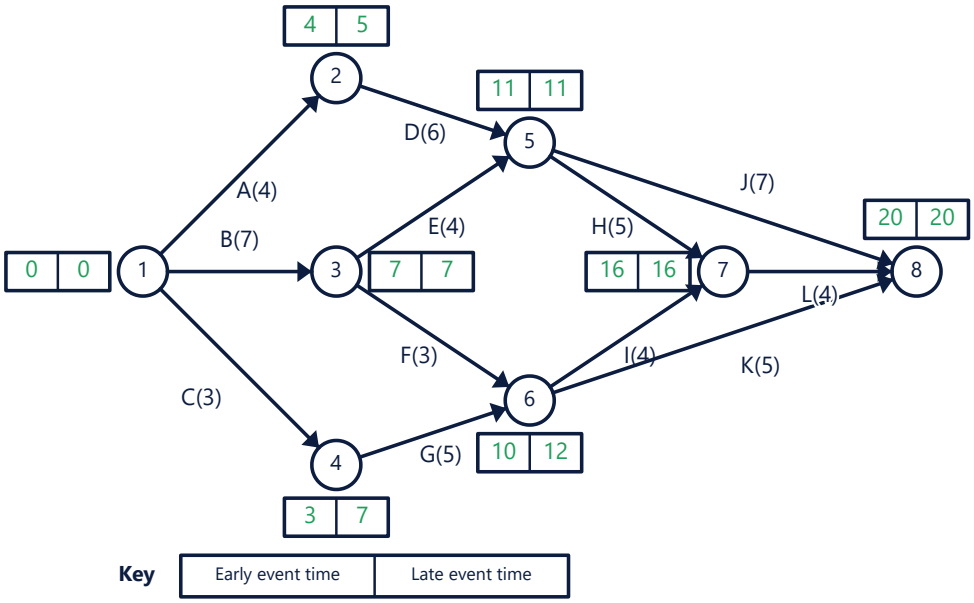
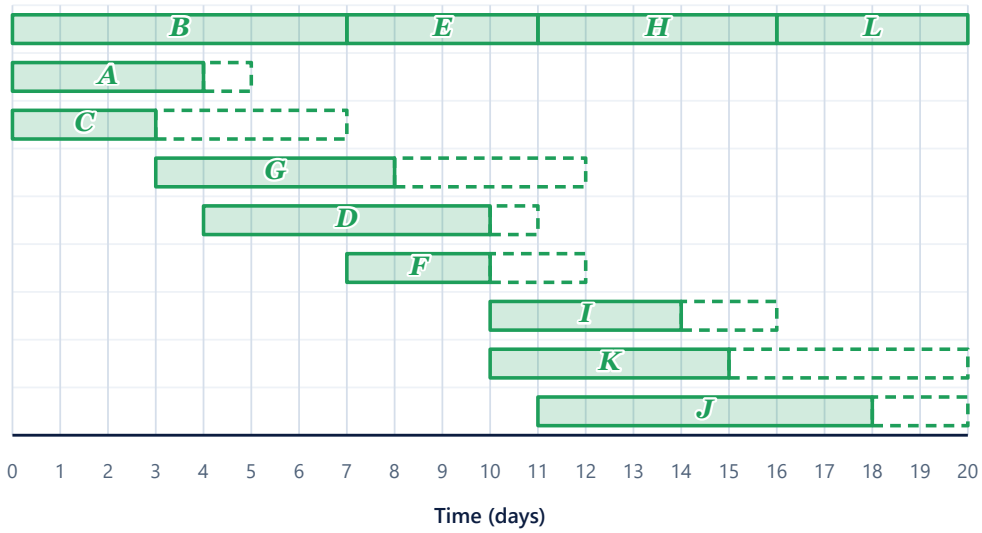


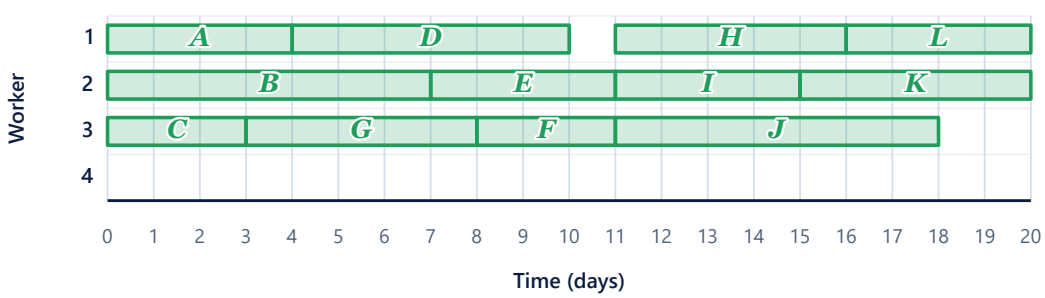
Diagram NOT accurately drawn

Activity network

(12 marks)



Grid 1



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Grid 2

a)

Early 1 = 0

Forward pass: early time = largest (early time at start + duration) over arcs in

Early 2 = 0 + 4 = 4

Early 3 = 0 + 7 = 7

Early 4 = 0 + 3 = 3

Early 5 = max(4 + 6, 7 + 4) = 11

Early 6 = max(7 + 3, 3 + 5) = 10

Early 7 = max(11 + 5, 10 + 4) = 16

Early 8 = max(11 + 7, 10 + 5, 16 + 4) = 20

Backward pass: late time = smallest (late time at end - duration) over arcs out

Late 8 = early 8 = 20

Late 7 = 20 - 4 = 16

Late 6 = min(16 - 4, 20 - 5) = 12

Late 5 = min(16 - 5, 20 - 7) = 11

Late 4 = 12 - 5 = 7

Late 3 = min(11 - 4, 12 - 3) = 7

Late 2 = 11 - 6 = 5

Late 1 = min(5 - 4, 7 - 7, 7 - 3) = 0

1: (0, 0); 2: (4, 5); 3: (7, 7); 4: (3, 7); 5: (11, 11); 6: (10, 12); 7: (16, 16); 8: (20, 20)

b)

Total float = late time at end - duration - early time at start

Float A = 5 - 4 - 0 = 1

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Float C = 7 − 3 − 0 = 4

Float D = 11 − 6 − 4 = 1

Float F = 12 − 3 − 7 = 2

Float G = 12 − 5 − 3 = 4

Float I = 16 − 4 − 10 = 2

Float J = 20 − 7 − 11 = 2

Float K = 20 − 5 − 10 = 5

Critical activities on the top row; each other bar starts at its early start time, float dashed after it

B: 0 to 7, critical

E: 7 to 11, critical

H: 11 to 16, critical

L: 16 to 20, critical

A: 0 to 4, float to 5

C: 0 to 3, float to 7

G: 3 to 8, float to 12

D: 4 to 10, float to 11

F: 7 to 10, float to 12

I: 10 to 14, float to 16

K: 10 to 15, float to 20

J: 11 to 18, float to 20

Cascade chart drawn on Grid 1

c)

Lower bound = sum of all activity times ÷ critical path length, rounded up

Sum = 4 + 7 + 3 + 6 + 4 + 3 + 5 + 5 + 4 + 7 + 5 + 4 = 57

57 ÷ 20 = 2.85

Round up: 3 workers

3 workers

d)

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When a worker becomes free, start the available activity with the smallest latest finish time (critical first)

Time 0: A to worker 1, B to worker 2, C to worker 3

Time 3: G to worker 3

Time 4: D to worker 1

Time 7: E to worker 2 (available E, F)

Time 8: F to worker 3

Time 10: worker 1 is free, but H, I, J and K all wait for E or F (end 11), so worker 1 waits

Time 11: H to worker 1, I to worker 2, J to worker 3 (available H, I, J, K)

Time 15: K to worker 2

Time 16: L to worker 1

Worker 1: A (0 to 4), D (4 to 10), H (11 to 16), L (16 to 20)

Worker 2: B (0 to 7), E (7 to 11), I (11 to 15), K (15 to 20)

Worker 3: C (0 to 3), G (3 to 8), F (8 to 11), J (11 to 18)

Schedule drawn on Grid 2: finished in 20 days with 3 workers

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