The simplex algorithm

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** 3x + 2y + z + s1 = 60, x + 4y + 2z + s2 = 48, 2x + y + 3z + s3 = 45 (s1, s2, s3 ≥ 0) **b)** P - 4x - 5y - 3z = 0 **c)** x = y = z = 0, s1 = 60, s2 = 48, s3 = 45, P = 0: the origin	4
2	**a)** s1: 1, 1, 1, 0, 0 | 12; s2: 3, 2, 0, 1, 0 | 30; s3: 1, 3, 0, 0, 1 | 27; P: -6, -5, 0, 0, 0 | 0 **b)** x column (most negative, -6); θ values 12, 10, 27, so the pivot is the 3 in the s2 row	5
3	**a)** z has the most negative entry (-5) in the objective row **b)** z: 1/2, 1, 1, 1/2, 0, 0 | 20 (R1 ÷ 2); s2: 2, -1, 0, -1, 1, 0 | 5 (R2 - 2R1); s3: 3/2, 1, 0, -1/2, 0, 1 | 30 (R3 - R1); P: -3/2, 2, 0, 5/2, 0, 0 | 100 (R4 + 5R1) **c)** Not optimal: -3/2 in the x column of the objective row	6
4	**a)** s1: 1, 1, 1, 0, 0 | 10; s2: 3, 1, 0, 1, 0 | 24; s3: 1, 2, 0, 0, 1 | 16; P: -5, -4, 0, 0, 0 | 0 **b)** Two iterations: x enters (pivot 3 in R2), then y enters (pivot 2/3 in R1) **c)** No negative values in the objective row; x = 7, y = 3, P = 47, s1 = 0, s2 = 0, s3 = 3	10
5	**a)** There are no negative values in the profit (objective) row **b)** x = 40, y = 15, z = 0; profit £260 **c)** s1 = 0 and s2 = 0: all mixing and oven time used; s3 = 30: 30 minutes of shaping time unused **d)** P = 260 - 1/2 z - s1 - 3/2 s2: every batch of spelt would reduce P by £0.50	7
6	**a)** First iteration: y enters, pivot 3 in R1; second iteration: z enters, pivot 5/3 in R2 **b)** No negative values in the profit row; x = 0, y = 52/5, z = 24/5, P = 356/5	8
7	**a)** The y column, as y is now a basic variable **b)** Maximise P = 6x + 8y + 2z subject to 2x + 4y + 4z ≤ 100, x + 4y + z ≤ 80, x + y + z ≤ 95, x, y, z ≥ 0 **c)** x = 20, y = 15, s3 = 60, P = 240 **d)** Not optimal: -2 in the s2 column in the objective row	10
8	**a)** x = 4, y = 6, P = 4 + 6k **b)** 1/3 ≤ k ≤ 2 **c)** s2 column, since (2 - k)/5 = -1/5 is the only negative entry; only the x row has a positive entry (θ = 4 ÷ 2/5 = 10), so the pivot is the 2/5 in the x row	8
9	**a)** Maximise P = -C = -2x + 3y + 4z **b)** s1: 2, 1, 1, 1, 0, 0 | 30; s2: 3, 1, 2, 0, 1, 0 | 75; s3: 2, 2, 3, 0, 0, 1 | 80; P: 2, -3, -4, 0, 0, 0 | 0 **c)** x = 0, y = 10, z = 20, C = -110	9

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10	**a)** Maximise P = 9x + 5y + 6z subject to 2x + y + z ≤ 65, 2x + 3y + z ≤ 80, 3x + 2y + 2z ≤ 110, x, y, z ≥ 0 **b)** Two iterations: x enters (pivot 2 in R1), then z enters (pivot 1/2 in R3); x = 20, z = 25, s2 = 15, P = 330 **c)** Make 20 chairs, no stools and 25 benches for a profit of £330; cutting and finishing time are used up, 15 hours of sanding time are spare	12
11	**a)** The lower bounds x ≥ 2, y ≥ 3, z ≥ 1 mean the origin is not feasible, so there is no starting basic feasible solution **b)** Maximise Q = 3X + 2Y + 4Z subject to 2X + Y + Z ≤ 45, X + 3Y + Z ≤ 25, X + 2Y + 2Z ≤ 40, X, Y, Z ≥ 0 **c)** X = 10, Y = 0, Z = 15, Q = 90 **d)** x = 12, y = 3, z = 16, P = 106	12
12	**a)** y enters, pivot 3 in R1: y = 10, s2 = 2, s3 = 8, P = 60 **b)** No negative values in the objective row **c)** The non-basic variable x has a zero in the objective row, so it can enter without changing P **d)** Pivot 1/3 in R2 (θ = 15, 6, 8): x = 6, y = 6, s3 = 2, P = 60 **e)** All points on 2x + 3y = 30 between (0, 10) and (6, 6), with P = 60	11

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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