The simplex algorithm

A-level Decision Mathematics 1 | Edexcel 9FM0 | 12 questions, 102 marks

Further Maths topic practice

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1 A linear programming problem is given as

Maximise $P = 4x + 5y + 3z$

subject to

$3x + 2y + z \leq 60$

$x + 4y + 2z \leq 48$

$2x + y + 3z \leq 45$

$x, y, z \geq 0$

The problem is to be solved using the Simplex algorithm.

a. Using slack variables s_1, s_2 and s_3, write the three constraints as equations.

b. Write the objective function in the form used for the objective row of a Simplex tableau.

c. State the values of x, y, z, s_1, s_2, s_3 and P at the start of the algorithm, and the vertex of the feasible region this represents.

(4 marks)

Answer _____

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2 A linear programming problem is given as

Maximise $P = 6x + 5y$

subject to

$x + y \leq 12$

$3x + 2y \leq 30$

$x + 3y \leq 27$

$x, y \geq 0$

a. Using slack variables s_1, s_2 and s_3, set up the initial tableau for solving this problem using the Simplex algorithm. Use the template below.

b. Identify the pivot for the first iteration. You must give reasons for your choice of column and row, showing your θ values.

Initial tableau

b.v.	x	y	s_1	s_2	s_3	Value

(5 marks)

Answer

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3 The initial tableau for a linear programming problem in x, y and z is shown below. The objective is to maximise P.

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	1	2	2	1	0	0	40
s_2	3	1	2	0	1	0	45
s_3	2	2	1	0	0	1	50
P	−4	−3	−5	0	0	0	0

a. Explain why the z column is chosen as the pivot column for the first iteration.

b. Perform one iteration of the Simplex algorithm. You should show your θ values and state the row operations you use.

c. Explain whether the tableau you obtained in part (b) is optimal.

Tableau after 1st iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

(6 marks)

Answer

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4 A linear programming problem is given as

Maximise $P = 5x + 4y$

subject to

$x + y \leq 10$
$3x + y \leq 24$
$x + 2y \leq 16$
$x, y \geq 0$

a. Using slack variables, set up the initial tableau for solving this problem with the Simplex algorithm.

b. Solve the problem using the Simplex algorithm. Take the most negative number in the profit row to indicate the pivot column each time, and state the row operations you use.

c. Explain how you know your final tableau is optimal, and state the optimal values of x, y, P and the slack variables.

Use the templates below.

Initial tableau

b.v.	x	y	s_1	s_2	s_3	Value	Row ops

Tableau after 1st iteration

b.v.	x	y	s_1	s_2	s_3	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	s_1	s_2	s_3	Value	Row ops

(10 marks)

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Answer

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5 A bakery makes loaves of three types: wholemeal, rye and spelt, in batches. It makes x batches of wholemeal, y batches of rye and z batches of spelt each day. The profit, P, in pounds, is to be maximised, where $P = 5x + 4y + 3z$.

The constraints are

mixing time (minutes): $2x + y + 2z \leq 95$
oven time (minutes): $2x + 2y + z \leq 110$
shaping time (minutes): $x + 3y + 3z \leq 115$

where s_1, s_2 and s_3 are the slack variables for these constraints. After applying the Simplex algorithm the tableau below is obtained.

b.v.	x	y	z	s_1	s_2	s_3	**Value**
x	1	0	$\frac{3}{2}$	1	$-\frac{1}{2}$	0	40
y	0	1	-1	-1	1	0	15
s_3	0	0	$\frac{9}{2}$	2	$-\frac{5}{2}$	1	30
P	0	0	$\frac{1}{2}$	1	$\frac{3}{2}$	0	260

a. Explain how you know that this tableau is optimal.

b. State the number of batches of each type the bakery should make and the maximum daily profit.

c. State the values of the slack variables and explain what they mean in context.

d. Write down the equation represented by the profit row and use it to explain why making any batches of spelt would reduce the profit.

(7 marks)

Answer

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6 The initial tableau for a linear programming problem in x, y and z is shown below. The objective is to maximise P.

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	2	3	1	1	0	0	36
s_2	1	1	2	0	1	0	20
s_3	3	2	4	0	0	1	48
P	−3	−5	−4	0	0	0	0

a. Solve the problem using the Simplex algorithm, taking the most negative number in the profit row to indicate the pivot column at each stage. You must show your θ values and state the row operations you use. Use the templates below.

b. Explain how you know your final tableau is optimal and state the optimal values of x, y, z and P.

Tableau after 1st iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

(8 marks)

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Answer

7 A maximisation linear programming problem in x, y and z is being solved using the Simplex algorithm. The slack variables are s_1, s_2 and s_3. The tableau after the first iteration is shown below.

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	1	0	3	1	−1	0	20
y	$\frac{1}{4}$	1	$\frac{1}{4}$	0	$\frac{1}{4}$	0	20
s_3	$\frac{3}{4}$	0	$\frac{3}{4}$	0	$-\frac{1}{4}$	1	75
P	−4	0	0	0	2	0	160

a. State the column that contained the pivot for the first iteration. You must give a reason.

b. By considering the row operations that were used, formulate the original linear programming problem. State the objective and list the constraints as inequalities with integer coefficients.

c. Perform the second iteration, showing your θ values and the row operations you use.

d. State whether the tableau you obtained in part (c) is optimal, giving a reason.

Tableau after 2nd iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

(10 marks)

Answer

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8 A linear programming problem is given as

Maximise $P = x + ky$

subject to

$x + 2y \leq 16$
$3x + y \leq 18$
$x - y \leq 4$
$x, y \geq 0$

where k is a positive constant. After two iterations of the Simplex algorithm a possible tableau is

b.v.	x	y	s_1	s_2	s_3	Value
y	0	1	$\frac{3}{5}$	$-\frac{1}{5}$	0	6
x	1	0	$-\frac{1}{5}$	$\frac{2}{5}$	0	4
s_3	0	0	$\frac{4}{5}$	$-\frac{3}{5}$	1	6
P	0	0	$\frac{-1+3k}{5}$	$\frac{2-k}{5}$	0	$4 + 6k$

a. State the values of x, y and P given by this tableau, in terms of k where necessary.

b. Find the range of values of k for which this tableau is optimal.

c. Given that $k = 3$, identify the pivot for the next iteration. You must give reasons for your answer.

(8 marks)

Answer _____

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9 A linear programming problem is given as

Minimise $C = 2x - 3y - 4z$

subject to

$2x + y + z \leq 30$

$3x + y + 2z \leq 75$

$2x + 2y + 3z \leq 80$

$x, y, z \geq 0$

a. Explain how this problem can be rewritten as a maximisation problem, stating the new objective.

b. Set up the initial tableau for solving the problem with the Simplex algorithm, using slack variables s_1, s_2 and s_3.

c. Solve the problem, taking the most negative number in the objective row to indicate the pivot column each time. State the row operations you use and the optimal values of x, y, z and C.

Use the templates below.

Initial tableau

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

Tableau after 1st iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

(9 marks)

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Answer

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10 A workshop makes garden chairs, stools and benches. Each item passes through three processes. The time, in hours, each item needs for each process and the time available each week are shown in the table.

	Chair	Stool	Bench	Hours available
Cutting	2	1	1	65
Sanding	2	3	1	80
Finishing	3	2	2	110

The profit is £9 on each chair, £5 on each stool and £6 on each bench. The workshop wants to maximise its weekly profit, £P, by making x chairs, y stools and z benches.

a. Formulate this as a linear programming problem.

b. Using slack variables s_1, s_2 and s_3, set up the initial tableau and solve the problem using the Simplex algorithm, taking the most negative number in the profit row to indicate the pivot column each time. State the row operations you use. Use the templates below.

c. Interpret your final tableau in context, including the value of any slack variable that is not zero.

Initial tableau

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

Tableau after 1st iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

(12 marks)

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Answer

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11 Consider the linear programming problem

Maximise $P = 3x + 2y + 4z$

subject to

$2x + y + z \leq 53$

$x + 3y + z \leq 37$

$x + 2y + 2z \leq 50$

$x \geq 2, y \geq 3, z \geq 1$

a. Explain why the Simplex algorithm cannot be used to solve this problem in its current form.

b. Use the substitutions $x = X + 2$, $y = Y + 3$, $z = Z + 1$ and $P = Q + 16$ to reformulate the problem in X, Y and Z so that the Simplex algorithm can be used. Write the constraints as simplified inequalities with integer coefficients.

c. Using slack variables s_1, s_2 and s_3, solve the reformulated problem with the Simplex algorithm, taking the most negative number in the objective row each time. State the row operations you use. Use the templates below.

d. Hence state the optimal values of x, y, z and P.

Initial tableau

b.v.	X	Y	Z	s_1	s_2	s_3	Value	Row ops

Tableau after 1st iteration

b.v.	X	Y	Z	s_1	s_2	s_3	Value	Row ops

Tableau after 2nd iteration

b.v.	X	Y	Z	s_1	s_2	s_3	Value	Row ops

(12 marks)

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Answer

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12 A linear programming problem is given as

Maximise $P = 4x + 6y$

subject to

$2x + 3y \leq 30$

$x + y \leq 12$

$x \leq 8$

$x, y \geq 0$

a. Using slack variables s_1, s_2 and s_3, set up the initial tableau and perform one iteration of the Simplex algorithm, taking the most negative number in the objective row to indicate the pivot column. State the row operations you use.

b. Explain why the tableau you obtained in part (a) is optimal.

c. Explain how the tableau shows that the optimal solution is not unique.

d. Perform a further iteration, with x entering the basis, to find a second optimal vertex.

e. Describe the complete set of optimal solutions.

Use the templates below for parts (a) and (d).

Initial tableau

b.v.	x	y	s_1	s_2	s_3	Value	Row ops

Tableau after 1st iteration

b.v.	x	y	s_1	s_2	s_3	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	s_1	s_2	s_3	Value	Row ops

(11 marks)

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Answer

End of questions | Total 102 marks

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