Travelling salesman: lower bounds

A-level Decision Mathematics 1 | Edexcel 9FM0 | 12 questions, 78 marks

Further Maths topic practice

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1 A travelling salesman problem has seven vertices, A to G. Vertex D and all of its arcs are deleted, and a minimum spanning tree for the remaining six vertices has weight 74

The arcs from D have weights 23, 18, 31, 40, 27 and 35

Find a lower bound for the length of the optimal tour.

(2 marks)

Answer _____

2 The table shows the least distances, in km, between five bus garages, A to E, in Bristol.

	A	**B**	**C**	**D**	**E**
A	–	29	13	48	18
B	29	–	20	70	47
C	13	20	–	56	31
D	48	70	56	–	55
E	18	47	31	55	–

By deleting E and all of its arcs, find a lower bound for the length of a tour of the five garages. Complete the table below.

Deleted vertex method

Deleted vertex	**RMST weight**	**Two least arcs**	**Lower bound**
E			

(3 marks)

Answer _____

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3 The deleted vertex method is used to find lower bounds for a travelling salesman problem.

a. Explain why the method gives a lower bound for the length of the optimal tour.

Deleting different vertices gives lower bounds of 147, 152 and 139. The nearest neighbour algorithm from two vertices gives upper bounds of 186 and 179

b. Write down the smallest interval that must contain the length, L, of the optimal tour, giving reasons for your choice of each end.

(4 marks)

Answer

4 The table below shows the least distances, in miles, between six market towns, A to F, in Norfolk.

a. By deleting A and all of its arcs, use Prim's algorithm, starting at B, to find a minimum spanning tree for the remaining five towns. State the order in which you select the arcs and the weight of the tree.

b. Hence find a lower bound for the length of a tour of all six towns.

	A	**B**	**C**	**D**	**E**	**F**
A	-	40	52	48	42	68
B	40	-	55	53	57	67
C	52	55	-	11	32	16
D	48	53	11	-	21	25
E	42	57	32	21	-	46
F	68	67	16	25	46	-

Deleted vertex method

Deleted vertex	**RMST weight**	**Two least arcs**	**Lower bound**
A			

(5 marks)

Answer

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5 The table shows the least distances, in km, between six offices, A to F, of a firm in Glasgow.

	A	**B**	**C**	**D**	**E**	**F**
A	–	19	58	45	51	31
B	19	–	39	40	32	18
C	58	39	–	42	60	36
D	45	40	42	–	71	22
E	51	32	60	71	–	50
F	31	18	36	22	50	–

a. By deleting A, find a lower bound for the length of a tour of the six offices.

b. By deleting E, find a second lower bound.

c. State the better lower bound, giving a reason.

Deleted vertex method

Deleted vertex	RMST weight	Two least arcs	Lower bound
A			
E			

(6 marks)

Answer

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6 The table shows the least times, in minutes, for a minibus to travel between six schools, A to F, in Swansea.

	A	B	C	D	E	F
A	–	19	30	41	11	46
B	19	–	49	35	25	29
C	30	49	–	65	37	72
D	41	35	65	–	31	51
E	11	25	37	31	–	48
F	46	29	72	51	48	–

a. Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of a tour of the schools.

b. By deleting F, find a lower bound for the length of the tour.

c. Hence write down an interval within which the length, T minutes, of the optimal tour must lie.

Upper bound

Start	Tour	Length
A		

Lower bound

Deleted vertex	RMST weight	Two least arcs	Lower bound
F			

(6 marks)

Answer _____

7 The table below shows the least distances, in km, between six lighthouses, A to F, on the coast of Cornwall. An engineer must inspect every lighthouse and return to her start.

a. By deleting F, and using Prim's algorithm starting at A, find a lower bound for the length of her tour.

b. Show that the arcs you used in (a) form a tour, and state this tour.

c. Explain what you can deduce about this tour.

	A	B	C	D	E	F
A	-	43	16	33	52	60
B	43	-	41	27	13	58
C	16	41	-	31	48	61
D	33	27	31	-	23	67
E	52	13	48	23	-	65
F	60	58	61	67	65	-

Deleted vertex method

Deleted vertex	RMST weight	Two least arcs	Lower bound
F			

(7 marks)

Answer

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8 The table shows the least distances, in miles, between six garden centres, A to F, in Hampshire. The least distance between A and C is x miles, where $52 < x < 65$

	A	B	C	D	E	F
A	–	54	x	14	47	55
B	54	–	72	53	18	12
C	x	72	–	65	74	84
D	14	53	65	–	35	41
E	47	18	74	35	–	10
F	55	12	84	41	10	–

a. By deleting C, find a lower bound, in terms of x, for the length of a tour of the garden centres.

b. By deleting B, find a second lower bound in terms of x

c. State which of the two lower bounds is better, giving a reason.

d. The tour A - C - B - F - E - D - A has length $(143 + x)$ miles. Write down an interval, in terms of x, that must contain the length, L, of the optimal tour.

Deleted vertex method

Deleted vertex	RMST weight	Two least arcs	Lower bound
C			
B			

(7 marks)

Answer _____

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9 The table shows the least distances, in km, between seven campsites, A to G, in the Lake District. A warden visits every campsite, starting and finishing at A.

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	–	32	71	22	77	61	48
B	32	–	39	30	45	34	20
C	71	39	–	63	8	12	23
D	22	30	63	–	62	51	40
E	77	45	8	62	–	18	31
F	61	34	12	51	18	–	16
G	48	20	23	40	31	16	–

a. Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of the route.

b. By deleting A, find a lower bound for the length of the route.

c. By deleting B, find a second lower bound.

d. Using your answers, write down the smallest interval that must contain the length, L km, of the optimal route.

Upper bound

Start	**Tour**	**Length**
A		

Lower bounds

Deleted vertex	**RMST weight**	**Two least arcs**	**Lower bound**
A			
B			

(9 marks)

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Answer

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10 The network below shows the cycle paths between six stations, A to F, in Cambridge. The numbers on the arcs are lengths in hundreds of metres.

a. Complete the table of least distances below.

b. By deleting A, find a lower bound for the length of a tour of the six stations.

c. By deleting C, find a second lower bound.

d. State the better lower bound, giving a reason.

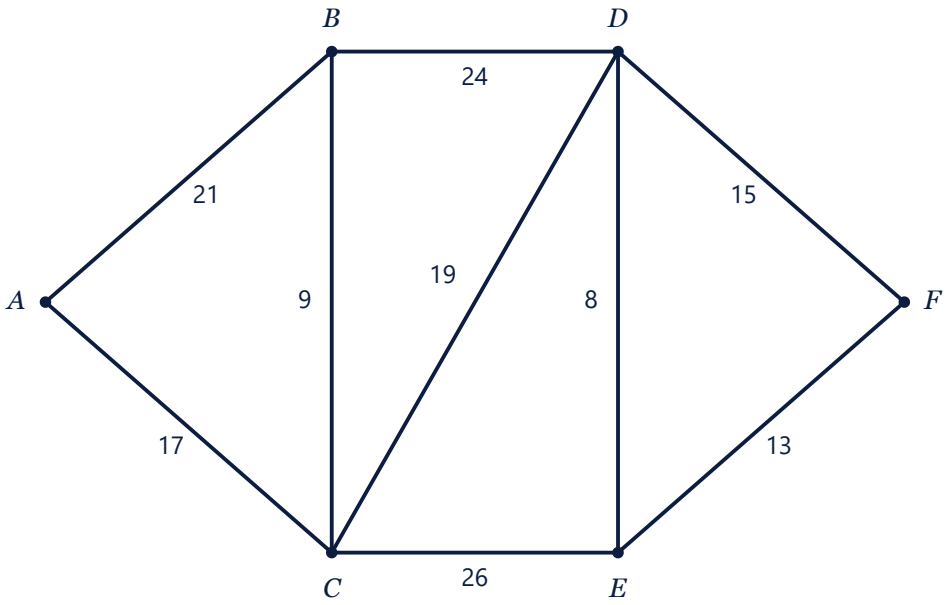


Table of least distances

	A	B	C	D	E	F
A	-	21	17		43	
B	21	-	9	24	32	
C	17	9	-	19	26	
D		24	19	-	8	15
E	43	32	26	8	-	13
F				15	13	-

Lower bounds

Deleted vertex	RMST weight	Two least arcs	Lower bound
A			
C			

Diagram NOT accurately drawn

(8 marks)

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Answer

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11 The table shows the least distances, in km, between six farms, A to F, in Devon. The least distance between B and D is x km, where $36 < x < 49$

	A	B	C	D	E	F
A	–	8	21	44	17	19
B	8	–	13	x	14	11
C	21	13	–	49	27	16
D	44	x	49	–	35	47
E	17	14	27	35	–	25
F	19	11	16	47	25	–

a. By deleting A, find a lower bound for the length of a tour of the farms.

b. By deleting D, find a lower bound in terms of x when $x < 44$, and the lower bound when $x \geq 44$

c. Use the nearest neighbour algorithm, starting at A, to find an upper bound.

The better of the lower bounds in (a) and (b) is 120 km.

d. Find the value of x and write down an interval that must contain the length, L km, of the optimal tour.

Lower bounds

Deleted vertex	RMST weight	Two least arcs	Lower bound
A			
D (x < 44)			
D (x ≥ 44)			

Upper bound

Start	Tour	Length
A		

(10 marks)

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Answer

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12 A heritage inspector must visit eight castles in Wales, A to H, starting and finishing at A. The table below shows the least driving distances between them, in miles.

a. By deleting B, and using Prim's algorithm starting at A, find a lower bound for the length of the inspector's route. State the order in which you select the arcs.

Deleting A gives a lower bound of 177 miles and deleting C gives a lower bound of 188 miles.

b. State the best lower bound, giving a reason.

c. Use the nearest neighbour algorithm, starting at A, to find an upper bound.

d. Write down the smallest interval that must contain the length, L miles, of the optimal route.

e. A colleague claims to have found a route of 198 miles. Explain why the colleague must be wrong.

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
A	-	43	34	13	14	50	33	45
B	43	-	64	56	51	11	48	68
C	34	64	-	21	39	70	67	76
D	13	56	21	-	18	59	46	55
E	14	51	39	18	-	57	32	37
F	50	11	70	59	57	-	58	66
G	33	48	67	46	32	58	-	20
H	45	68	76	55	37	66	20	-

Lower bounds

Deleted vertex	**RMST weight**	**Two least arcs**	**Lower bound**
B			
A			177
C			188

Upper bound

Start	**Tour**	**Length**
A		

(11 marks)

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Answer

End of questions | Total 78 marks

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