Travelling salesman: upper bounds

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** Classical: each vertex must be visited exactly once before returning to the start. Practical: each vertex must be visited at least once before returning to the start. **b)** The optimal route is the shortest of all possible tours, so any particular tour is at least as long as it: its length is an upper bound.	3
2	A - E - C - D - B - A, 137 miles	3
3	**a)** AD = 28, AE = 28, BE = 20, CD = 19 **b)** A - C - B - D - E - A, 65 km	4
4	**a)** A - B - D - F - C - E - A, 209 minutes **b)** D - B - A - F - C - E - D, 204 minutes **c)** 204 minutes, as it is the smaller	5
5	**a)** A, B, C, E, D, F reached, but there is no arc from F back to A (every neighbour of F is visited), so the algorithm cannot complete a tour. **b)** A - B - C - E - D - F - A, 76 (7600 m) **c)** A - B - C - E - D - F - D - B - A	7
6	**a)** AB, BC, BE, AD, DF; weight 103 km **b)** 206 km **c)** e.g. A - D - F - C - B - E - A, 155 km	7
7	**a)** $20 \leq x < 28$ **b)** A - D - F - E - B - C - A, 208 + x **c)** A - D - E - B - C - F - A, 253 miles	6
8	**a)** C - D - F - E - B - A - C, 188 km **b)** 170 km **c)** 170 km; it is an upper bound only, so the optimal tour could still be shorter (a lower bound of 170 would be needed to show it is optimal).	6
9	**a)** AG = 48, BG = 37, CE = 27, DG = 28 **b)** A - C - D - B - F - G - E - A, 123 km **c)** A - C - D - B - D - F - G - E - D - A **d)** 110 km; 110 km is better as it is smaller	9
10	**a)** A: A - C - D - B - E - A 150; B: B - C - A - D - E - B 160; C: C - A - D - B - E - C 155; D: D - C - A - B - E - D 154; E: E - C - A - D - B - E 155 **b)** 150 minutes **c)** A tour is a cycle through every vertex, so the same cycle can be started at any vertex with the same length.	7
11	**a)** A - C - F - B - D - E - A, 212 + x **b)** A - C - F - B - E - D - A, 222 km **c)** $25 < x < 28$ or $35 < x < 63$	8
12	**a)** AD, AF, FG, GB, BC, CE; weight 97 minutes **b)** 194 minutes **c)** e.g. A - F - G - B - C - E - D - A, 128 minutes **d)** A - D - F - G - B - C - E - A, 146 minutes **e)** 128 minutes, the smallest upper bound	11

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Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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