Travelling salesman: upper bounds

A-level Decision Mathematics 1 | Edexcel 9FM0 | 12 questions, 76 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 76**

1 A van driver must deliver to six shops in Cardiff, starting and finishing at the depot, and wants to keep the total distance as small as possible.

a. Explain the difference between the classical and the practical travelling salesman problems.

b. Explain why the length of any tour that visits every shop and returns to the depot is an upper bound for the length of the optimal route.

(3 marks)

Answer _____

2 The table shows the least distances, in miles, between five villages in Cumbria, A, B, C, D and E. A district nurse starts and finishes at A and visits every village.

	A	**B**	**C**	**D**	**E**
A	–	27	31	33	23
B	27	–	48	58	37
C	31	48	–	16	13
D	33	58	16	–	21
E	23	37	13	21	–

Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of her route. Write your tour and its length in the table below.

Nearest neighbour

Start	**Tour**	**Length**
A		

(3 marks)

Answer _____

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3 The network below shows the direct roads between five farms, A to E. The number on each arc is the length of the road in km.

a. Complete the table of least distances below.

b. Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of a tour that visits every farm and returns to A.

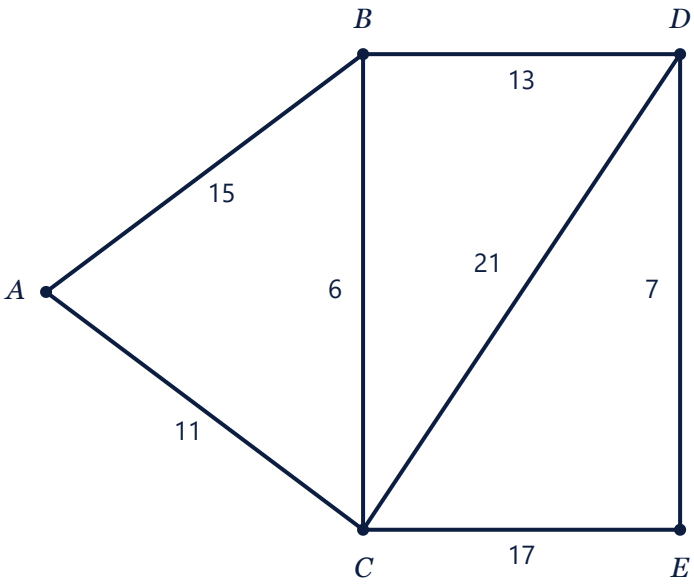


Table of least distances (km)

	A	B	C	D	E
A	-	15	11		
B	15	-	6	13	
C	11	6	-		17
D		13		-	7
E			17	7	-

Nearest neighbour

Start	Tour	Length
A		

Diagram NOT accurately drawn

(4 marks)

Answer _____

4 The table shows the least travel times, in minutes, between six depots, A to F, used by a parcel company in Kent.

	A	B	C	D	E	F
A	–	24	49	25	69	47
B	24	–	41	9	53	36
C	49	41	–	32	61	19
D	25	9	32	–	44	27
E	69	53	61	44	–	42
F	47	36	19	27	42	–

a. Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of a tour of the six depots.

b. Repeat the nearest neighbour algorithm, starting at D.

c. State the better upper bound, giving a reason for your answer.

Nearest neighbour tours

Start	Tour	Length
A		
D		

(5 marks)

Answer _____

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5 The network below shows the footpaths between six viewpoints, A to F, in a country park in Yorkshire. The numbers on the arcs are lengths in hundreds of metres. A ranger must visit every viewpoint, starting and finishing at A.

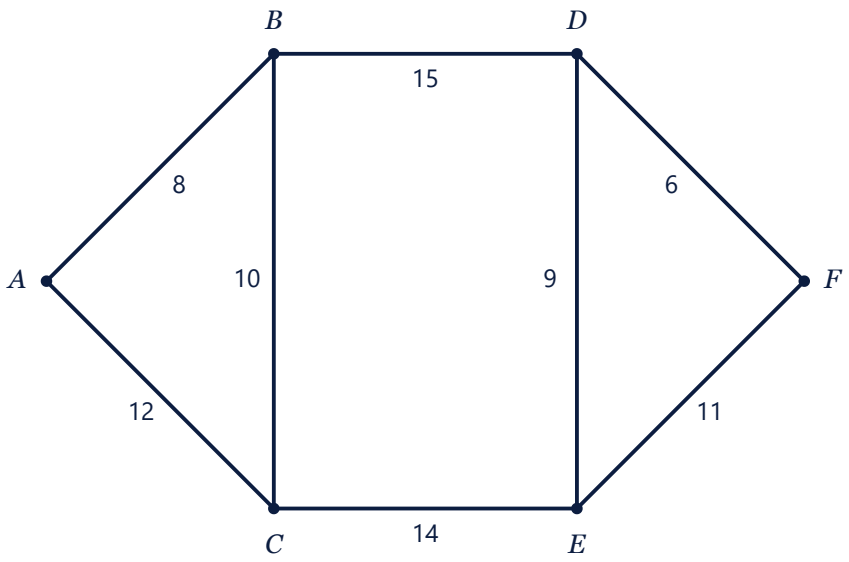
a. Apply the nearest neighbour algorithm, starting at A, directly to the network. Explain why it does not give a tour.

The table shows the least distances between the viewpoints.

	A	**B**	**C**	**D**	**E**	**F**
A	–	8	12	23	26	29
B	8	–	10	15	24	21
C	12	10	–	23	14	25
D	23	15	23	–	9	6
E	26	24	14	9	–	11
F	29	21	25	6	11	–

b. Use the nearest neighbour algorithm on the table, starting at A, to find an upper bound for the length of the ranger's route.

c. Write down the route the ranger actually walks in the network, listing every viewpoint passed through.



Answer

	Order of viewpoints	**Length**
Tour (b)		
Route (c)		

Diagram NOT accurately drawn

(7 marks)

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Answer

6 The table below shows the least distances, in km, between six sports centres, A to F, in a borough of London.

a. Use Prim's algorithm, starting at A, to find a minimum spanning tree for the table. You must state the order in which you select the arcs.

b. Hence find an initial upper bound for the length of a tour of the six centres.

c. Starting from the route that travels along every arc of the tree twice, use shortcuts to reduce the upper bound to below 160 km. State your tour and its length.

	A	B	C	D	E	F
A	-	22	31	40	28	54
B	22	-	9	49	11	51
C	31	9	-	58	14	46
D	40	49	58	-	60	21
E	28	11	14	60	-	52
F	54	51	46	21	52	-

Upper bounds

Method	Tour	Length
2 × MST		
Shortcuts		

(7 marks)

Answer

7 The table shows the least distances, in miles, between six towns, A to F, in Lincolnshire. The least distance between D and F is x miles, where $x \geq 20$

	A	**B**	**C**	**D**	**E**	**F**
A	–	63	49	31	38	46
B	63	–	56	53	25	72
C	49	56	–	48	40	67
D	31	53	48	–	28	x
E	38	25	40	28	–	47
F	46	72	67	x	47	–

The nearest neighbour algorithm, starting at A, is applied to the table. The tour begins A, D, F.

a. Find the range of possible values of x

b. Complete the tour and find, in terms of x, the upper bound it gives.

c. Given instead that $x = 35$, use the nearest neighbour algorithm, starting at A, to find an upper bound.

Nearest neighbour from A

Part	**Tour**	**Length**
(b)		
(c)		

(6 marks)

Answer

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8 The table shows the least distances, in km, between six branches of a bank in Northern Ireland, A to F. An auditor visits every branch and returns to her starting point.

	A	B	C	D	E	F
A	–	37	60	52	51	46
B	37	–	59	49	39	30
C	60	59	–	10	22	29
D	52	49	10	–	24	23
E	51	39	22	24	–	19
F	46	30	29	23	19	–

a. Use the nearest neighbour algorithm, starting at C, to find an upper bound for the length of her tour.

Kofi suggests the tour A - B - F - E - C - D - A.

b. Find the length of Kofi's tour.

c. State the best upper bound you now have, and explain whether you can say that it is the length of the optimal tour.

Tours

Start	Tour	Length
C (NN)		
Kofi	A - B - F - E - C - D - A	

(6 marks)

Answer _____

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9 The network below shows the roads between seven villages, A to G, on Anglesey. The numbers on the arcs are distances in km. A mobile library must visit every village, starting and finishing at A.

a. Complete the table of least distances below.

b. Use the nearest neighbour algorithm on the completed table, starting at A, to find an upper bound for the length of the route.

c. Write down the route in the network that the library van actually drives for your tour in (b).

The nearest neighbour algorithm starting at G gives the tour G - E - D - B - A - C - F - G.

d. Find the length of this tour and state, with a reason, which upper bound is better.

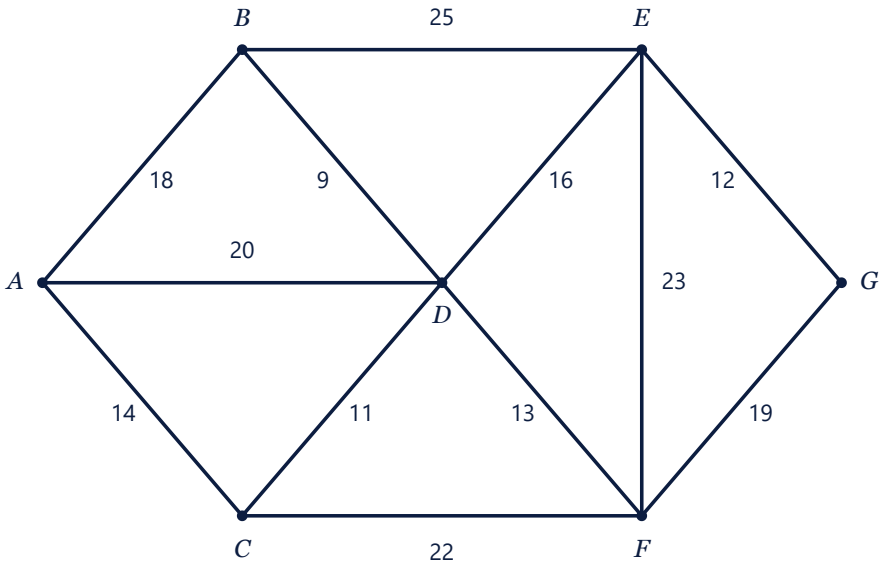


Table of least distances (km)

	A	B	C	D	E	F	G
A	-	18	14	20	36	33	
B	18	-	20	9	25	22	
C	14	20	-	11		22	39
D	20	9	11	-	16	13	
E	36	25		16	-	23	12
F	33	22	22	13	23	-	19
G			39		12	19	-

Answer

	Order of villages	Length
Tour (b)		
Route (c)		
From G (d)	G - E - D - B - A - C - F - G	

Diagram NOT accurately drawn

(9 marks)

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Answer

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10 The table shows the least travel times, in minutes, between five care homes, A to E, in Leicester. A visiting doctor plans a tour of all five homes, returning to the first.

	A	**B**	**C**	**D**	**E**
A	–	29	11	22	33
B	29	–	27	35	57
C	11	27	–	14	30
D	22	35	14	–	43
E	33	57	30	43	–

a. Use the nearest neighbour algorithm, starting at each of the five homes in turn, to find five tours. State the length of each tour.

b. Hence state the best upper bound for the length of the optimal tour.

c. Explain why the doctor may start her tour at any of the homes.

Nearest neighbour tours

Start	**Tour**	**Length**
A		
B		
C		
D		
E		

(7 marks)

Answer _____

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11 The table shows the least distances, in km, between six warehouses, A to F, near Birmingham. The least distance between B and D is x km, where $25 < x < 63$

	A	B	C	D	E	F
A	–	54	50	60	85	59
B	54	–	20	x	35	26
C	50	20	–	44	40	9
D	60	x	44	–	42	47
E	85	35	40	42	–	49
F	59	26	9	47	49	–

a. Given that $x < 35$, use the nearest neighbour algorithm, starting at A, to find an upper bound, in terms of x, for the length of a tour of the warehouses.

b. Given instead that $x > 35$, use the nearest neighbour algorithm, starting at A, to find an upper bound.

c. Find the set of values of x for which the nearest neighbour algorithm, starting at A, gives an upper bound of less than 240 km.

Nearest neighbour from A

Case	Tour	Length
x < 35		
x > 35		

(8 marks)

Answer

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12 A tour guide in Edinburgh plans a walking tour of seven sites, A to G, starting and finishing at the hotel at A. The table below shows the least walking times, in minutes, between the sites.

a. Use Prim's algorithm, starting at A, to find a minimum spanning tree. You must state the order in which you select the arcs, and its weight.

b. Hence find an initial upper bound for the length of the tour.

c. Use a shortcut to reduce this upper bound to below 140 minutes. State the tour you use.

d. Use the nearest neighbour algorithm, starting at A, to find a second upper bound.

e. State, with a reason, the best upper bound you have found.

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	-	37	52	12	43	21	29
B	37	-	18	34	24	32	10
C	52	18	-	40	14	50	28
D	12	34	40	-	31	27	30
E	43	24	14	31	-	42	33
F	21	32	50	27	42	-	22
G	29	10	28	30	33	22	-

Upper bounds

Method	**Tour**	**Length**
2 × MST		
Shortcut		
NN from A		

(11 marks)

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Answer

End of questions | Total 76 marks

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