Two-stage simplex and big-M

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** The ≥ constraint x + y ≥ 6 means the origin is not in the feasible region, so the algorithm has no basic feasible solution to start from **b)** x + 2y + s1 = 20, x + y - s2 + a1 = 6; s1 slack, s2 surplus, a1 artificial **c)** With x = y = 0 the second equation would need s2 = -6, which is not allowed; a1 = 6 gives a starting basic feasible solution	4
2	M is an arbitrarily large positive real number; P = 2x + 3y + z - Ma1 with a1 = 8 - 2x - y + s2 gives the result	4
3	**a)** s1: 3, 1, 2, 1, 0, 0, 0, 0 | 40; a1: 1, 1, 1, 0, -1, 0, 1, 0 | 10; a2: 0, 1, -1, 0, 0, -1, 0, 1 | 2; P: -5 - M, -3 - 2M, -2, 0, M, M, 0, 0 | -12M **b)** y column (-3 - 2M is most negative); θ = 40, 10, 2 so the pivot is the 1 in the a2 row	7
4	**a)** x + 2y + z + s1 = 30, 2x + y - s2 + a1 = 10, x + y + z + a2 = 15 **b)** s1: 1, 2, 1, 1, 0, 0, 0 | 30; a1: 2, 1, 0, 0, -1, 1, 0 | 10; a2: 1, 1, 1, 0, 0, 0, 1 | 15; P: -4, -1, -3, 0, 0, 0, 0 | 0; A: -3, -2, -1, 0, 1, 0, 0 | -25	7
5	**a)** x + 2y + z ≤ 24, x - y + 2z ≥ 6, 3x + y ≥ 9, x, y, z ≥ 0; maximise P = 2x - y + 3z (or minimise -P = -2x + y - 3z) **b)** x column: -2 - 4M is the most negative (largest multiple of M); θ = 24, 6, 3, so the pivot is the 3 in the a2 row	7
6	**a)** A: -5, -2, -2, 0, 1, 1, 0, 0 | -28 **b)** Two iterations: x enters (pivot 3 in the a2 row), then s3 enters (pivot 2/3 in the a1 row); A = 0 **c)** x = 6, y = 0, z = 0, s1 = 11, s2 = 0, s3 = 2	9
7	**a)** s1: 1, 1, 1, 0, 0, 0 | 12; s2: 1, 2, 0, 1, 0, 0 | 14; a1: 0, 1, 0, 0, -1, 1 | 4; P: -6, -4 - M, 0, 0, M, 0 | -4M **b)** y enters first (pivot 1 in the a1 row), then x (pivot 1 in the s2 row); x = 6, y = 4, P = 52	10
8	**a)** The artificial variable a1 is basic and equal to 3, but must be 0 in a feasible solution **b)** x = 4, y = 0 **c)** y column (most negative entry); smallest θ is in the a1 row, so the pivot is 1 **d)** x = 4, y = 3, P = 24 **e)** Feasible: no artificial variable is basic, so both are zero	9
9	**a)** The value of A is 0, so the artificial variable is zero **b)** Remove the A row and a1 column; x enters (pivot 2 in R1), then z enters (pivot 1 in R2)	9

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	c) x = 9, y = 5, z = 9, P = 104	
10	**a)** Maximise P = -4x - 3y subject to 2x + y ≥ 10, x + 2y ≥ 11, x + y ≤ 12, x, y ≥ 0 **b)** a1: 2, 1, -1, 0, 0, 1, 0 | 10; a2: 1, 2, 0, -1, 0, 0, 1 | 11; s3: 1, 1, 0, 0, 1, 0, 0 | 12; P: 4 - 3M, 3 - 3M, M, M, 0, 0, 0 | -21M **c)** x = 3, y = 4; minimum cost £24	12
11	**a)** s1: 1, 1, 1, 0, 0 | 6; a1: 1, 2, 0, -1, 1 | 16; P: -2, -3, 0, 0, 0 | 0; A: -1, -2, 0, 1, 0 | -16 **b)** y enters, pivot 1 in the s1 row; the A row is then 1, 0, 2, 1, 0 | -4: optimal for stage 1 with A = 4 ≠ 0, so the problem is infeasible **c)** If x + y ≤ 6 and x, y ≥ 0 then x + 2y ≤ 12 < 16, so no point satisfies both constraints	8
12	**a)** Minimise C = 4x + 5y + 4z subject to x + y + z ≥ 40, 2x + 4y + 3z ≥ 130, x ≤ 25, x, y, z ≥ 0 **b)** a1: 1, 1, 1, -1, 0, 0, 1, 0 | 40; a2: 2, 4, 3, 0, -1, 0, 0, 1 | 130; s3: 1, 0, 0, 0, 0, 1, 0, 0 | 25; P: 4 - 3M, 5 - 5M, 4 - 4M, M, M, 0, 0, 0 | -170M **c)** There are no negative values in the objective row **d)** x = 0, y = 10, z = 30; minimum cost £170 **e)** s1 = s2 = 0: exactly 40 kg of feed and exactly 130 units of protein; s3 = 25: no Alfa is stored	12

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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