Two-stage simplex and big-M

A-level Decision Mathematics 1 | Edexcel 9FM0 | 12 questions, 98 marks

Further Maths topic practice

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1 A linear programming problem is given as

Maximise $P = 3x + 4y$

subject to

$x + 2y \leq 20$

$x + y \geq 6$

$x, y \geq 0$

a. Explain why the Simplex algorithm cannot be used directly to solve this problem.

b. Write each constraint as an equation, using exactly one slack variable, one surplus variable and one artificial variable. State which of your variables is which.

c. Explain why the artificial variable is needed.

(4 marks)

Answer _____

2 A linear programming problem is given as

Maximise $P = 2x + 3y + z$

subject to

$x + y + z \leq 30$

$2x + y \geq 8$

$x, y, z \geq 0$

The problem is to be solved using the big-M method, with slack variable s_1, surplus variable s_2 and artificial variable a_1.

a. Define, in this context, what M represents.

b. Show that the objective row of the initial tableau represents the equation

$P - (2 + 2M)x - (3 + M)y - z + Ms_2 = -8M$

(4 marks)

Answer _____

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3 A linear programming problem is given as

Maximise $P = 5x + 3y + 2z$

subject to

$3x + y + 2z \leq 40$

$x + y + z \geq 10$

$y - z \geq 2$

$x, y, z \geq 0$

The problem is to be solved using the big-M method.

a. Using slack variable s_1, surplus variables s_2 and s_3, and artificial variables a_1 and a_2, set up the initial tableau. You must show how you obtain the objective row. Use the template below.

b. Identify the pivot for the first iteration of the big-M method, giving reasons for your choice.

Initial tableau

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value

(7 marks)

Answer

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4 A linear programming problem is given as

Maximise $P = 4x + y + 3z$

subject to

$x + 2y + z \leq 30$

$2x + y \geq 10$

$x + y + z = 15$

$x, y, z \geq 0$

The problem is to be solved using the two-stage Simplex method.

a. Write the three constraints as equations, using slack variable s_1, surplus variable s_2 and artificial variables a_1 and a_2.

b. Set up the initial tableau for the first stage, including the objective row for P and the row for the new objective A. You should make your method for the A row clear. Use the template below.

Initial tableau

b.v.	x	y	z	s_1	s_2	a_1	a_2	Value

(7 marks)

Answer

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5 A linear programming problem in x, y and z is to be solved using the big-M method. The initial tableau is shown below.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	1	2	1	1	0	0	0	0	24
a_1	1	−1	2	0	−1	0	1	0	6
a_2	3	1	0	0	0	−1	0	1	9
P	−2 − 4M	1	−3 − 2M	0	M	M	0	0	−15M

a. Using the information in the tableau, formulate the linear programming problem. You should list each constraint as an inequality and state two possible ways of writing the objective.

b. Obtain the pivot for the first iteration of the big-M method. You must give reasons for your answer.

(7 marks)

Answer

6 A maximisation linear programming problem in x, y and z is to be solved using the two-stage Simplex method. The partially completed initial tableau is shown in the template below; the objective for the first stage is to minimise $A = a_1 + a_2$.

a. Complete the bottom row of the initial tableau. You should make your method clear.

b. Carry out the first stage of the two-stage Simplex method, taking the most negative number in the A row to indicate the pivot column each time. State the row operations you use.

c. State the basic feasible solution found at the end of the first stage.

Initial tableau

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value	Row ops
s_1	2	1	2	1	0	0	0	0	23	
a_1	2	0	1	0	−1	0	1	0	12	
a_2	3	2	1	0	0	−1	0	1	16	
P	−1	−4	−1	0	0	0	0	0	0	

Tableau after 1st iteration

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value	Row ops

(9 marks)

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Answer

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7 A linear programming problem is given as

Maximise $P = 6x + 4y$

subject to

$x + y \leq 12$

$x + 2y \leq 14$

$y \geq 4$

$x, y \geq 0$

a. Using slack variables s_1 and s_2, surplus variable s_3 and artificial variable a_1, set up the initial tableau for solving this problem using the big-M method.

b. Solve the problem using the big-M method, taking the most negative number in the objective row to indicate the pivot column each time. State the row operations you use and the optimal values of x, y and P.

Use the templates below.

Initial tableau

b.v.	x	y	s_1	s_2	s_3	a_1	Value	Row ops

Tableau after 1st iteration

b.v.	x	y	s_1	s_2	s_3	a_1	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	s_1	s_2	s_3	a_1	Value	Row ops

(10 marks)

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Answer

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8 A linear programming problem in x and y is being solved using the big-M method. The constraints are $3x + y \leq 35$, $x + y \geq 7$ and $x \geq 4$, and the objective is to maximise $P = 3x + 4y$. After the first iteration the tableau is

b.v.	x	y	s_1	s_2	s_3	a_1	a_2	Value
s_1	0	1	1	0	3	0	−3	23
a_1	0	1	0	−1	1	1	−1	3
x	1	0	0	0	−1	0	1	4
P	0	$-4 - M$	0	M	$-3 - M$	0	3 + 2M	12 − 3M

a. Explain why the solution given by this tableau is not feasible.

b. State the values of x and y given by this tableau.

c. Identify the pivot for the second iteration. You must give reasons for your answer.

d. Perform the second iteration, stating the row operations you use. Use the template below.

e. Explain whether the tableau you obtained in part (d) gives a feasible solution.

Tableau after 2nd iteration

b.v.	x	y	s_1	s_2	s_3	a_1	a_2	Value	Row ops

(9 marks)

Answer _____

9 A linear programming problem is given as

Maximise $P = 6x + y + 5z$

subject to

$2x + 4y \leq 38$

$x + 4y + z \leq 38$

$y \geq 5$

$x, y, z \geq 0$

It is being solved using the two-stage Simplex method, with slack variables s_1 and s_2, surplus variable s_3 and artificial variable a_1. The tableau at the end of the first stage is

b.v.	x	y	z	s_1	s_2	s_3	a_1	**Value**
s_1	2	0	0	1	0	4	−4	18
s_2	1	0	1	0	1	4	−4	18
y	0	1	0	0	0	−1	1	5
P	−6	0	−5	0	0	−1	1	5
A	0	0	0	0	0	0	1	0

a. Explain how the tableau shows that a basic feasible solution has been found.

b. Write down the initial tableau for the second stage and complete the second stage, taking the most negative number in the profit row to indicate the pivot column each time. State the row operations you use. Use the templates below.

c. State the optimal values of x, y, z and P.

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Second stage initial tableau

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

(9 marks)

Tableau after 1st iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	z	s_1	s_2	s_3	Value	Row ops

Answer _____

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10 A park manager mixes two fertilisers, Growfast and Bloomwell, for the flower beds. Each bag of Growfast contains 2 units of nitrogen and 1 unit of potash and costs £4. Each bag of Bloomwell contains 1 unit of nitrogen and 2 units of potash and costs £3. The mixture must contain at least 10 units of nitrogen and at least 11 units of potash, and there is room to store at most 12 bags.

The manager buys x bags of Growfast and y bags of Bloomwell and wants to minimise the cost, £C.

a. Formulate this as a linear programming problem in which the objective is to maximise P.

b. Set up the initial tableau for solving the problem using the big-M method, using surplus variables s_1 and s_2, slack variable s_3 and artificial variables a_1 and a_2. You must show how you obtain the objective row.

c. Solve the problem using the big-M method, stating the row operations you use. State how many bags of each fertiliser should be bought and the minimum cost.

Use the templates below.

Initial tableau

b.v.	x	y	s_1	s_2	s_3	a_1	a_2	Value	Row ops

Tableau after 1st iteration

b.v.	x	y	s_1	s_2	s_3	a_1	a_2	Value	Row ops

Tableau after 2nd iteration

b.v.	x	y	s_1	s_2	s_3	a_1	a_2	Value	Row ops

(12 marks)

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Answer

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11 A linear programming problem is given as

Maximise $P = 2x + 3y$

subject to

$x + y \leq 6$

$x + 2y \geq 16$

$x, y \geq 0$

a. Using slack variable s_1, surplus variable s_2 and artificial variable a_1, set up the initial tableau for the first stage of the two-stage Simplex method, including the A row.

b. Carry out the first stage, stating the row operations you use, and explain what your final tableau shows about the problem.

c. Explain, by considering the constraints, why this result is correct.

Use the templates below.

Initial tableau

b.v.	x	y	s_1	s_2	a_1	Value	Row ops

Tableau after 1st iteration

b.v.	x	y	s_1	s_2	a_1	Value	Row ops

(8 marks)

Answer

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12 An animal sanctuary mixes three dry feeds, Alfa, Bran and Corn. Each kilogram of Alfa contains 2 units of protein and costs £4, each kilogram of Bran contains 4 units of protein and costs £5, and each kilogram of Corn contains 3 units of protein and costs £4. Each day the sanctuary needs at least 40 kg of feed containing at least 130 units of protein in total. It can store at most 25 kg of Alfa.

The sanctuary uses x kg of Alfa, y kg of Bran and z kg of Corn each day, and wants to minimise the daily cost, £C.

a. Formulate this as a linear programming problem.

b. The problem is to be solved by maximising $P = -C$ using the big-M method, with surplus variables s_1 and s_2, slack variable s_3 and artificial variables a_1 and a_2. Set up the initial tableau, showing how you obtain the objective row. Use the template below.

After three iterations of the big-M method the tableau is

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	**Value**
z	2	0	1	−4	1	0	4	−1	30
y	−1	1	0	3	−1	0	−3	1	10
s_3	1	0	0	0	0	1	0	0	25
P	1	0	0	1	1	0	$-1 + M$	$-1 + M$	−170

c. Explain how you know this tableau is optimal.

d. State how much of each feed the sanctuary should use each day and the minimum daily cost.

e. Interpret the values of the slack and surplus variables in context.

Initial tableau

b.v.	**x**	**y**	**z**	**s_1**	**s_2**	**s_3**	**a_1**	**a_2**	**Value**

(12 marks)

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Answer _____

End of questions | Total 98 marks

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