Direct impact and restitution

Worked solutions

Further Maths topic practice

This topic online

1 A small smooth sphere P of mass 0.4 kg is moving in a straight line on a smooth horizontal floor with speed 6 m s^{-1}. It strikes a fixed smooth vertical wall directly, so that its direction of motion is perpendicular to the wall. The coefficient of restitution between P and the wall is 0.5

a. Find the speed of P immediately after it strikes the wall.

b. Find the magnitude of the impulse exerted on P by the wall.

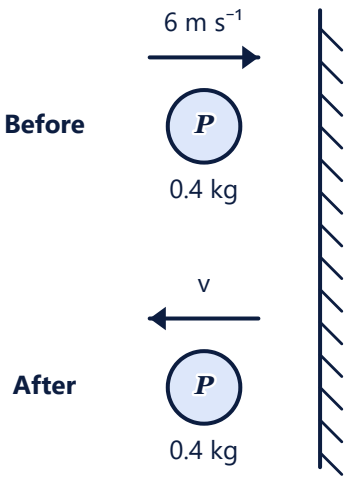


Diagram NOT accurately drawn

(4 marks)

a) Newton's law of restitution for a fixed wall

speed after = e × speed before

$v = 0.5 \times 6$

$v = 3 \text{ m s}^{-1}$

b) Impulse = change in momentum, away from the wall positive

$I = 0.4(3) - 0.4(-6)$

$I = 1.2 + 2.4$

$I = 3.6 \text{ N s}$

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2 Two small smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving in the same direction along the same straight line on a smooth horizontal surface. A has speed 5 m s^{-1} and B has speed 1 m s^{-1}, with B in front of A. The spheres collide directly. Immediately after the collision A is moving with speed 2 m s^{-1} in its original direction.

a. Find the speed of B immediately after the collision.

b. Find the coefficient of restitution between A and B.

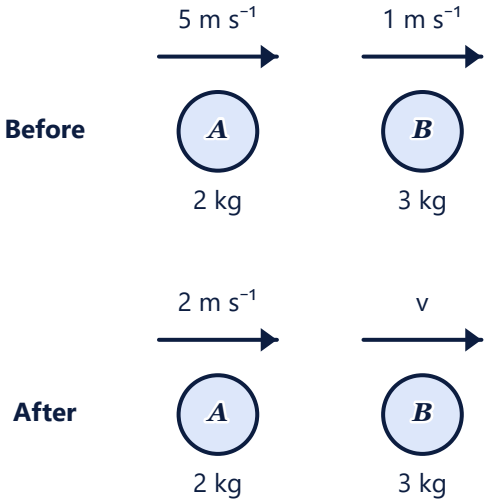


Diagram NOT accurately drawn

(4 marks)

Take the original direction of motion as positive

a) Conservation of momentum

$$2(5) + 3(1) = 2(2) + 3v$$

$$13 = 4 + 3v$$

$$3v = 9$$

$$\boxed{v = 3 \text{ m s}^{-1}}$$

b) Newton's law of restitution

$$\text{speed of separation} = e \times \text{speed of approach}$$

$$3 - 2 = e(5 - 1)$$

$$1 = 4e$$

$$\boxed{e = 0.25}$$

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3 Two small smooth spheres A and B, of equal radius, have masses 0.5 kg and 1.5 kg respectively. They are moving towards each other along the same straight line on a smooth horizontal table. Immediately before they collide directly, A has speed 8 m s^{-1} and B has speed 2 m s^{-1}. The coefficient of restitution between the spheres is 0.6

a. Find the speed and direction of motion of each sphere immediately after the collision.

b. Find the kinetic energy lost in the collision.

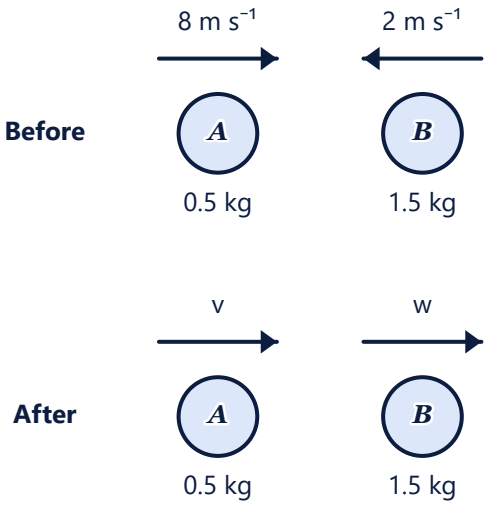


Diagram NOT accurately drawn

(7 marks)

a) Take A's original direction as positive

Conservation of momentum

$$0.5(8) + 1.5(-2) = 0.5v + 1.5w$$

$$0.5v + 1.5w = 1$$

Newton's law of restitution

$$w - v = 0.6(8 - (-2))$$

$$w - v = 6$$

$$w = v + 6$$

Substitute into the momentum equation

$$0.5v + 1.5(v + 6) = 1$$

$$2v + 9 = 1$$

$$v = -4$$

$$w = -4 + 6 = 2$$

A: 4 m s^{-1}, B: 2 m s^{-1}, both directions reversed

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b) $KE = \frac{1}{2}mv^2$ before and after

$$KE \text{ before} = \frac{1}{2}(0.5)(8^2) + \frac{1}{2}(1.5)(2^2) = 16 + 3 = 19$$

$$KE \text{ after} = \frac{1}{2}(0.5)(4^2) + \frac{1}{2}(1.5)(2^2) = 4 + 3 = 7$$

$$KE \text{ lost} = 19 - 7$$

$\boxed{12 \text{ J}}$

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4 A small ball of mass 0.2 kg is released from rest at a height of 2.5 m above a horizontal floor. It falls freely, hits the floor and rebounds vertically to a greatest height of 1.6 m above the floor. The ball is modelled as a particle and air resistance is ignored.

a. Find the coefficient of restitution between the ball and the floor.

b. Find the magnitude of the impulse exerted on the ball by the floor.

c. Find the kinetic energy lost in the impact.

(7 marks)

a) Falling: $u = 0$, $a = 9.8$, $s = 2.5$

$$v^2 = 0 + 2(9.8)(2.5)$$

$$v^2 = 49$$

$$v = 7$$

Rising: $v = 0$ at the top, $a = -9.8$, $s = 1.6$

$$0 = w^2 - 2(9.8)(1.6)$$

$$w^2 = 31.36$$

$$w = 5.6$$

Speed after = e x speed before

$$e = \frac{5.6}{7}$$

$$\boxed{e = 0.8}$$

b) Impulse = change in momentum, upwards positive

$$I = 0.2(5.6) - 0.2(-7)$$

$$I = 1.12 + 1.4$$

$$\boxed{I = 2.52 \text{ N s}}$$

c) KE lost = KE just before - KE just after

$$\text{KE lost} = \frac{1}{2}(0.2)(49) - \frac{1}{2}(0.2)(31.36)$$

$$\text{KE lost} = 4.9 - 3.136$$

$$\text{KE lost} = 1.764$$

$$\boxed{1.76 \text{ J}}$$

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5 A small smooth puck of mass 0.5 kg slides on smooth horizontal ice with speed 8 m s^{-1} and strikes a fixed smooth vertical barrier directly. The puck rebounds along the same line. In the impact the puck loses 36% of its kinetic energy.

a. Show that the coefficient of restitution between the puck and the barrier is 0.8

b. Find the magnitude of the impulse exerted on the puck by the barrier. (5 marks)

a) 36% lost, so 64% of the KE remains

$$\frac{1}{2}(0.5)v^2 = 0.64 \times \frac{1}{2}(0.5)(8^2)$$

$$v^2 = 0.64 \times 64$$

$$v^2 = 40.96$$

$$v = 6.4$$

Speed after = e x speed before

$$e = \frac{6.4}{8}$$

$$\boxed{e = 0.8}$$

b) Impulse = change in momentum, away from the barrier positive

$$I = 0.5(6.4) - 0.5(-8)$$

$$I = 3.2 + 4$$

$$\boxed{I = 7.2 \text{ N s}}$$

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6 Two small smooth spheres P and Q, of equal radius, have masses 0.3 kg and 0.2 kg respectively. They move towards each other along the same straight line on a smooth horizontal surface, P with speed 6 m s^{-1} and Q with speed 4 m s^{-1}. They collide directly. The magnitude of the impulse exerted on Q by P in the collision is 2.1 N s.

a. Find the speed of each sphere immediately after the collision and state whether its direction of motion has been reversed.

b. Find the coefficient of restitution between P and Q.

c. Find the kinetic energy lost in the collision.

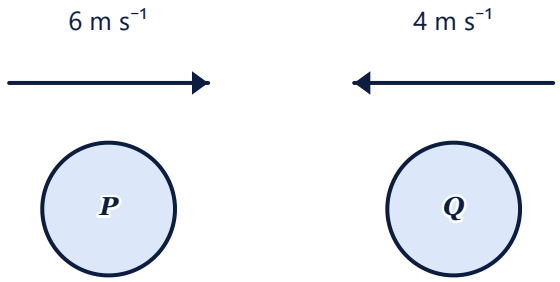


Diagram NOT accurately drawn

(7 marks)

Take P's original direction as positive

a) Impulse on Q = +2.1 N s (in P's direction)

$0.2(v_Q - (-4)) = 2.1$

$v_Q + 4 = 10.5$

$v_Q = 6.5$

Impulse on P = −2.1 N s (equal and opposite)

$0.3(v_P - 6) = -2.1$

$v_P - 6 = -7$

$v_P = -1$

P: 1 m s^{-1}, reversed; Q: 6.5 m s^{-1}, reversed

b) Newton's law of restitution

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}$$

$$e = \frac{6.5 - (-1)}{6 - (-4)}$$

$$e = \frac{7.5}{10}$$

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$e = 0.75$

c) KE before and after

$$\text{KE before} = \frac{1}{2}(0.3)(6^2) + \frac{1}{2}(0.2)(4^2) = 5.4 + 1.6 = 7$$

$$\text{KE after} = \frac{1}{2}(0.3)(1^2) + \frac{1}{2}(0.2)(6.5^2) = 0.15 + 4.225 = 4.375$$

$$\text{KE lost} = 7 - 4.375 = 2.625$$

2.63 J

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7 Two small smooth spheres A and B, of equal radius, are moving towards each other along the same straight line on a smooth horizontal floor. A has mass 4 kg and speed 3 m s^{-1}. B has mass m kg and speed 6 m s^{-1}. The spheres collide directly and the coefficient of restitution between them is 0.5. Immediately after the collision the direction of motion of A has been reversed and its speed is 1.5 m s^{-1}.

a. Find the speed of B immediately after the collision.

b. Find the value of m.

c. Find the total kinetic energy lost in the collision.

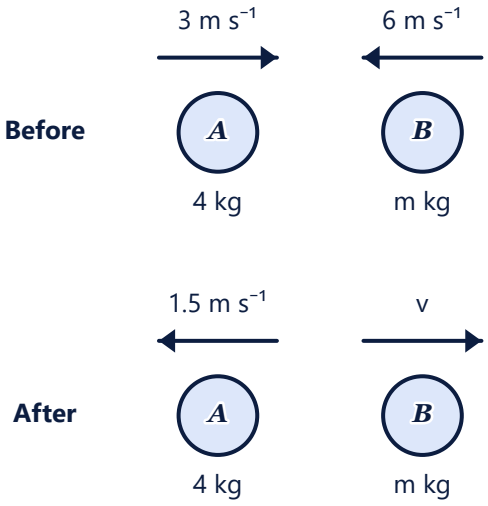


Diagram NOT accurately drawn

(7 marks)

Take A's original direction as positive: $v_A = -1.5$

a) Newton's law of restitution

$$v_B - v_A = e(u_A - u_B)$$
$$v_B - (-1.5) = 0.5(3 - (-6))$$
$$v_B + 1.5 = 4.5$$
$$v_B = 3$$

$\boxed{3 \text{ m s}^{-1}}$

b) Conservation of momentum

$$4(3) + m(-6) = 4(-1.5) + m(3)$$
$$12 - 6m = -6 + 3m$$
$$18 = 9m$$

$\boxed{m = 2}$

c) KE before and after

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$$\text{KE before} = \frac{1}{2}(4)(3^2) + \frac{1}{2}(2)(6^2) = 18 + 36 = 54$$

$$\text{KE after} = \frac{1}{2}(4)(1.5^2) + \frac{1}{2}(2)(3^2) = 4.5 + 9 = 13.5$$

$$\text{KE lost} = 54 - 13.5$$

$$\boxed{40.5 \text{ J}}$$

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8 A small smooth sphere A of mass m is moving with speed u on a smooth horizontal surface when it collides directly with a small smooth sphere B of mass $3m$, which is at rest. The spheres have equal radius and the coefficient of restitution between them is e.

a. Show that the speed of B immediately after the collision is $\frac{1}{4}u(1 + e)$ and find the velocity of A.

b. Given that the direction of motion of A is reversed by the collision, find the range of possible values of e.

c. Given instead that $e = \frac{2}{3}$, find, in terms of m and u, the magnitude of the impulse exerted on B in the collision.

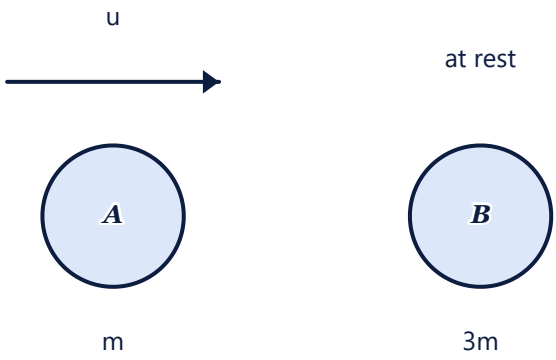


Diagram NOT accurately drawn

(8 marks)

a) Take A's direction of motion as positive

Conservation of momentum

$$mu = mv_A + 3mv_B$$

$$v_A + 3v_B = u$$

Newton's law of restitution

$$v_B - v_A = eu$$

Add the equations

$$4v_B = u + eu$$

$$\boxed{v_B = \frac{1}{4}u(1 + e)}$$

$$v_A = v_B - eu$$

$$v_A = \frac{u + eu - 4eu}{4}$$

$$\boxed{v_A = \frac{1}{4}u(1 - 3e)}$$

b) Reversed means $v_A < 0$

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$\frac{1}{4}u(1-3e) < 0$

$1 - 3e < 0$

$e > \frac{1}{3}$

Also $0 \leq e \leq 1$

$$\boxed{\frac{1}{3} < e \leq 1}$$

c) Impulse on B = change in momentum of B

$v_B = \frac{1}{4}u(1+\frac{2}{3}) = \frac{5}{12}u$

$I = 3m \times \frac{5}{12}u - 0$

$$\boxed{I = \frac{5}{4}mu}$$

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9 A small smooth sphere A of mass $2m$ is moving with speed u on a smooth horizontal plane. It collides directly with a small smooth sphere B of mass m which is at rest. The spheres have equal radius and the coefficient of restitution between them is e.

a. Find, in terms of u and e, the speed of each sphere immediately after the collision.

b. Show that the kinetic energy lost in the collision is $\frac{1}{3}mu^2(1 - e^2)$.

c. Given that one quarter of the original kinetic energy is lost in the collision, find the value of e.

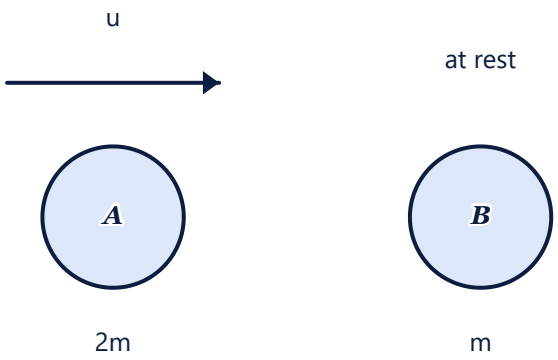


Diagram NOT accurately drawn

(8 marks)

a) Take A's direction as positive

Conservation of momentum

$$2mu = 2mv_A + mv_B$$

$$2v_A + v_B = 2u$$

Newton's law of restitution

$$v_B - v_A = eu$$

Subtract to remove v_B

$$3v_A = 2u - eu$$

$$\boxed{v_A = \frac{1}{3}u(2 - e)}$$

$$v_B = v_A + eu = \frac{2u - eu + 3eu}{3}$$

$$\boxed{v_B = \frac{2}{3}u(1 + e)}$$

b) KE before

$$\text{KE before} = \frac{1}{2}(2m)u^2 = mu^2$$

KE after

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$$\text{KE after} = \frac{1}{2}(2m) \times \frac{u^2(2-e)^2}{9} + \frac{1}{2}m \times \frac{4u^2(1+e)^2}{9}$$

$$\text{KE after} = \frac{mu^2}{9}[(2-e)^2 + 2(1+e)^2]$$

$$\text{KE after} = \frac{mu^2}{9}[4 - 4e + e^2 + 2 + 4e + 2e^2]$$

$$\text{KE after} = \frac{mu^2}{9}(6 + 3e^2) = \frac{1}{3}mu^2(2 + e^2)$$

$$\text{KE lost} = mu^2 - \frac{1}{3}mu^2(2 + e^2)$$

$$\boxed{\text{KE lost} = \frac{1}{3}mu^2(1 - e^2)}$$

c) KE lost = $\frac{1}{4}$ of mu^2

$$\frac{1}{3}mu^2(1 - e^2) = \frac{1}{4}mu^2$$

$$1 - e^2 = \frac{3}{4}$$

$$e^2 = \frac{1}{4}$$

$$\boxed{e = \frac{1}{2}}$$

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10 A small ball is dropped from rest at a point 3.6 m above a horizontal floor. Each time it hits the floor it rebounds vertically. The coefficient of restitution between the ball and the floor is 0.5. The ball is modelled as a particle moving freely under gravity.

a. Find the greatest height reached by the ball after its first bounce.

b. Show that, according to the model, the total distance travelled by the ball before it comes to rest is 6 m.

c. Find the total time, according to the model, from the instant the ball is dropped until it comes to rest.

d. Give one reason why the answer to part (c) is likely to differ from the time taken in reality.

(9 marks)

a) First fall: $u = 0$, $a = 9.8$, $s = 3.6$

$$v^2 = 2(9.8)(3.6) = 70.56$$

$$v = 8.4$$

Rebound speed = e × 8.4

$$w_1 = 0.5 \times 8.4 = 4.2$$

Rises until speed is 0

$$0 = 4.2^2 - 2(9.8)h_1$$

$$h_1 = \frac{17.64}{19.6}$$

$$\boxed{h_1 = 0.9 \text{ m}}$$

b) Each rebound speed is 0.5 × the one before, and h is proportional to w^2

$$h_{n+1} = 0.25h_n$$

The ball goes up and down each height

$$d = 3.6 + 2(0.9 + 0.225 + 0.05625 + \ldots)$$

Sum to infinity of a geometric series, $a = 0.9$, $r = 0.25$

$$d = 3.6 + 2 \times \frac{0.9}{1 - 0.25}$$

$$d = 3.6 + 2 \times 1.2$$

$$\boxed{d = 6 \text{ m}}$$

c) Time for the first fall

$$t_0 = \frac{8.4}{9.8} = 0.85714\ldots$$

After rebounding at speed w the ball is in the air for $\frac{2w}{g}$

$$T = \frac{2}{9.8}(4.2 + 2.1 + 1.05 + \ldots)$$

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$$T = \frac{2}{9.8} \times \frac{4.2}{1 - 0.5}$$

$$T = \frac{2}{9.8} \times 8.4 = 1.7142...$$

$$\text{total} = 0.85714... + 1.7142...$$

$$\text{total} = 2.5714...$$

2.57 s

d) Limitation of the model

Air resistance is ignored, and e will not stay constant for very small bounces

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11 A small block B of mass 0.5 kg is projected along a rough horizontal floor towards a fixed smooth vertical wall. It moves along a line perpendicular to the wall. When it is projected, B is 4 m from the wall and has speed 6 m s^{-1}. The coefficient of friction between B and the floor is 0.25. B hits the wall and rebounds. The coefficient of restitution between B and the wall is 0.6. B is modelled as a particle.

a. Find the speed of B immediately before it hits the wall.

b. Find the speed of B immediately after it hits the wall.

c. Find the distance of B from the wall when it comes to rest.

d. Find the kinetic energy lost in the impact with the wall.

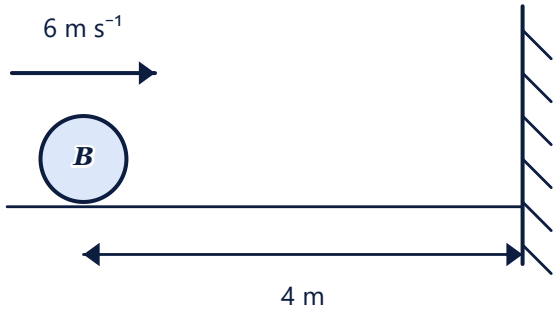


Diagram NOT accurately drawn

(10 marks)

a) $R = 0.5g$, so friction $F = 0.25 \times 0.5g$

$F = 1.225$

Work-energy principle from projection to the wall

$$\frac{1}{2}(0.5)v^2 = \frac{1}{2}(0.5)(6^2) - 1.225 \times 4$$

$$0.25v^2 = 9 - 4.9$$

$$v^2 = 16.4$$

$$v = 4.0496...$$

$\boxed{4.05 \text{ m s}^{-1}}$

b) Speed after = e x speed before

$w = 0.6 \times 4.0496...$

$w = 2.4298...$

$\boxed{2.43 \text{ m s}^{-1}}$

c) Friction does work until B stops

$w^2 = 0.36 \times 16.4 = 5.904$

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$$1.225d = \frac{1}{2}(0.5)(5.904)$$

$$d = \frac{1.476}{1.225}$$

$$d = 1.2048...$$

1.20 m from the wall

d) KE lost in the impact

$$\text{KE lost} = \frac{1}{2}(0.5)(16.4) - \frac{1}{2}(0.5)(5.904)$$

$$\text{KE lost} = 4.1 - 1.476$$

$$\text{KE lost} = 2.624$$

2.62 J

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12 Two small smooth spheres A and B, of equal radius, have masses $3m$ and $2m$ respectively. They are moving towards each other along the same straight line on a smooth horizontal surface. Immediately before they collide directly, A has speed $2u$ and B has speed u. The coefficient of restitution between the spheres is e.

a. Show that, immediately after the collision, the velocity of A is $\frac{2u}{5}(2 - 3e)$ in its original direction of motion.

b. Find, in terms of u and e, the velocity of B immediately after the collision, and explain why the direction of motion of B is always reversed.

c. Find the range of values of e for which the direction of motion of A is also reversed.

d. Given that A is brought to rest by the collision, find, in terms of m and u, the kinetic energy lost.

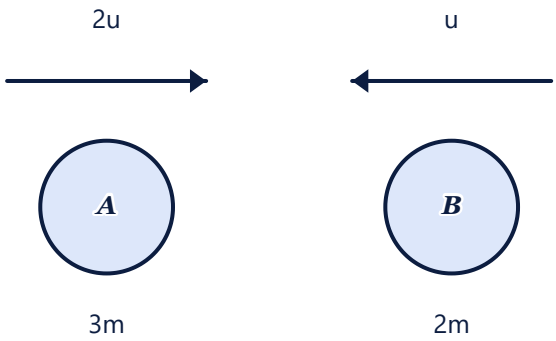


Diagram NOT accurately drawn

(11 marks)

a) Take A's original direction as positive

Conservation of momentum

$$3m(2u) + 2m(-u) = 3mv_A + 2mv_B$$

$$3v_A + 2v_B = 4u$$

Newton's law of restitution

$$v_B - v_A = e(2u - (-u)) = 3eu$$

$$v_B = v_A + 3eu$$

Substitute

$$3v_A + 2v_A + 6eu = 4u$$

$$5v_A = 4u - 6eu$$

$$\boxed{v_A = \frac{2u}{5}(2 - 3e)}$$

b)

$$v_B = \frac{2u}{5}(2 - 3e) + 3eu$$

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$$v_B = \frac{4u - 6eu + 15eu}{5}$$

$$\boxed{v_B = \frac{u}{5}(4 + 9e)}$$

$e \geqslant 0$, so $4 + 9e > 0$: B moves in A's original direction

$$\boxed{\text{B's direction is always reversed}}$$

c) A reversed means $v_A < 0$

$$2 - 3e < 0$$

$$e > \frac{2}{3}$$

$$\boxed{\frac{2}{3} < e \leqslant 1}$$

d) A at rest: $2 - 3e = 0$

$$e = \frac{2}{3}$$

$$v_B = \frac{u}{5}(4 + 6) = 2u$$

$$\text{KE before} = \frac{1}{2}(3m)(2u)^2 + \frac{1}{2}(2m)u^2 = 6mu^2 + mu^2 = 7mu^2$$

$$\text{KE after} = \frac{1}{2}(2m)(2u)^2 = 4mu^2$$

$$\text{KE lost} = 7mu^2 - 4mu^2$$

$$\boxed{3mu^2}$$

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