Elastic potential energy

Worked solutions

Further Maths topic practice

This topic online

1 A light elastic string has natural length 1.2 m and modulus of elasticity 30 N.

a. Find the elastic potential energy stored in the string when its extension is 0.4 m.

b. Find the work done in stretching the string from an extension of 0.4 m to an extension of 0.6 m. (4 marks)

a) EPE formula

$$EPE = \frac{\lambda x^2}{2l}$$

$$EPE = \frac{30 \times 0.4^2}{2 \times 1.2}$$

$$\boxed{EPE = 2 \text{ J}}$$

b) Work done = increase in EPE

$$EPE \text{ at } 0.6 = \frac{30 \times 0.6^2}{2 \times 1.2}$$

$$EPE \text{ at } 0.6 = 4.5$$

$$\text{work done} = 4.5 - 2$$

$$\boxed{\text{work done} = 2.5 \text{ J}}$$

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2 A light spring of natural length 0.5 m and modulus of elasticity 80 N lies on a smooth horizontal surface with one end fixed to a vertical wall. A particle of mass 0.4 kg is held against the free end of the spring so that the spring is compressed to a length of 0.35 m. The particle is released from rest.

a. Find the elastic potential energy stored in the spring before the particle is released.

b. Find the speed of the particle when it leaves the spring.

(4 marks)

a) Compression

$$x = 0.5 - 0.35 = 0.15$$

$$EPE = \frac{80 \times 0.15^2}{2 \times 0.5}$$

$$\boxed{EPE = 1.8 \text{ J}}$$

b) Leaves the spring at its natural length

Smooth: EPE lost = KE gained

$$\frac{1}{2} \times 0.4 \times v^2 = 1.8$$

$$v^2 = 9$$

$$\boxed{v = 3 \text{ m s}^{-1}}$$

3 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 20 N. The other end of the string is attached to a fixed point O on a smooth horizontal table. P moves on the table directly away from O, and when the string becomes taut P has speed 3 m s^{-1}.

a. Find the greatest extension of the string in the motion that follows.

b. Find the greatest tension in the string.

(5 marks)

a) At the greatest extension P is instantaneously at rest

KE lost = EPE gained

$$\frac{1}{2} \times 0.5 \times 3^2 = \frac{20x^2}{2 \times 0.8}$$

$$2.25 = 12.5x^2$$

$$x^2 = 0.18$$

$$x = 0.42426...$$

$$\boxed{\text{greatest extension} = 0.424 \text{ m}}$$

b) Hooke's law at the greatest extension

$$T = \frac{20 \times 0.42426}{0.8}$$

$$\boxed{T = 10.6 \text{ N}}$$

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4 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 19.6 N. The other end of the string is attached to a fixed point O.

a. Find the extension of the string when P hangs in equilibrium vertically below O.

P is pulled vertically down until OP = 1.3 m and is released from rest.

b. Find the speed of P as it passes through its equilibrium position.

c. Find the speed of P at the instant the string becomes slack.

(8 marks)

a) Resolve vertically

$$\frac{19.6e}{0.8} = 0.5 \times 9.8$$

$$24.5e = 4.9$$

$$\boxed{e = 0.2 \text{ m}}$$

b) Initial extension

$$x = 1.3 - 0.8 = 0.5$$

From extension 0.5 to extension 0.2

$$\text{EPE lost} = \frac{19.6 \times 0.5^2}{1.6} - \frac{19.6 \times 0.2^2}{1.6}$$

$$\text{EPE lost} = 3.0625 - 0.49 = 2.5725$$

$$\text{GPE gained} = 0.5 \times 9.8 \times 0.3 = 1.47$$

Energy: KE gained = EPE lost - GPE gained

$$\frac{1}{2} \times 0.5 \times v^2 = 2.5725 - 1.47$$

$$0.25v^2 = 1.1025$$

$$v^2 = 4.41$$

$$\boxed{v = 2.1 \text{ m s}^{-1}}$$

c) String slack at its natural length: P has risen 0.5 m

$$\text{EPE lost} = 3.0625$$

$$\text{GPE gained} = 0.5 \times 9.8 \times 0.5 = 2.45$$

$$\frac{1}{2} \times 0.5 \times v^2 = 3.0625 - 2.45$$

$$0.25v^2 = 0.6125$$

$$v^2 = 2.45$$

$$\boxed{v = 1.57 \text{ m s}^{-1}}$$

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5 A particle P of mass 0.4 kg is attached to one end of a light elastic string of natural length 1.5 m and modulus of elasticity 29.4 N. The other end of the string is attached to a fixed point O. The particle is held at O and released from rest, and falls vertically.

a. Show that the greatest extension of the string is 1 m.

b. Find the greatest speed of P during its fall.

(8 marks)

a) At the lowest point P is at rest, fallen $1.5 + x$

GPE lost = EPE gained

$$0.4 \times 9.8 \times (1.5 + x) = \frac{29.4x^2}{2 \times 1.5}$$

$$3.92(1.5 + x) = 9.8x^2$$

$$9.8x^2 - 3.92x - 5.88 = 0$$

Divide by 1.96

$$5x^2 - 2x - 3 = 0$$

$$(5x + 3)(x - 1) = 0$$

$x > 0$

$$\boxed{x = 1 \text{ m}}$$

b) Greatest speed when acceleration = 0, at the equilibrium position

$$\frac{29.4e}{1.5} = 0.4 \times 9.8$$

$$19.6e = 3.92$$

$$e = 0.2$$

P has fallen $1.5 + 0.2 = 1.7$ m

$$\frac{1}{2} \times 0.4 \times v^2 = 0.4 \times 9.8 \times 1.7 - \frac{29.4 \times 0.2^2}{3}$$

$$0.2v^2 = 6.664 - 0.392$$

$$0.2v^2 = 6.272$$

$$v^2 = 31.36$$

$$\boxed{v = 5.6 \text{ m s}^{-1}}$$

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6 A light spring of natural length 0.6 m and modulus of elasticity 48 N lies on a rough horizontal floor with one end fixed to a vertical wall. A particle P of mass 0.8 kg is placed against the free end of the spring, and is held at rest with the spring compressed by 0.2 m, as shown in the diagram. The coefficient of friction between P and the floor is 0.25. The particle is not attached to the spring.

P is released from rest.

a. Find the speed of P at the instant it leaves the spring.

b. Find the total distance that P moves before it comes to rest.

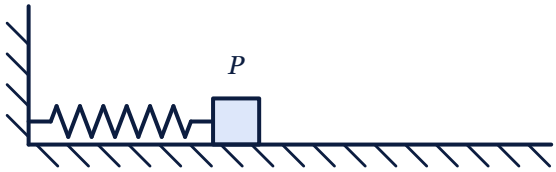


Diagram NOT accurately drawn

(7 marks)

a) Initial EPE

$$EPE = \frac{48 \times 0.2^2}{2 \times 0.6}$$

$$EPE = 1.6$$

Friction

$$R = 0.8 \times 9.8 = 7.84$$

$$F = 0.25 \times 7.84 = 1.96$$

Leaves the spring after moving 0.2 m

$$\text{work against friction} = 1.96 \times 0.2 = 0.392$$

Work-energy: KE = EPE lost - work against friction

$$\frac{1}{2} \times 0.8 \times v^2 = 1.6 - 0.392$$

$$0.4v^2 = 1.208$$

$$v^2 = 3.02$$

$$\boxed{v = 1.74 \text{ m s}^{-1}}$$

b) All the EPE is used against friction

$$1.96d = 1.6$$

$$d = 0.81632...$$

$$\boxed{\text{distance} = 0.816 \text{ m}}$$

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7 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 9.8 N. The other end of the string is attached to a fixed point A on a smooth plane inclined at 30° to the horizontal. P is held at A and released from rest. It slides down a line of greatest slope and first comes to instantaneous rest at the point B, as shown in the diagram.

a. Find the distance AB.

b. Find the magnitude and direction of the acceleration of P at B.

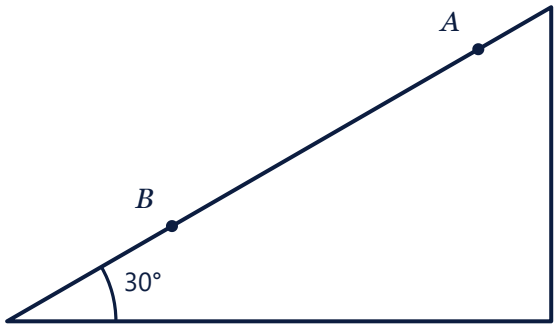


Diagram NOT accurately drawn

(7 marks)

a) Let AB = d, extension at B is d - 1

At rest at A and at B: GPE lost = EPE gained

$$0.5 \times 9.8 \times d \sin 30^\circ = \frac{9.8(d-1)^2}{2 \times 1}$$

$$2.45d = 4.9(d-1)^2$$

$$2.45d = 4.9d^2 - 9.8d + 4.9$$

$$4.9d^2 - 12.25d + 4.9 = 0$$

Divide by 2.45

$$2d^2 - 5d + 2 = 0$$

$$(2d-1)(d-2) = 0$$

$d = 0.5 < 1$ is not possible: the string would be slack

$$\boxed{AB = 2 \text{ m}}$$

b) Hooke's law at B

$$T = \frac{9.8 \times 1}{1} = 9.8$$

Newton's second law up the plane

$$9.8 - 0.5 \times 9.8 \sin 30^\circ = 0.5a$$

$$7.35 = 0.5a$$

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$a = 14.7 \text{ m s}^{-2}$ up the plane

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8 A rough plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1.5 m and modulus of elasticity 120 N. The other end of the string is attached to a fixed point O on the plane. The coefficient of friction between P and the plane is 0.25.

P is held at rest at the point B on the plane, where B is below O on a line of greatest slope and OB = 2.1 m, as shown in the diagram. P is released and moves up the plane.

a. Find the speed of P at the instant the string becomes slack.

b. Find the further distance that P moves up the plane before it comes to instantaneous rest.

c. Determine whether P remains at rest at this point, giving a reason.

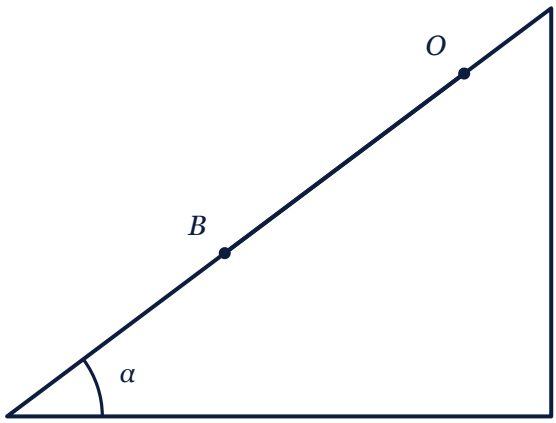


Diagram NOT accurately drawn

(9 marks)

$\tan\alpha = \frac{3}{4}$ so $\sin\alpha = 0.6$, $\cos\alpha = 0.8$

Resolve perpendicular to the plane

$$R = 2 \times 9.8 \times 0.8 = 15.68$$

$$F = 0.25 \times 15.68 = 3.92$$

$$2g\sin\alpha = 2 \times 9.8 \times 0.6 = 11.76$$

a) Initial extension and EPE

$$x = 2.1 - 1.5 = 0.6$$

$$EPE = \frac{120 \times 0.6^2}{2 \times 1.5}$$

$$EPE = 14.4$$

Slack after moving 0.6 m up the plane

$$\text{GPE gained} = 11.76 \times 0.6 = 7.056$$

$$\text{work against friction} = 3.92 \times 0.6 = 2.352$$

Work-energy

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$$\frac{1}{2} \times 2 \times v^2 = 14.4 - 7.056 - 2.352$$

$$v^2 = 4.992$$

$$\boxed{v = 2.23 \text{ m s}^{-1}}$$

b) String slack: Newton's second law down the plane

$$11.76 + 3.92 = 2a$$

$$a = 7.84$$

$v^2 = u^2 + 2as$ with final speed 0

$$0 = 4.992 - 2 \times 7.84 \times s$$

$$s = \frac{4.992}{15.68}$$

$$\boxed{s = 0.318 \text{ m}}$$

c) Compare the weight component with the maximum friction

$$11.76 > 3.92$$

$$\boxed{\text{P does not stay at rest: it slides back down the plane}}$$

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9 A light spring of natural length 0.5 m and modulus of elasticity 147 N stands vertically with its lower end fixed to a horizontal floor. A particle P of mass 1 kg is held at rest 0.9 m above the floor, directly above the spring, as shown in the diagram. P is released and falls onto the top of the spring.

a. Find the speed of P as it first hits the spring.

b. Find the least height of P above the floor in the motion that follows.

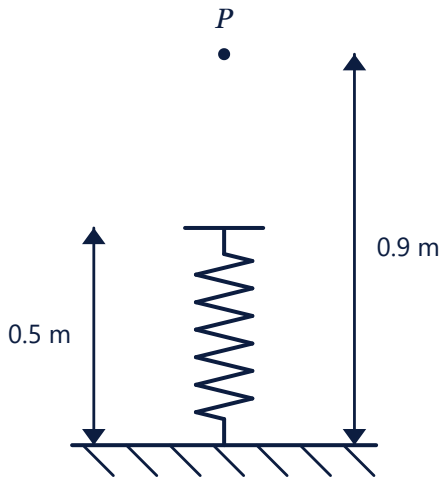


Diagram NOT accurately drawn

(7 marks)

a) Free fall of $0.9 - 0.5 = 0.4$ m

$$\frac{1}{2} \times 1 \times v^2 = 1 \times 9.8 \times 0.4$$

$$v^2 = 7.84$$

$$\boxed{v = 2.8 \text{ m s}^{-1}}$$

b) Let x = greatest compression; at rest at both ends

GPE lost = EPE gained

$$1 \times 9.8 \times (0.4 + x) = \frac{147x^2}{2 \times 0.5}$$

$$3.92 + 9.8x = 147x^2$$

$$147x^2 - 9.8x - 3.92 = 0$$

Divide by 9.8

$$15x^2 - x - 0.4 = 0$$

$$x = \frac{1 \pm \sqrt{(1 + 24)}}{30}$$

$$x = \frac{1 + 5}{30} = 0.2$$

Height of P above the floor

$0.5 - 0.2 = 0.3$

Least height = 0.3 m

10 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 19.6 N. The other end of the string is attached to a fixed point O on a rough horizontal floor. P is held at rest on the floor with OP = 1.5 m and is then released. P moves in a straight line towards O and comes to rest for the first time at a point 0.25 m from O.

a. Find the coefficient of friction between P and the floor.

b. Find the speed of P at the instant the string becomes slack.

(8 marks)

a) Initial EPE

$$EPE = \frac{19.6 \times 0.5^2}{2 \times 1}$$

$$EPE = 2.45$$

Distance moved

$$d = 1.5 - 0.25 = 1.25$$

$$F = \mu \times 0.5 \times 9.8 = 4.9\mu$$

At rest at both ends, string slack at the end: EPE lost = work against friction

$$4.9\mu \times 1.25 = 2.45$$

$$6.125\mu = 2.45$$

$$\mu = 0.4$$

b) Slack after moving 0.5 m

$$F = 4.9 \times 0.4 = 1.96$$

$$\frac{1}{2} \times 0.5 \times v^2 = 2.45 - 1.96 \times 0.5$$

$$0.25v^2 = 1.47$$

$$v^2 = 5.88$$

$$v = 2.42 \text{ m s}^{-1}$$

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11 A bungee jumper of mass 72 kg is attached to one end of a light elastic rope of natural length 16 m. The other end of the rope is fixed to a bridge. The jumper steps off the bridge from rest at the point where the rope is fixed and falls vertically. The jumper first comes to instantaneous rest 40 m below the bridge.

The jumper is modelled as a particle and air resistance is ignored.

a. Show that the modulus of elasticity of the rope is 1568 N.

b. Find the speed of the jumper when they are 30 m below the bridge.

c. Find the greatest speed of the jumper.

d. State one way in which ignoring air resistance affects your answer to part (c).

(10 marks)

a) Lowest point: extension 40 - 16 = 24, at rest

GPE lost = EPE gained

$$72 \times 9.8 \times 40 = \frac{\lambda \times 24^2}{2 \times 16}$$

$$28224 = 18\lambda$$

$$\boxed{\lambda = 1568 \text{ N}}$$

b) At 30 m below the bridge, extension = 14

$$\frac{1}{2} \times 72 \times v^2 = 72 \times 9.8 \times 30 - \frac{1568 \times 14^2}{2 \times 16}$$

$$36v^2 = 21168 - 9604$$

$$36v^2 = 11564$$

$$v^2 = 321.22...$$

$$\boxed{v = 17.9 \text{ m s}^{-1}}$$

c) Greatest speed when acceleration = 0: tension = weight

$$\frac{1568e}{16} = 72 \times 9.8$$

$$98e = 705.6$$

$$e = 7.2$$

Fallen 16 + 7.2 = 23.2 m

$$\frac{1}{2} \times 72 \times v^2 = 72 \times 9.8 \times 23.2 - \frac{1568 \times 7.2^2}{2 \times 16}$$

$$36v^2 = 16369.92 - 2540.16$$

$$36v^2 = 13829.76$$

$$v^2 = 384.16$$

$$\boxed{v = 19.6 \text{ m s}^{-1}}$$

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d) Air resistance does work against the motion

the true greatest speed would be less than 19.6 m s^{-1}

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12 A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity 29.4 N. The other end of the string is attached to a fixed point O on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

P is held at rest at the point A on the plane, where A is below O on a line of greatest slope and OA = 1.2 m. P is released and slides down the line of greatest slope. It comes to rest for the first time at the point B, where AB = 0.576 m, as shown in the diagram. The coefficient of friction between P and the plane is μ.

a. Show that μ = 0.3

b. Show that P remains at rest at B.

c. Find the greatest speed of P as it moves from A to B.

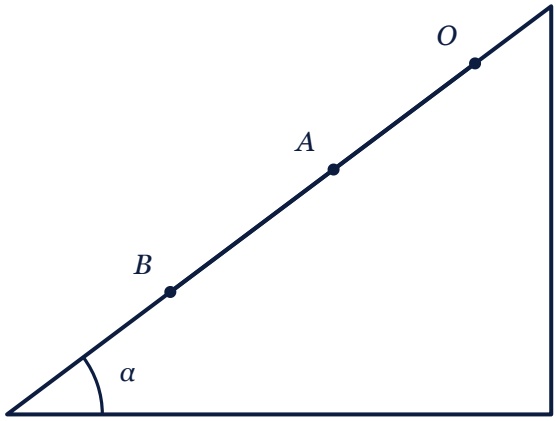


Diagram NOT accurately drawn

(12 marks)

$\tan \alpha = \frac{3}{4}$ so $\sin \alpha = 0.6$, $\cos \alpha = 0.8$

$R = 2 \times 9.8 \times 0.8 = 15.68$

$F = 15.68\mu$

a) A to B: at rest at both, string at natural length at A

$$\text{GPE lost} = 2 \times 9.8 \times 0.6 \times 0.576 = 6.77376$$

$$\text{EPE at B} = \frac{29.4 \times 0.576^2}{2 \times 1.2} = 4.064256$$

$$\text{work against friction} = 15.68\mu \times 0.576 = 9.03168\mu$$

Work-energy

$$6.77376 = 4.064256 + 9.03168\mu$$

$$9.03168\mu = 2.709504$$

$$\boxed{\mu = 0.3}$$

b) Forces at B along the plane

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$$T = \frac{29.4 \times 0.576}{1.2} = 14.112$$

$2g \sin\alpha = 11.76$

$14.112 - 11.76 = 2.352$ up the plane

maximum friction $= 0.3 \times 15.68 = 4.704$

$2.352 < 4.704$

friction can hold P, so P remains at rest at B

c) Greatest speed when acceleration = 0, friction acts up the plane

$T + 4.704 = 11.76$

$T = 7.056$

$\frac{29.4x}{1.2} = 7.056$

$x = 0.288$

Work-energy from A to this point

$$\frac{1}{2} \times 2 \times v^2 = 11.76 \times 0.288 - 4.704 \times 0.288 - \frac{29.4 \times 0.288^2}{2 \times 1.2}$$

$v^2 = 3.38688 - 1.354752 - 1.016064$

$v^2 = 1.016064$

$v = 1.01 \text{ m s}^{-1}$

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