Hooke's law and elastic strings

Worked solutions

Further Maths topic practice

This topic online

1 A light elastic string has natural length 0.8 m and modulus of elasticity 24 N. A light spring has natural length 0.5 m and modulus of elasticity 40 N.

a. Find the tension in the string when it is stretched to a length of 1.1 m.

b. Find the thrust in the spring when it is compressed to a length of 0.35 m.

(3 marks)

a) Hooke's law

$$T = \frac{\lambda x}{l}$$

$$x = 1.1 - 0.8 = 0.3$$

$$T = \frac{24 \times 0.3}{0.8}$$

$$\boxed{T = 9 \text{ N}}$$

b) Same law, x is now the compression

$$x = 0.5 - 0.35 = 0.15$$

$$T = \frac{40 \times 0.15}{0.5}$$

$$\boxed{\text{Thrust} = 12 \text{ N}}$$

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2 A particle P of mass 1.5 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 45 N. The other end of the string is attached to a fixed point O. The particle hangs in equilibrium vertically below O.

Find the length OP.

(3 marks)

Resolve vertically

$$T = 1.5 \times 9.8$$

$$T = 14.7$$

Hooke's law

$$\frac{45x}{0.6} = 14.7$$

$$x = \frac{14.7 \times 0.6}{45}$$

$$x = 0.196$$

$$OP = 0.6 + 0.196$$

$$\boxed{OP = 0.796 \text{ m}}$$

3 A light elastic string has one end attached to a fixed point. When a particle of mass 2 kg hangs in equilibrium on the other end, the length of the string is 1.2 m. When the 2 kg particle is replaced by a particle of mass 3 kg, the length of the string in equilibrium is 1.35 m.

Find the natural length and the modulus of elasticity of the string. (5 marks)

Tension = weight in each case

$$2g = \frac{\lambda(1.2 - l)}{l}$$

$$3g = \frac{\lambda(1.35 - l)}{l}$$

Divide the equations

$$\frac{2}{3} = \frac{(1.2 - l)}{(1.35 - l)}$$

$$2(1.35 - l) = 3(1.2 - l)$$

$$2.7 - 2l = 3.6 - 3l$$

$$\boxed{l = 0.9 \text{ m}}$$

Substitute into the first equation

$$19.6 = \frac{\lambda \times 0.3}{0.9}$$

$$\lambda = \frac{19.6 \times 0.9}{0.3}$$

$$\boxed{\lambda = 58.8 \text{ N}}$$

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4 A light spring has natural length 0.4 m and modulus of elasticity 49 N. The spring stands vertically with its lower end fixed to a horizontal floor. A particle P of mass m kg rests in equilibrium on the top of the spring, and the length of the spring is 0.3 m, as shown in the diagram.

a. Find the value of m.

A vertical force of magnitude F newtons is now applied to P, pushing it downwards, so that P rests in equilibrium with the length of the spring equal to 0.22 m.

b. Find the value of F.

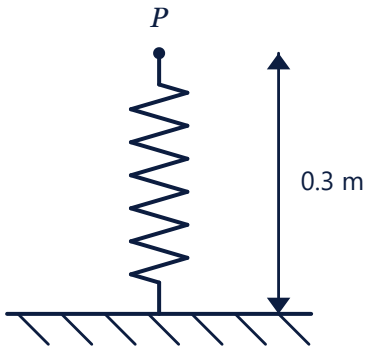


Diagram NOT accurately drawn

(5 marks)

a) Compression

$$x = 0.4 - 0.3 = 0.1$$

$$\text{Thrust} = \frac{49 \times 0.1}{0.4}$$

$$\text{Thrust} = 12.25$$

Resolve vertically

$$9.8m = 12.25$$

$$\boxed{m = 1.25}$$

b) New compression

$$x = 0.4 - 0.22 = 0.18$$

$$\text{Thrust} = \frac{49 \times 0.18}{0.4}$$

$$\text{Thrust} = 22.05$$

Resolve vertically: thrust up = weight + F

$$22.05 = 12.25 + F$$

$$\boxed{F = 9.8}$$

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5 A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity 30 N. The other end of the string is attached to a fixed point A on a smooth plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

P rests in equilibrium on the plane below A, with the string lying along a line of greatest slope, as shown in the diagram.

a. Find the length AP.

P is now pulled down the line of greatest slope and held in equilibrium with AP = 2 m by a force acting parallel to the plane.

b. Find the magnitude of this force.

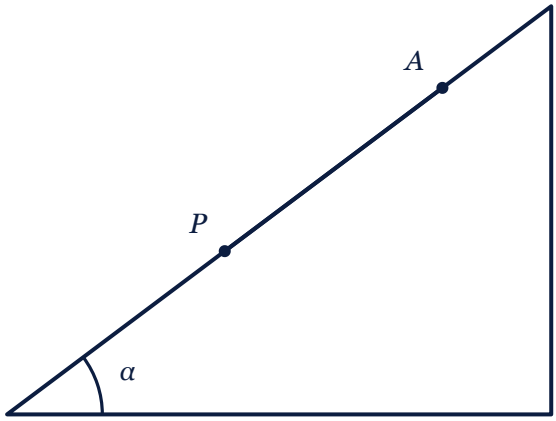


Diagram NOT accurately drawn

(6 marks)

$\tan \alpha = \frac{3}{4}$ so $\sin \alpha = \frac{3}{5}$

a) Resolve parallel to the plane

$T = 2g \sin \alpha$

$T = 2 \times 9.8 \times 0.6$

$T = 11.76$

Hooke's law

$\frac{30x}{1.2} = 11.76$

$x = 0.4704$

$AP = 1.2 + 0.4704$

$\boxed{AP = 1.67 \text{ m}}$

b) New extension

$x = 2 - 1.2 = 0.8$

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$$T = \frac{30 \times 0.8}{1.2}$$

$$T = 20$$

Resolve parallel to the plane: F acts down the slope

$$F + 11.76 = 20$$

$$\boxed{F = 8.24 \text{ N}}$$

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6 Two points A and B on a smooth horizontal floor are 1.8 m apart. A light spring of natural length 0.8 m and modulus of elasticity 20 N joins A to a particle P. A second light spring of natural length 1.2 m and modulus of elasticity 30 N joins P to B. The springs lie along the straight line AB and P rests in equilibrium, as shown in the diagram.

a. Find the distance AP.

b. Find the thrust in each spring.

c. Explain why P would not have a single equilibrium position if the two springs were replaced by light elastic strings with the same natural lengths and moduli.

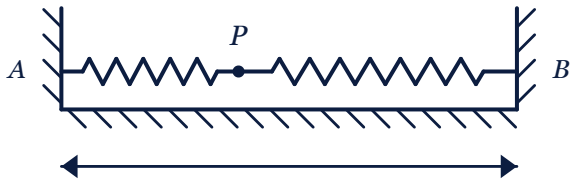


Diagram NOT accurately drawn

(7 marks)

a) 0.8 + 1.2 = 2 > 1.8, so both springs are compressed

Let AP = x

compression of AP = $0.8 - x$

compression of PB = $1.2 - (1.8 - x) = x - 0.6$

Horizontal forces on P balance

$$\frac{20(0.8 - x)}{0.8} = \frac{30(x - 0.6)}{1.2}$$

$$25(0.8 - x) = 25(x - 0.6)$$

$$20 - 25x = 25x - 15$$

$$50x = 35$$

AP = 0.7 m

b) Compression of AP = 0.8 − 0.7 = 0.1

$$\text{Thrust in AP} = \frac{20 \times 0.1}{0.8} = 2.5$$

Compression of PB = 0.7 − 0.6 = 0.1

$$\text{Thrust in PB} = \frac{30 \times 0.1}{1.2} = 2.5$$

Thrust = 2.5 N in each spring

c) A string can only pull, it cannot push

0.8 + 1.2 = 2 > 1.8, so at least one string must be slack

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One taut string alone cannot be balanced, so both would be slack

P could rest anywhere with $0.6 \leqslant AP \leqslant 0.8$

7 A light elastic string has natural length 1.2 m and modulus of elasticity λ N. Its ends are attached to two fixed points A and B which are 1.6 m apart on the same horizontal level. A particle P of mass 0.8 kg is attached to the midpoint of the string and hangs in equilibrium 0.6 m below the midpoint of AB, as shown in the diagram.

a. Find the tension in the string.

b. Show that λ = 9.8

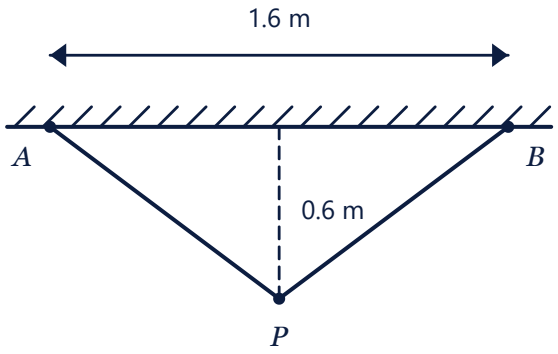


Diagram NOT accurately drawn

(6 marks)

a) Each half of the string by Pythagoras

$AP = \sqrt{(0.8^2 + 0.6^2)} = 1$

θ = angle of AP with the vertical

$\cos\theta = \frac{0.6}{1} = 0.6$

Resolve vertically

$2T\cos\theta = 0.8g$

$1.2T = 7.84$

$T = 6.5333...$

$T = 6.53$ N

b) Each half: natural length 0.6, modulus λ

$x = 1 - 0.6 = 0.4$

$6.5333... = \frac{\lambda \times 0.4}{0.6}$

$\lambda = \frac{6.5333... \times 0.6}{0.4}$

$\lambda = 9.8$

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8 A fixed point A is 3 m vertically above a fixed point B. A particle P of mass 2 kg is attached to one end of each of two light elastic strings. The other end of the first string is attached to A, and the other end of the second string is attached to B.

The string AP has natural length 1 m and modulus of elasticity 49 N. The string PB has natural length 1 m and modulus of elasticity 98 N. The particle rests in equilibrium on the line AB with both strings taut, as shown in the diagram.

a. Find the distance AP.

b. Find the tension in each string.

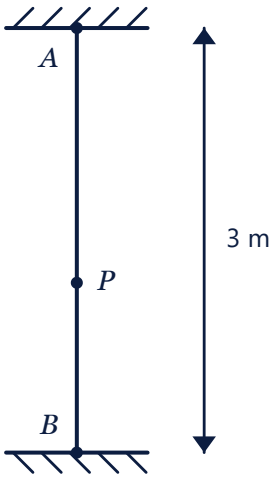


Diagram NOT accurately drawn

(7 marks)

a) Let AP = d, so PB = 3 − d

T_1 = tension in AP, T_2 = tension in PB

$$T_1 = \frac{49(d-1)}{1}$$

$$T_2 = \frac{98(3-d-1)}{1} = 98(2-d)$$

Resolve vertically: up = down

$$49(d-1) = 98(2-d) + 2 \times 9.8$$

$$49d - 49 = 196 - 98d + 19.6$$

$$147d = 264.6$$

$$\boxed{AP = 1.8 \text{ m}}$$

Check: PB = 1.2 > 1, so PB is taut

b)

$$T_1 = 49 \times 0.8$$

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$\boxed{T_1 = 39.2\ N}$

$T_2 = 98 \times 0.2$

$\boxed{T_2 = 19.6\ N}$

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9 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point O on a rough plane inclined at 30° to the horizontal. The coefficient of friction between P and the plane is 0.4.

P rests in equilibrium on the plane below O, with the string taut and lying along a line of greatest slope, as shown in the diagram.

Find the range of possible values of the length OP.

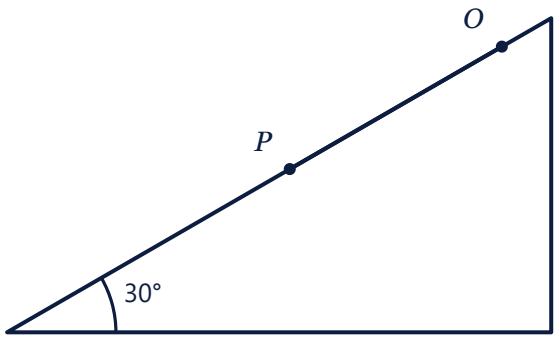


Diagram NOT accurately drawn

(8 marks)

Resolve perpendicular to the plane

$R = 0.5 \times 9.8 \times \cos 30^\circ$

$R = 4.2435...$

Limiting friction

$F = 0.4 \times 4.2435...$

$F = 1.6974...$

$0.5g \sin 30^\circ = 2.45$

Shortest OP: about to slide down, friction acts up the plane

$T + 1.6974 = 2.45$

$T = 0.75259...$

$\frac{12x}{0.8} = 0.75259...$

$x = 0.050173...$

$OP = 0.8 + 0.050173 = 0.850$

Longest OP: about to slide up, friction acts down the plane

$T = 2.45 + 1.6974$

$T = 4.1474...$

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$$\frac{12x}{0.8} = 4.1474...$$

$$x = 0.27649...$$

$$OP = 0.8 + 0.27649 = 1.08$$

$$\boxed{0.850 \text{ m} \leq OP \leq 1.08 \text{ m}}$$

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10 A bird feeder of mass 0.6 kg hangs from a fixed hook O by a light elastic cord of natural length 1.5 m and modulus of elasticity 29.4 N. On a windy day the wind exerts a constant horizontal force of magnitude 4.41 N on the feeder. The feeder hangs in equilibrium with the cord making an angle θ with the downward vertical, as shown in the diagram.

The feeder is modelled as a particle.

a. Find the tension in the cord.

b. Find the value of θ.

c. Find the vertical distance of the feeder below O.

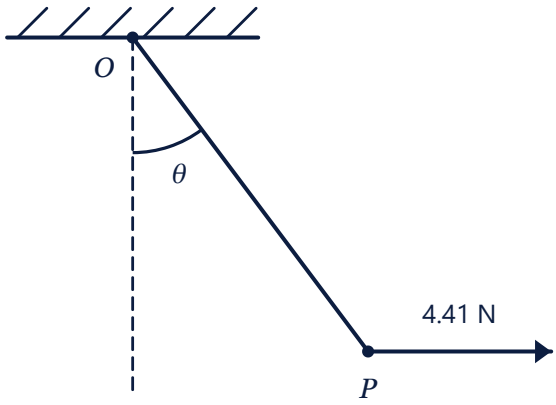


Diagram NOT accurately drawn

(8 marks)

a) Resolve vertically and horizontally

$T\cos\theta = 0.6 \times 9.8 = 5.88$

$T\sin\theta = 4.41$

Square and add

$T = \sqrt{(5.88^2 + 4.41^2)}$

$\boxed{T = 7.35\ \text{N}}$

b) Divide the equations

$\tan\theta = \dfrac{4.41}{5.88} = 0.75$

$\boxed{\theta = 36.9^\circ}$

c) Hooke's law for the extension

$$\frac{29.4x}{1.5} = 7.35$$

$$x = 0.375$$

Length of cord $= 1.5 + 0.375 = 1.875$

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$$\cos\theta = \frac{5.88}{7.35} = 0.8$$

$$\text{depth} = 1.875 \times 0.8$$

$$\boxed{\text{depth} = 1.5 \text{ m}}$$

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11 A particle A of mass 3 kg rests on a rough horizontal table. The coefficient of friction between A and the table is 0.5. One end of a light elastic string of natural length 0.9 m and modulus of elasticity 35 N is attached to A. The string passes over a small smooth pulley fixed at the edge of the table, and a particle B hangs freely from the other end, as shown in the diagram. The part of the string between A and the pulley is horizontal.

a. Given that B has mass 1.2 kg and the system is in equilibrium, find the extension of the string.

b. Find the greatest mass that B could have if A is to remain at rest, and the extension of the string in this case.

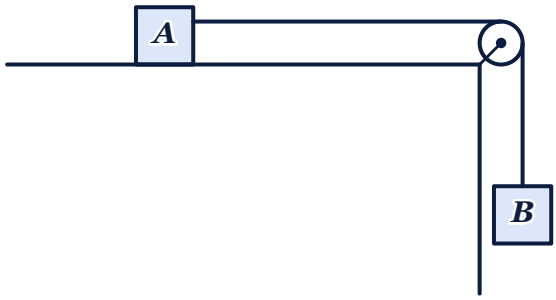


Diagram NOT accurately drawn

(7 marks)

a) Smooth pulley: same tension throughout

Resolve vertically for B

$T = 1.2 \times 9.8 = 11.76$

Check A: maximum friction

$F = 0.5 \times 3 \times 9.8 = 14.7$

$11.76 < 14.7$ so A stays at rest

Hooke's law

$$\frac{35x}{0.9} = 11.76$$

$$x = 0.3024$$

extension = 0.302 m

b) Limiting equilibrium: T = F

$T = 14.7$

$mg = 14.7$

$m = 1.5$ kg

$$\frac{35x}{0.9} = 14.7$$

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extension = 0.378 m

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12 Two fixed points A and B are 2 m apart on the same horizontal level. A particle P of mass 2.5 kg is attached to A by a light elastic string of natural length 1 m and to B by a light elastic string of natural length 1.2 m. The particle hangs in equilibrium below AB with AP = 1.2 m and BP = 1.6 m, as shown in the diagram.

a. Show that angle APB = 90°.

b. Find the tension in each string.

c. Find the modulus of elasticity of each string.

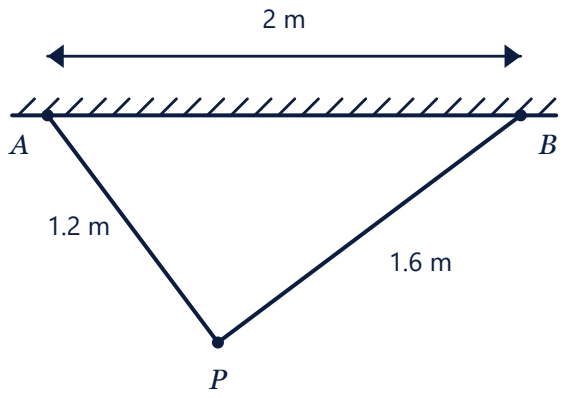


Diagram NOT accurately drawn

(10 marks)

a) Converse of Pythagoras

$1.2^2 + 1.6^2 = 1.44 + 2.56 = 4$

$2^2 = 4$

angle APB = 90°

b) T_1 = tension in AP, T_2 = tension in BP

Angles with the horizontal

$\cos A = \frac{1.2}{2} = 0.6$, $\sin A = 0.8$

$\cos B = \frac{1.6}{2} = 0.8$, $\sin B = 0.6$

Resolve horizontally

$T_1 \cos A = T_2 \cos B$

$0.6T_1 = 0.8T_2$

Resolve vertically

$T_1 \sin A + T_2 \sin B = 2.5g$

$0.8T_1 + 0.6T_2 = 24.5$

From the first equation $T_2 = 0.75T_1$

$0.8T_1 + 0.45T_1 = 24.5$

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$$1.25T_1 = 24.5$$

$$\boxed{T_1 = 19.6\ N}$$

$$T_2 = 0.75 \times 19.6$$

$$\boxed{T_2 = 14.7\ N}$$

c) Hooke's law for each string

$$19.6 = \frac{\lambda_1 \times 0.2}{1}$$

$$\boxed{\lambda_1 = 98\ N}$$

$$14.7 = \frac{\lambda_2 \times 0.4}{1.2}$$

$$\boxed{\lambda_2 = 44.1\ N}$$

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