Momentum and impulse as vectors

Worked solutions

Further Maths topic practice

This topic online

1 A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} - 2\mathbf{j})$ m s^{-1} when it receives an impulse of $(3\mathbf{i} + 5\mathbf{j})$ N s.

Find the speed of P immediately after receiving the impulse. (3 marks)

Impulse = change in momentum

$$I = mv - mu$$

$$3i + 5j = 0.5v - 0.5(4i - 2j)$$

$$0.5v = 3i + 5j + 2i - j$$

$$0.5v = 5i + 4j$$

$$v = 10i + 8j$$

Speed = magnitude of the velocity

$$|v| = \sqrt{(10^2 + 8^2)}$$

$$|v| = \sqrt{164} = 12.8062...$$

$$\boxed{12.8 \text{ m s}^{-1}}$$

2 A ball of mass 0.4 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j})$ m s^{-1} when it is struck by a racket. Immediately after being struck, the ball has velocity $(-2\mathbf{i} + 6\mathbf{j})$ m s^{-1}.

a. Find the impulse exerted on the ball by the racket, giving your answer in the form $(a\mathbf{i} + b\mathbf{j})$ N s.

b. Find the magnitude of this impulse. (4 marks)

a) Impulse = change in momentum

$$I = m(v - u)$$

$$I = 0.4((-2i + 6j) - (5i + 3j))$$

$$I = 0.4(-7i + 3j)$$

$$\boxed{I = (-2.8i + 1.2j) \text{ N s}}$$

b) Magnitude by Pythagoras

$$|I| = \sqrt{((-2.8)^2 + 1.2^2)}$$

$$|I| = \sqrt{9.28} = 3.0463...$$

$$\boxed{3.05 \text{ N s}}$$

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3 A particle of mass 0.25 kg is moving with velocity $(2\mathbf{i} + 8\mathbf{j})$ m s^{-1}. It receives an impulse of magnitude 6.5 N s in the direction of the vector $(5\mathbf{i} - 12\mathbf{j})$.

a. Write the impulse in the form $(a\mathbf{i} + b\mathbf{j})$ N s.

b. Find the speed of the particle immediately after the impulse. (4 marks)

a) Unit vector in the direction $5i - 12j$

$$|5i - 12j| = \sqrt{(25 + 144)} = 13$$

$$I = \frac{6.5}{13}(5i - 12j)$$

$$I = 0.5(5i - 12j)$$

$$\boxed{I = (2.5i - 6j)\ \text{N s}}$$

b) $I = mv - mu$

$$2.5i - 6j = 0.25v - 0.25(2i + 8j)$$

$$0.25v = 2.5i - 6j + 0.5i + 2j$$

$$0.25v = 3i - 4j$$

$$v = 12i - 16j$$

$$|v| = \sqrt{(12^2 + 16^2)} = \sqrt{400}$$

$$\boxed{20\ \text{m s}^{-1}}$$

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4 A particle of mass 2 kg is moving on a smooth horizontal surface with velocity $(6\mathbf{i} + \mathbf{j})$ m s^{-1}. It receives an impulse of $(-4\mathbf{i} + 6\mathbf{j})$ N s.

a. Find the velocity of the particle immediately after the impulse.

b. Find the angle through which the direction of motion of the particle is turned by the impulse. (5 marks)

a) $I = mv - mu$

$-4i + 6j = 2v - 2(6i + j)$

$2v = -4i + 6j + 12i + 2j$

$2v = 8i + 8j$

$\boxed{v = (4i + 4j) \text{ m s}^{-1}}$

b) Angle each velocity makes with i

before: $\tan\alpha = \frac{1}{6}$, $\alpha = 9.462...^\circ$

after: $\tan\beta = \frac{4}{4}$, $\beta = 45^\circ$

Both are in the first quadrant, so subtract

$\theta = 45^\circ - 9.462...^\circ = 35.538...^\circ$

Check with the scalar product

$\cos\theta = \frac{(6)(4) + (1)(4)}{(\sqrt{37} \times \sqrt{32})} = 0.8137...$

$\boxed{35.5^\circ}$

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5 A particle Q of mass 0.5 kg is moving with velocity $(8\mathbf{i} + 2\mathbf{j})$ m s^{-1}. It receives an impulse of $(-3\mathbf{i} + \lambda\mathbf{j})$ N s, where λ is a constant. Immediately after the impulse, Q is moving parallel to the vector $(\mathbf{i} + 2\mathbf{j})$.

a. Find the value of λ.

b. Find the kinetic energy lost by Q as a result of the impulse.

(6 marks)

a) $I = mv - mu$

$-3i + \lambda j = 0.5v - 0.5(8i + 2j)$

$0.5v = -3i + \lambda j + 4i + j$

$0.5v = i + (1 + \lambda)j$

$v = 2i + (2 + 2\lambda)j$

Parallel to $i + 2j$: j-component is twice the i-component

$2 + 2\lambda = 2 \times 2$

$2\lambda = 2$

$\boxed{\lambda = 1}$

b) $v = 2i + 4j$; $KE = \frac{1}{2}m|v|^2$

KE before $= \frac{1}{2}(0.5)(8^2 + 2^2) = 17$ J

KE after $= \frac{1}{2}(0.5)(2^2 + 4^2) = 5$ J

KE lost $= 17 - 5$

$\boxed{12 \text{ J}}$

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6 A cricket ball of mass 0.16 kg is moving horizontally with speed 25 m s^{-1} when it is hit by a bat. Immediately after being hit, the ball moves horizontally with speed 30 m s^{-1}. The angle between the direction of motion of the ball before it is hit and its direction of motion after it is hit is 120°, as shown in the diagram.

a. Find the magnitude of the impulse exerted on the ball by the bat.

b. Find the angle between the direction of this impulse and the original direction of motion of the ball.

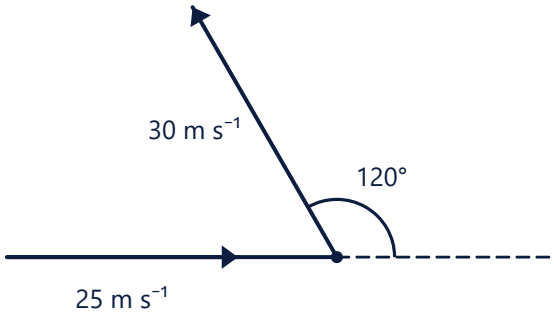


Diagram NOT accurately drawn

(6 marks)

a) i along the original direction, j perpendicular to it

$u = 25i$

$v = 30\cos 120^\circ\, i + 30\sin 120^\circ\, j$

$v = -15i + 15\sqrt{3}\, j$

$I = m(v - u)$

$I = 0.16(-15i + 15\sqrt{3}\, j - 25i)$

$I = 0.16(-40i + 15\sqrt{3}\, j)$

$I = -6.4i + 2.4\sqrt{3}\, j$

$|I| = \sqrt{(6.4^2 + (2.4\sqrt{3})^2)}$

$|I| = \sqrt{(40.96 + 17.28)} = \sqrt{58.24} = 7.6315\ldots$

7.63 N s

b) I points backwards (negative i) and upwards (positive j)

$\tan\beta = \frac{2.4\sqrt{3}}{6.4}$, $\beta = 33.004\ldots^\circ$

β is measured from $-i$, so from the original direction

angle $= 180^\circ - 33.004\ldots^\circ$

147°

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7 A particle P of mass 0.2 kg is moving with velocity $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1} when it receives an impulse of $\lambda(\mathbf{i} - 2\mathbf{j})$ N s, where λ is a non-zero constant. The speed of P immediately after the impulse is the same as its speed immediately before the impulse.

a. Show that the velocity of P immediately after the impulse is $((3 + 5\lambda)\mathbf{i} + (4 - 10\lambda)\mathbf{j})$ m s^{-1}.

b. Find the value of λ.

c. Find the angle through which the direction of motion of P is turned by the impulse. (7 marks)

a) $I = mv - mu$

$$\lambda(i - 2j) = 0.2v - 0.2(3i + 4j)$$

$$0.2v = 0.6i + 0.8j + \lambda i - 2\lambda j$$

Multiply by 5

$$\boxed{v = (3 + 5\lambda)i + (4 - 10\lambda)j}$$

b) Speed before $= \sqrt{(3^2 + 4^2)} = 5$

$$(3 + 5\lambda)^2 + (4 - 10\lambda)^2 = 25$$

$$9 + 30\lambda + 25\lambda^2 + 16 - 80\lambda + 100\lambda^2 = 25$$

$$125\lambda^2 - 50\lambda = 0$$

$$25\lambda(5\lambda - 2) = 0$$

λ is non-zero

$$\boxed{\lambda = 0.4}$$

c) $v = (3 + 2)i + (4 - 4)j = 5i$

Scalar product of the two velocities

$$\cos\theta = \frac{(3)(5) + (4)(0)}{(5 \times 5)}$$

$$\cos\theta = 0.6$$

$$\theta = 53.130...^\circ$$

$$\boxed{53.1^\circ}$$

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8 Two particles A and B, of masses 2 kg and 3 kg, are moving on a smooth horizontal plane. The velocity of A is $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of B is $(-\mathbf{i} + 4\mathbf{j})$ m s^{-1}. The particles collide and coalesce to form a single particle C.

a. Find the velocity of C.

b. Find the speed of C.

c. Find the magnitude of the impulse exerted on A in the collision.

d. Find the kinetic energy lost in the collision.

(8 marks)

a) Conservation of momentum (vectors)

$$2(3\mathbf{i} + 2\mathbf{j}) + 3(-\mathbf{i} + 4\mathbf{j}) = 5\mathbf{v}$$

$$6\mathbf{i} + 4\mathbf{j} - 3\mathbf{i} + 12\mathbf{j} = 5\mathbf{v}$$

$$5\mathbf{v} = 3\mathbf{i} + 16\mathbf{j}$$

$$\mathbf{v} = (0.6\mathbf{i} + 3.2\mathbf{j}) \text{ m s}^{-1}$$

b) Speed

$$|\mathbf{v}| = \sqrt{(0.6^2 + 3.2^2)}$$

$$|\mathbf{v}| = \sqrt{10.6} = 3.2558...$$

$$3.26 \text{ m s}^{-1}$$

c) Impulse on A = change in momentum of A

$$\mathbf{I} = 2(0.6\mathbf{i} + 3.2\mathbf{j}) - 2(3\mathbf{i} + 2\mathbf{j})$$

$$\mathbf{I} = -4.8\mathbf{i} + 2.4\mathbf{j}$$

$$|\mathbf{I}| = \sqrt{(4.8^2 + 2.4^2)} = \sqrt{28.8} = 5.3666...$$

$$5.37 \text{ N s}$$

d) $KE = \frac{1}{2}m|\mathbf{v}|^2$

$$KE \text{ before} = \frac{1}{2}(2)(3^2 + 2^2) + \frac{1}{2}(3)(1^2 + 4^2)$$

$$KE \text{ before} = 13 + 25.5 = 38.5 \text{ J}$$

$$KE \text{ after} = \frac{1}{2}(5)(10.6) = 26.5 \text{ J}$$

$$KE \text{ lost} = 38.5 - 26.5$$

$$12 \text{ J}$$

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9 A particle of mass 0.5 kg is moving with velocity $(6\mathbf{i} - 2\mathbf{j})$ m s^{-1}. It receives an impulse of $(a\mathbf{i} + 4\mathbf{j})$ N s, where a is a constant. Immediately after the impulse, the particle is moving at right angles to its original direction of motion.

a. Find the value of a.

b. Find the speed of the particle immediately after the impulse.

c. Find the angle between the direction of the impulse and the original direction of motion of the particle.

(7 marks)

a) $I = mv - mu$

$$a\mathbf{i} + 4\mathbf{j} = 0.5\mathbf{v} - 0.5(6\mathbf{i} - 2\mathbf{j})$$

$$0.5\mathbf{v} = a\mathbf{i} + 4\mathbf{j} + 3\mathbf{i} - \mathbf{j}$$

$$0.5\mathbf{v} = (3 + a)\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{v} = (6 + 2a)\mathbf{i} + 6\mathbf{j}$$

Perpendicular: scalar product = 0

$$(6 + 2a)(6) + (6)(-2) = 0$$

$$36 + 12a - 12 = 0$$

$$12a = -24$$

$$\boxed{a = -2}$$

b) $\mathbf{v} = 2\mathbf{i} + 6\mathbf{j}$

$$|\mathbf{v}| = \sqrt{(2^2 + 6^2)} = \sqrt{40} = 6.3246\ldots$$

$$\boxed{6.32 \text{ m s}^{-1}}$$

c) Impulse $-2\mathbf{i} + 4\mathbf{j}$, original velocity $6\mathbf{i} - 2\mathbf{j}$

$$\cos\theta = \frac{(-2)(6) + (4)(-2)}{(\sqrt{20} \times \sqrt{40})}$$

$$\cos\theta = \frac{-20}{\sqrt{800}} = -\frac{1}{\sqrt{2}}$$

$$\boxed{135^\circ}$$

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10 [In this question **i** and **j** are horizontal unit vectors due east and due north respectively, and position vectors are given relative to a fixed origin O.]

A puck of mass 0.25 kg slides on smooth horizontal ice with constant velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. At time $t = 0$ seconds the puck is at the point with position vector $(\mathbf{i} + 10\mathbf{j})$ m. At time $t = 2$ the puck is struck by a hockey stick and receives an impulse of $(1.5\mathbf{i} + 2\mathbf{j})$ N s. The puck is modelled as a particle.

a. Find the position vector of the puck at time $t = 2$.

b. Find the velocity of the puck immediately after it is struck.

c. Show that the puck passes through the point with position vector $(21\mathbf{i} + 14\mathbf{j})$ m, and find the time at which it does so.

d. Find the kinetic energy gained by the puck when it is struck.

(9 marks)

a) $r = r(0) + vt$ for constant velocity

$r = (i + 10j) + 2(2i - 3j)$

$\boxed{(5i + 4j)\text{ m}}$

b) $I = mv - mu$

$1.5i + 2j = 0.25v - 0.25(2i - 3j)$

$0.25v = 1.5i + 2j + 0.5i - 0.75j$

$0.25v = 2i + 1.25j$

$\boxed{v = (8i + 5j)\text{ m s}^{-1}}$

c) For $t \geq 2$, the time since the impulse is $t - 2$

$r = (5i + 4j) + (t - 2)(8i + 5j)$

i-component = 21

$5 + 8(t - 2) = 21$

$t - 2 = 2$

Check the j-component at the same time

$4 + 5(2) = 14$ ✓

$\boxed{\text{passes through } (21i + 14j) \text{ at } t = 4}$

d) $KE = \frac{1}{2}m|v|^2$

KE before $= \frac{1}{2}(0.25)(2^2 + 3^2) = 1.625$ J

KE after $= \frac{1}{2}(0.25)(8^2 + 5^2) = 11.125$ J

gain $= 11.125 - 1.625$

$\boxed{9.5\text{ J}}$

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11 A particle P of mass 2 kg is moving on a smooth horizontal surface with velocity $(\mathbf{i} + 2\mathbf{j})$ m s^{-1}. It receives an impulse of $(4\mathbf{i} - 2\mathbf{j})$ N s.

a. Find the velocity of P immediately after this impulse.

Some time later P receives a second impulse, of $\lambda(\mathbf{i} + 3\mathbf{j})$ N s, where λ is a positive constant. Immediately after the second impulse, the speed of P is $4\sqrt{2}$ m s^{-1}.

b. Find the value of λ.

c. Find the angle between the direction of motion of P before the first impulse and its direction of motion after the second impulse.

(10 marks)

a) $I = mv - mu$

$$4i - 2j = 2v - 2(i + 2j)$$

$$2v = 4i - 2j + 2i + 4j$$

$$2v = 6i + 2j$$

$$\boxed{v = (3i + j) \text{ m s}^{-1}}$$

b) Second impulse, final velocity w

$$\lambda(i + 3j) = 2w - 2(3i + j)$$

$$2w = (6 + \lambda)i + (2 + 3\lambda)j$$

$$w = (3 + \frac{\lambda}{2})i + (1 + \frac{3\lambda}{2})j$$

Speed $4\sqrt{2}$, so $|w|^2 = 32$

$$(3 + \frac{\lambda}{2})^2 + (1 + \frac{3\lambda}{2})^2 = 32$$

$$9 + 3\lambda + \frac{\lambda^2}{4} + 1 + 3\lambda + \frac{9\lambda^2}{4} = 32$$

$$\frac{5}{2}\lambda^2 + 6\lambda + 10 = 32$$

$$5\lambda^2 + 12\lambda - 44 = 0$$

$$(\lambda - 2)(5\lambda + 22) = 0$$

λ is positive

$$\boxed{\lambda = 2}$$

c) $w = 4i + 4j$; first velocity $i + 2j$

$$\cos\theta = \frac{(1)(4) + (2)(4)}{(\sqrt{5} \times \sqrt{32})}$$

$$\cos\theta = \frac{12}{\sqrt{160}} = 0.9487...$$

$$\theta = 18.435...^\circ$$

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18.4°

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12 Two smooth spheres A and B, of equal radius, have masses 3 kg and 2 kg respectively. They are moving on a smooth horizontal surface. Immediately before they collide, the velocity of A is $(4\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of B is $(-3\mathbf{i} + \mathbf{j})$ m s^{-1}. Immediately after the collision, the velocity of A is $(\mathbf{i} + 2\mathbf{j})$ m s^{-1}.

a. Find the velocity of B immediately after the collision.

b. Find the impulse exerted on B by A in the collision.

c. Explain what your answer to part (b) tells you about the line of centres of the spheres at the moment of impact.

d. Find the kinetic energy lost in the collision.

e. Find the angle through which the direction of motion of B is turned by the collision.

(12 marks)

a) Conservation of momentum (vectors)

$$3(4\mathbf{i} + 2\mathbf{j}) + 2(-3\mathbf{i} + \mathbf{j}) = 3(\mathbf{i} + 2\mathbf{j}) + 2\mathbf{v}$$

$$12\mathbf{i} + 6\mathbf{j} - 6\mathbf{i} + 2\mathbf{j} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{v}$$

$$6\mathbf{i} + 8\mathbf{j} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{v}$$

$$2\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$$

$$\mathbf{v} = (1.5\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$$

b) Impulse on B = change in momentum of B

$$\mathbf{I} = 2(1.5\mathbf{i} + \mathbf{j}) - 2(-3\mathbf{i} + \mathbf{j})$$

$$\mathbf{I} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{i} - 2\mathbf{j}$$

$$\mathbf{I} = 9\mathbf{i} \text{ N s}$$

c) Smooth spheres: the impulse acts along the line of centres

The impulse is parallel to $\mathbf{i}$

Line of centres is parallel to $\mathbf{i}$

d) $KE = \frac{1}{2}m|\mathbf{v}|^2$

$$KE \text{ before} = \frac{1}{2}(3)(4^2 + 2^2) + \frac{1}{2}(2)(3^2 + 1^2)$$

$$KE \text{ before} = 30 + 10 = 40 \text{ J}$$

$$KE \text{ after} = \frac{1}{2}(3)(1^2 + 2^2) + \frac{1}{2}(2)(1.5^2 + 1^2)$$

$$KE \text{ after} = 7.5 + 3.25 = 10.75 \text{ J}$$

$$KE \text{ lost} = 40 - 10.75$$

29.25 J

e) Angle between $-3\mathbf{i} + \mathbf{j}$ and $1.5\mathbf{i} + \mathbf{j}$

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$$\cos\theta = \frac{(-3)(1.5) + (1)(1)}{(\sqrt{10} \times \sqrt{3.25})}$$

$$\cos\theta = \frac{-3.5}{\sqrt{32.5}} = -0.6139...$$

$$\theta = 127.875...^\circ$$

$\boxed{128^\circ}$

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