Momentum and impulse

Worked solutions

Further Maths topic practice

This topic online

1 A ball of mass 0.15 kg is moving horizontally with speed 12 m s^{-1} when it hits a fixed vertical wall at right angles. It rebounds along the same line with speed 8 m s^{-1}.

a. Find the magnitude of the impulse exerted on the ball by the wall.

b. The ball is in contact with the wall for 0.04 s. Find the magnitude of the average force exerted on the ball by the wall.

(3 marks)

a) Take away from the wall as positive

$u = -12, v = 8$

Impulse = change in momentum

$I = mv - mu$

$I = 0.15(8) - 0.15(-12)$

$I = 1.2 + 1.8$

$\boxed{3 \text{ N s}}$

b) Impulse = force x time

$3 = F \times 0.04$

$F = \frac{3}{0.04}$

$\boxed{75 \text{ N}}$

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2 A particle P of mass 2.5 kg is moving in a straight line on a smooth horizontal surface with speed 4 m s^{-1}. A constant horizontal force of magnitude 6 N then acts on P for 3 seconds, in the direction of motion of P.

a. Find the speed of P at the end of the 3 seconds.

Later, a constant horizontal force of magnitude F newtons acts on P for 4 seconds, opposite to its direction of motion. At the end of the 4 seconds P is still moving in the same direction, with speed 3.2 m s^{-1}.

b. Find the value of F.

(4 marks)

a) Impulse = Ft = change in momentum

$6 \times 3 = 2.5v - 2.5(4)$

$18 = 2.5v - 10$

$2.5v = 28$

$\boxed{v = 11.2 \text{ m s}^{-1}}$

b) The force opposes the motion, so its impulse is negative

$-F \times 4 = 2.5(3.2) - 2.5(11.2)$

$-4F = 8 - 28$

$-4F = -20$

$\boxed{F = 5}$

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3 Two particles A and B, of masses 3 kg and 2 kg, are moving towards each other along the same straight line on a smooth horizontal floor. Just before they collide, A has speed 5 m s^{-1} and B has speed 2 m s^{-1}, as shown in the diagram. The particles coalesce in the collision to form a single particle.

a. Find the speed of the combined particle immediately after the collision and state its direction of motion.

b. Find the kinetic energy lost in the collision.

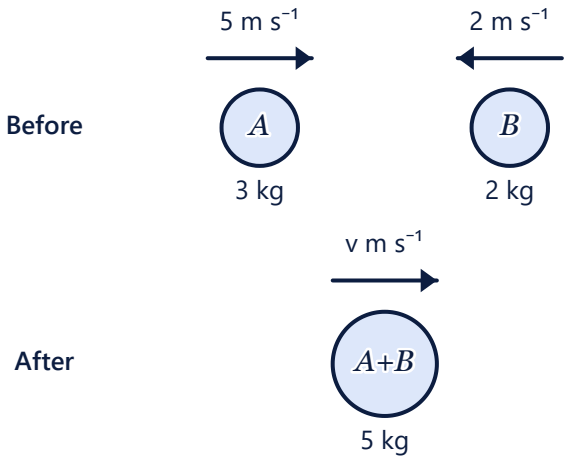


Diagram NOT accurately drawn

(5 marks)

a) Conservation of momentum, A's direction positive

$$3(5) + 2(-2) = 5v$$
$$15 - 4 = 5v$$
$$5v = 11$$
$$v = 2.2$$

2.2 m s^{-1} in the direction A was moving

b) $KE = \frac{1}{2}mv^2$

$$KE \text{ before} = \frac{1}{2}(3)(5^2) + \frac{1}{2}(2)(2^2)$$
$$KE \text{ before} = 37.5 + 4 = 41.5 \text{ J}$$
$$KE \text{ after} = \frac{1}{2}(5)(2.2^2) = 12.1 \text{ J}$$
$$KE \text{ lost} = 41.5 - 12.1$$

29.4 J

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4 Two small smooth spheres P and Q have masses 0.8 kg and m kg. They are moving in opposite directions along the same straight line on a smooth horizontal table when they collide directly. Immediately before the collision P has speed 5 m s^{-1} and Q has speed 3 m s^{-1}. In the collision the direction of motion of each sphere is reversed. Immediately after the collision P has speed 1 m s^{-1} and Q has speed 2 m s^{-1}.

a. Find the value of m.

b. Find the magnitude of the impulse exerted on P by Q.

c. Find the kinetic energy lost in the collision.

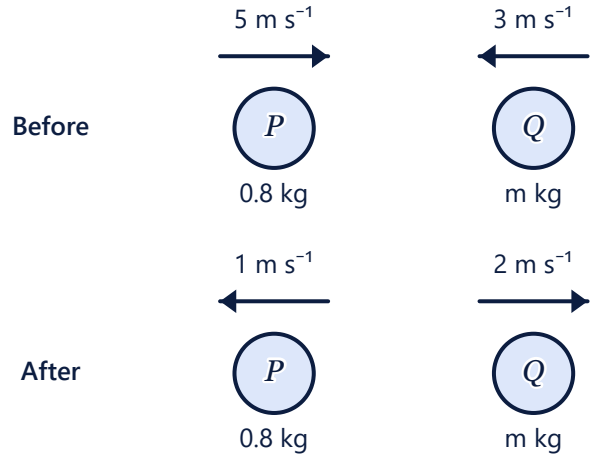


Diagram NOT accurately drawn

(7 marks)

a) Conservation of momentum, P's original direction positive

$$0.8(5) + m(-3) = 0.8(-1) + m(2)$$

$$4 - 3m = -0.8 + 2m$$

$$4.8 = 5m$$

$$\boxed{m = 0.96}$$

b) Impulse on P = change in momentum of P

$$I = 0.8(-1) - 0.8(5)$$

$$I = -4.8$$

$$\boxed{4.8 \text{ N s}}$$

c) $KE = \frac{1}{2}mv^2$

$$KE \text{ before} = \frac{1}{2}(0.8)(5^2) + \frac{1}{2}(0.96)(3^2)$$

$$KE \text{ before} = 10 + 4.32 = 14.32 \text{ J}$$

$$KE \text{ after} = \frac{1}{2}(0.8)(1^2) + \frac{1}{2}(0.96)(2^2)$$

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KE after = 0.4 + 1.92 = 2.32 J

KE lost = 14.32 - 2.32

12 J

5 A ball of mass 0.2 kg is released from rest at a height of 2.5 m above horizontal ground. It falls freely, hits the ground and rebounds vertically upwards, reaching a maximum height of 1.6 m above the ground. The ball is modelled as a particle and air resistance is ignored.

a. Find the speed of the ball immediately before it hits the ground.

b. Find the speed of the ball immediately after it leaves the ground.

c. Find the magnitude of the impulse exerted on the ball by the ground.

(6 marks)

a) Falling: $u = 0$, $a = 9.8$, $s = 2.5$

$v^2 = u^2 + 2as$

$v^2 = 0 + 2(9.8)(2.5) = 49$

7 m s^{-1}

b) Rising: $v = 0$, $a = -9.8$, $s = 1.6$

$0 = u^2 - 2(9.8)(1.6)$

$u^2 = 31.36$

5.6 m s^{-1}

c) Upwards positive: before -7, after +5.6

$I = mv - mu$

$I = 0.2(5.6) - 0.2(-7)$

$I = 1.12 + 1.4$

2.52 N s

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6 Two particles A and B, of masses 3 kg and 5 kg, are at rest on a smooth horizontal surface with a light compressed spring between them. The spring is released and the particles move apart in opposite directions along the same straight line. The spring then falls away. The energy stored in the spring, 21.6 J, is all converted into kinetic energy of the particles.

a. Show that the speed of A is $\frac{5}{3}$ times the speed of B.

b. Find the speed of each particle.

c. Find the magnitude of the impulse exerted on A by the spring.

(7 marks)

a) Momentum before = 0 (both at rest)

Speed of A is v, speed of B is w, in opposite directions

$$0 = 3v - 5w$$

$$3v = 5w$$

$$\boxed{v = \frac{5}{3}w}$$

b) All 21.6 J becomes kinetic energy

$$\frac{1}{2}(3)v^2 + \frac{1}{2}(5)w^2 = 21.6$$

$$\frac{3}{2}\left(\frac{25}{9}\right)w^2 + \frac{5}{2}w^2 = 21.6$$

$$\frac{25}{6}w^2 + \frac{15}{6}w^2 = 21.6$$

$$\frac{20}{3}w^2 = 21.6$$

$$w^2 = 3.24$$

$$w = 1.8$$

$$v = \frac{5}{3}(1.8) = 3$$

$$\boxed{\text{A: } 3 \text{ m s}^{-1}, \text{ B: } 1.8 \text{ m s}^{-1}}$$

c) Impulse on A = change in momentum of A

$$I = 3(3) - 3(0)$$

$$\boxed{9 \text{ N s}}$$

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7 Two particles P and Q, of masses 2 kg and 3 kg, lie at rest on a smooth horizontal surface. They are joined by a light inextensible string, which is slack. P is projected along the surface, directly away from Q, with speed 5 m s^{-1}. When the string becomes taut, Q is jerked into motion and from then on P and Q move with the same speed v m s^{-1}, as shown in the diagram.

a. State where in your working you use the fact that the string is inextensible.

b. Find the value of v.

c. Find the magnitude of the impulse in the string when it jerks taut.

d. Find the kinetic energy lost when the string jerks taut.

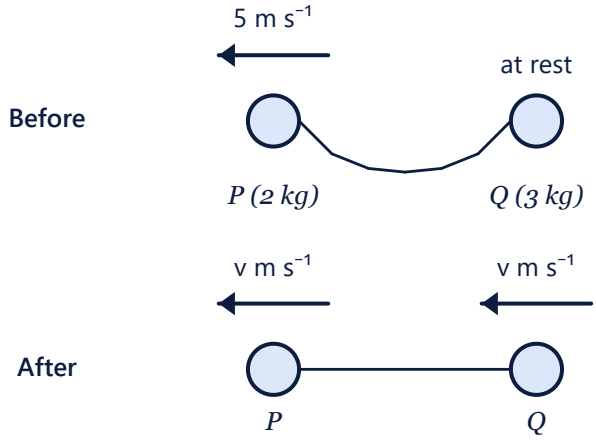


Diagram NOT accurately drawn

(7 marks)

a) Inextensible: once taut, P and Q move with the same speed v

b) Momentum is conserved (tension is internal)

$$2(5) + 3(0) = 2v + 3v$$

$$10 = 5v$$

$$\boxed{v = 2}$$

c) Impulse on Q = change in momentum of Q

$$J = 3(2) - 3(0)$$

$$\boxed{6\ \text{N s}}$$

Check with P: $2(2) - 2(5) = -6$ ✓

d) $KE = \frac{1}{2}mv^2$

$$KE\ \text{before} = \frac{1}{2}(2)(5^2) = 25\ \text{J}$$

$$KE\ \text{after} = \frac{1}{2}(5)(2^2) = 10\ \text{J}$$

$$KE\ \text{lost} = 25 - 10$$

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15 J

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8 Two smooth spheres P and Q have equal radii and masses $2m$ and $3m$. They are moving in opposite directions along the same straight line on a smooth horizontal surface and collide directly. Immediately before the collision P has speed $4u$ and Q has speed u.

The magnitude of the impulse exerted on Q by P in the collision is $\frac{15}{2}mu$.

a. Find the speed of Q immediately after the collision and state its direction of motion.

b. Find the speed of P immediately after the collision.

c. Find, in terms of m and u, the kinetic energy lost in the collision.

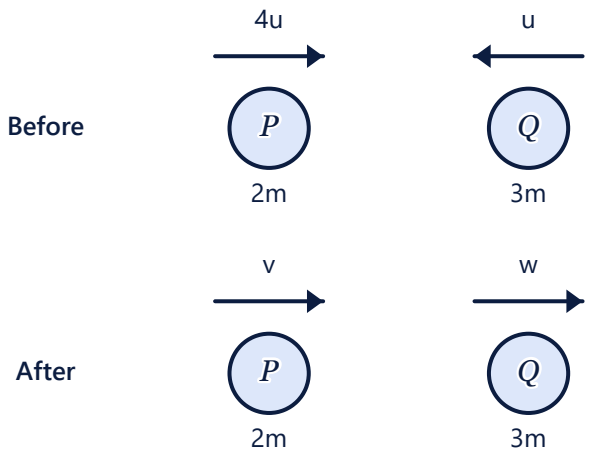


Diagram NOT accurately drawn

(8 marks)

a) P's direction positive; after: P has velocity v, Q has velocity w

P pushes Q in the positive direction

Impulse on Q = change in momentum of Q

$$\frac{15}{2}mu = 3mw - 3m(-u)$$

$$\frac{15}{2}u = 3w + 3u$$

$$3w = \frac{9}{2}u$$

$$w = \frac{3}{2}u$$

$\frac{3}{2}u$ in the direction P was moving

b) Impulse on P is equal and opposite

$$-\frac{15}{2}mu = 2mv - 2m(4u)$$

$$-\frac{15}{2}u = 2v - 8u$$

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$$2v = \frac{1}{2}u$$

$$\boxed{v = \frac{1}{4}u}$$

c) $KE = \frac{1}{2}mv^2$

$$KE \text{ before} = \frac{1}{2}(2m)(4u)^2 + \frac{1}{2}(3m)u^2$$

$$KE \text{ before} = 16mu^2 + \frac{3}{2}mu^2 = \frac{35}{2}mu^2$$

$$KE \text{ after} = \frac{1}{2}(2m)\left(\frac{1}{4}u\right)^2 + \frac{1}{2}(3m)\left(\frac{3}{2}u\right)^2$$

$$KE \text{ after} = \frac{1}{16}mu^2 + \frac{27}{8}mu^2 = \frac{55}{16}mu^2$$

$$KE \text{ lost} = \frac{280}{16}mu^2 - \frac{55}{16}mu^2$$

$$\boxed{\frac{225}{16}mu^2}$$

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9 A bullet of mass 20 grams is fired horizontally with speed 400 m s^{-1} into a wooden block of mass 1.98 kg, which is at rest on a rough horizontal floor. The bullet becomes embedded in the block and the block then slides in a straight line along the floor. The coefficient of friction between the block and the floor is 0.5. The bullet and block are modelled as particles.

a. Find the speed of the block immediately after the bullet becomes embedded in it.

b. Find the kinetic energy lost when the bullet becomes embedded in the block.

c. Find the distance the block slides before it comes to rest.

d. Explain why it is reasonable to ignore the friction from the floor during the very short time the bullet takes to become embedded.

(9 marks)

a) Conservation of momentum, mass after = 0.02 + 1.98 = 2 kg

$$0.02(400) + 1.98(0) = 2v$$

$$8 = 2v$$

$$\boxed{4 \text{ m s}^{-1}}$$

b) $KE = \frac{1}{2}mv^2$

$$KE \text{ before} = \frac{1}{2}(0.02)(400^2) = 1600 \text{ J}$$

$$KE \text{ after} = \frac{1}{2}(2)(4^2) = 16 \text{ J}$$

$$KE \text{ lost} = 1600 - 16$$

$$\boxed{1584 \text{ J}}$$

c) Resolve vertically: $R = 2g$

$$F = \mu R = 0.5 \times 2(9.8) = 9.8 \text{ N}$$

$F = ma$, friction opposes the motion

$$-9.8 = 2a$$

$$a = -4.9$$

$u = 4$, $v = 0$, $a = -4.9$

$$0 = 4^2 + 2(-4.9)s$$

$$9.8s = 16$$

$$s = 1.6327...$$

$$\boxed{1.63 \text{ m}}$$

d) Friction is a finite force; the time of impact is tiny

So the impulse of friction, Ft, is negligible during the impact

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10 Two particles A and B, of masses $3m$ and km, where k is a positive constant, are moving towards each other along the same straight line on a smooth horizontal plane. Immediately before they collide, A has speed $2u$ and B has speed u. In the collision the direction of motion of A is reversed and immediately afterwards A has speed $\frac{1}{2}u$.

a. Show that immediately after the collision the velocity of B, in the direction in which A was originally moving, is $\frac{(15 - 2k)u}{2k}$

b. Given that the direction of motion of B is also reversed by the collision, find the set of possible values of k.

c. By considering kinetic energy, show that $k \geq \frac{15}{7}$

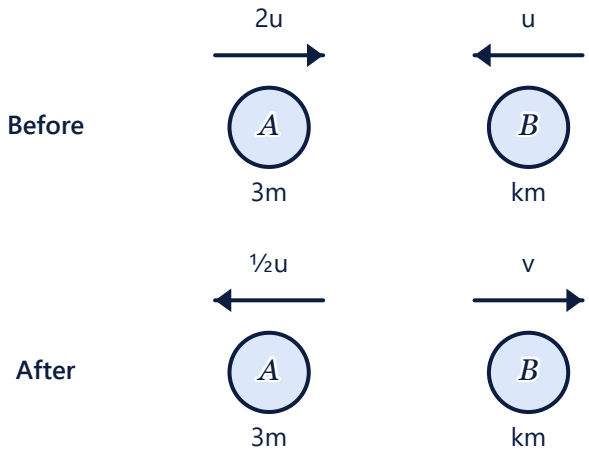


Diagram NOT accurately drawn

(10 marks)

a) A's original direction positive; B after has velocity v

$$3m(2u) + km(-u) = 3m\left(-\frac{1}{2}u\right) + kmv$$

$$6u - ku = -\frac{3}{2}u + kv$$

$$kv = \frac{15}{2}u - ku$$

$$kv = \frac{(15 - 2k)u}{2}$$

$$\boxed{v = \frac{(15 - 2k)u}{2k}}$$

b) B reversed: $v > 0$

$$\frac{(15 - 2k)u}{2k} > 0$$

$k > 0$ and $u > 0$, so we need

$$15 - 2k > 0$$

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$$\boxed{0 < k < \frac{15}{2}}$$

c) KE after $\leqslant$ KE before

$$\text{KE before} = \frac{1}{2}(3m)(2u)^2 + \frac{1}{2}(km)u^2 = 6mu^2 + \frac{1}{2}kmu^2$$

$$\text{KE after} = \frac{1}{2}(3m)\left(\frac{1}{2}u\right)^2 + \frac{1}{2}(km)v^2$$

$$\text{KE after} = \frac{3}{8}mu^2 + \frac{(15-2k)^2mu^2}{8k}$$

$$\frac{3}{8}mu^2 + \frac{(15-2k)^2mu^2}{8k} \leqslant 6mu^2 + \frac{1}{2}kmu^2$$

Multiply through by $8k/(mu^2)$, which is positive

$$3k + (15-2k)^2 \leqslant 48k + 4k^2$$

$$3k + 225 - 60k + 4k^2 \leqslant 48k + 4k^2$$

$$225 \leqslant 105k$$

$$\boxed{k \geqslant \frac{15}{7}}$$

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11 A pile driver of mass 800 kg is released from rest at a height of 1.8 m above the top of a vertical pile of mass 400 kg. The pile driver falls freely and strikes the pile. It does not rebound, and the pile driver and pile move downwards together. The pile is driven 0.15 m into the ground before coming to rest.

The resistance of the ground is modelled as a constant force of magnitude R newtons. The pile driver and the pile are modelled as particles.

a. Find the speed of the pile driver immediately before it strikes the pile.

b. Find the common speed of the pile driver and pile immediately after the impact.

c. Find the magnitude of the impulse exerted on the pile by the pile driver in the impact.

d. Find the value of R.

(10 marks)

a) Free fall: $u = 0$, $a = 9.8$, $s = 1.8$

$v^2 = 0 + 2(9.8)(1.8) = 35.28$

$v = 5.9397...$

$\boxed{5.94 \text{ m s}^{-1}}$

b) Conservation of momentum, coalescing

$800(5.9397...) = 1200w$

$w = 3.9598...$

$\boxed{3.96 \text{ m s}^{-1}}$

c) Impulse on pile = change in momentum of the pile

$I = 400(3.9598...) - 0$

$I = 1583.9192...$

$\boxed{1580 \text{ N s}}$

d) $w^2 = (\frac{2}{3})^2 \times 35.28 = 15.68$

Moving down 0.15 m to rest: $u^2 = 15.68$, $v = 0$

$0 = 15.68 + 2a(0.15)$

$a = -52.2667...$

$F = ma$ downwards for the combined 1200 kg

$1200(9.8) - R = 1200(-52.2667...)$

$R = 11760 + 62720$

$R = 74480$

$\boxed{R = 74\,500}$

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12 Two particles A and B, of masses 0.4 kg and 0.6 kg, are attached to the ends of a light inextensible string which passes over a small smooth pulley fixed above a horizontal floor. Particle B rests on the floor and the string is slack. Particle A is held at rest, with the string hanging vertically on both sides of the pulley, as shown in the diagram.

A is released and falls freely through a distance of 0.9 m, at which point the string becomes taut. B is jerked off the floor and immediately after the jerk A and B move with the same speed. In the subsequent motion A does not reach the floor and B does not reach the pulley.

a. Find the speed of A immediately before the string becomes taut.

b. Find the common speed of A and B immediately after the string becomes taut.

c. Find the magnitude of the impulse exerted on B by the string when it becomes taut.

d. Find the kinetic energy lost when the string becomes taut.

e. Find the further distance that A falls before it first comes to instantaneous rest.

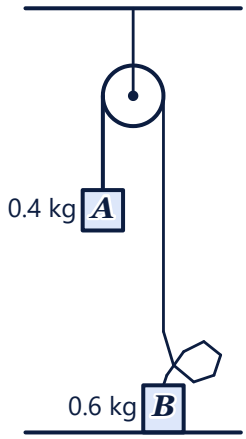


Diagram NOT accurately drawn

(12 marks)

a) Free fall: $u = 0$, $a = 9.8$, $s = 0.9$

$v^2 = 0 + 2(9.8)(0.9) = 17.64$

$\boxed{4.2 \text{ m s}^{-1}}$

b) Impulse J in the string acts up on A and up on B

A, downwards positive: impulse = change in momentum

$-J = 0.4v - 0.4(4.2)$

B, upwards positive (starts at rest)

$J = 0.6v - 0$

Add to remove J

$0 = v - 1.68$

$\boxed{1.68 \text{ m s}^{-1}}$

c) Impulse on B

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$J = 0.6(1.68)$

$J = 1.008$

$\boxed{1.01 \text{ N s}}$

d) $KE = \frac{1}{2}mv^2$

$KE \text{ before} = \frac{1}{2}(0.4)(4.2^2) = 3.528 \text{ J}$

$KE \text{ after} = \frac{1}{2}(0.4 + 0.6)(1.68^2) = 1.4112 \text{ J}$

$KE \text{ lost} = 3.528 - 1.4112 = 2.1168$

$\boxed{2.12 \text{ J}}$

e) F = ma for each, A downwards positive

A: $0.4g - T = 0.4a$

B: $T - 0.6g = 0.6a$

Add to remove T

$-0.2g = 1.0a$

$a = -1.96$

A: $u = 1.68$, $v = 0$, $a = -1.96$

$0 = 1.68^2 + 2(-1.96)s$

$3.92s = 2.8224$

$s = 0.72$

$\boxed{0.72 \text{ m}}$

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