Oblique collisions of smooth spheres

Worked solutions

Further Maths topic practice

This topic online

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1 Two smooth uniform spheres A and B have equal radii. Sphere A has mass $2m$ and sphere B has mass m. Sphere B is at rest on a smooth horizontal surface. Sphere A moves on the surface with speed 5 m s^{-1} and collides with B. Immediately before the collision, the direction of motion of A makes an angle of 30° with the line of centres, as shown in the diagram.

The coefficient of restitution between A and B is $\frac{1}{2}$

a. Find the speed of B immediately after the collision.

b. Find the speed of A immediately after the collision and the angle between its direction of motion and the line of centres.

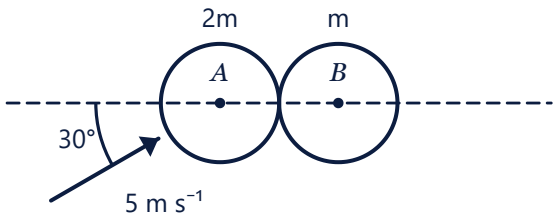


Diagram NOT accurately drawn

(7 marks)

a) Resolve A's velocity along and perpendicular to the line of centres

along: $5\cos 30^\circ = \frac{5\sqrt{3}}{2}$

perpendicular: $5\sin 30^\circ = 2.5$ (unchanged)

Along the line of centres: after, A has a and B has b

Conservation of momentum

$$2m \times \frac{5\sqrt{3}}{2} = 2ma + mb$$

$$5\sqrt{3} = 2a + b$$

Newton's law of restitution

$$b - a = \frac{1}{2} \times \frac{5\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}$$

$$a = b - \frac{5\sqrt{3}}{4}$$

$$5\sqrt{3} = 2b - \frac{5\sqrt{3}}{2} + b$$

$$3b = \frac{15\sqrt{3}}{2}$$

$$b = \frac{5\sqrt{3}}{2} = 4.3301\ldots$$

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B: $4.33\ m\ s^{-1}$

b) Component of A along the line of centres

$$a = \frac{5\sqrt{3}}{2} - \frac{5\sqrt{3}}{4} = \frac{5\sqrt{3}}{4} = 2.1650...$$

speed of A $= \sqrt{(2.1650...^2 + 2.5^2)}$

speed of A $= 3.3071...$

$3.31\ m\ s^{-1}$

$$\tan\theta = \frac{2.5}{2.1650...} = 1.1547...$$

$\theta = 49.1^\circ$ to the line of centres

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2 [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

Two smooth uniform spheres A and B, of equal radius, are moving on a smooth horizontal surface. Sphere A has mass 0.3 kg and sphere B has mass 0.2 kg. The spheres collide.

Immediately before the collision the velocity of A is $(4\mathbf{i} + 3\mathbf{j})$ m s^{-1} and the velocity of B is $(-2\mathbf{i} + 5\mathbf{j})$ m s^{-1}. When they collide, the line joining their centres is parallel to **i**. The coefficient of restitution between the spheres is $\frac{1}{2}$

a. Find the velocity of each sphere immediately after the collision.

b. Find the magnitude of the impulse exerted on B by A in the collision.

(7 marks)

a) Perpendicular to the line of centres (j): unchanged

A keeps 3j, B keeps 5j

Along the line of centres (i): conservation of momentum

$$0.3(4) + 0.2(-2) = 0.3a + 0.2b$$

$$0.8 = 0.3a + 0.2b$$

Newton's law of restitution

$$b - a = \frac{1}{2}(4 - (-2)) = 3$$

$$b = a + 3$$

$$0.8 = 0.3a + 0.2a + 0.6$$

$$0.5a = 0.2$$

$$a = 0.4,\ b = 3.4$$

A: $(0.4\mathbf{i} + 3\mathbf{j})$ m s^{-1}, B: $(3.4\mathbf{i} + 5\mathbf{j})$ m s^{-1}

b) Impulse on B = change in momentum of B

$$\mathbf{I} = 0.2((3.4\mathbf{i} + 5\mathbf{j}) - (-2\mathbf{i} + 5\mathbf{j}))$$

$$\mathbf{I} = 0.2 \times 5.4\mathbf{i} = 1.08\mathbf{i}$$

$|\mathbf{I}| = 1.08$ N s

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3 Two smooth uniform spheres P and Q have equal mass and equal radius. Sphere Q is at rest on a smooth horizontal table. Sphere P moves on the table with speed u and collides with Q. Immediately before the collision the direction of motion of P makes an angle α with the line of centres, where $0 < \alpha < 90°$.

The collision is perfectly elastic.

Show that, immediately after the collision, P and Q are moving at right angles to each other.

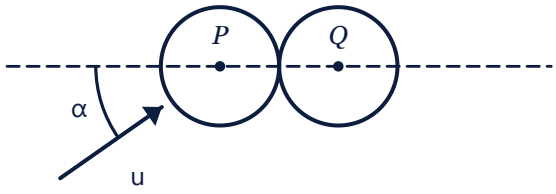


Diagram NOT accurately drawn

(5 marks)

Masses m; P moves u cos α along and u sin α perpendicular to the line of centres

Smooth spheres: the impulse is along the line of centres

P keeps u sin α perpendicular; Q moves along the line of centres

Along the line of centres: P has x and Q has y after

$$mu\cos\alpha = mx + my$$

$$u\cos\alpha = x + y$$

Perfectly elastic: $e = 1$

$$y - x = u\cos\alpha$$

Subtract

$$2x = 0$$

$$x = 0,\ y = u\cos\alpha$$

P moves with speed $u\sin\alpha$ perpendicular to the line of centres

Q moves with speed $u\cos\alpha$ along the line of centres

So P and Q move at right angles

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4 Two smooth uniform spheres A and B, of equal radius, each have mass 0.5 kg. They are moving on a smooth horizontal surface when they collide.

Immediately before the collision, A has speed 6 m s^{-1} in a direction making an angle of 60° with the line of centres, and B has speed 2 m s^{-1} along the line of centres, moving towards A, as shown in the diagram.

The coefficient of restitution between the spheres is 0.6

a. Find the components of the velocities of A and B along the line of centres immediately after the collision.

b. Find the speed of A immediately after the collision and the angle between its direction of motion and the line of centres.

c. Find the kinetic energy lost in the collision.

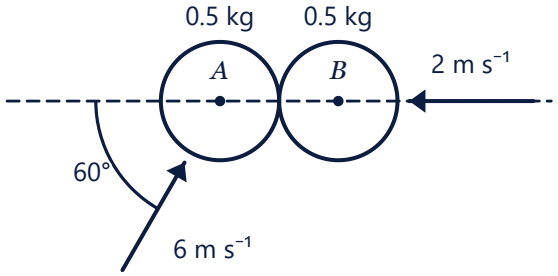


Diagram NOT accurately drawn

(9 marks)

a) Positive direction from A to B

A along: $6\cos 60^\circ = 3$

A perpendicular: $6\sin 60^\circ = 3\sqrt{3}$ (unchanged)

B along: -2

Conservation of momentum along the line of centres

$0.5(3) + 0.5(-2) = 0.5a + 0.5b$

$a + b = 1$

Newton's law of restitution

$b - a = 0.6(3 - (-2)) = 3$

Add the equations

$2b = 4$

$b = 2,\ a = -1$

A: 1 m s^{-1} reversed, B: 2 m s^{-1} reversed

b) Combine the components of A

speed $= \sqrt{(1^2 + (3\sqrt{3})^2)} = \sqrt{28}$

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speed $= 5.2915...$

$\boxed{5.29 \text{ m s}^{-1}}$

$\tan\theta = \frac{3\sqrt{3}}{1}$

$\boxed{\theta = 79.1^\circ \text{ to the line of centres}}$

c) B has speed 2 before and after

KE before $= \frac{1}{2}(0.5)(6^2) + \frac{1}{2}(0.5)(2^2) = 10$

KE after $= \frac{1}{2}(0.5)(28) + \frac{1}{2}(0.5)(2^2) = 8$

$\boxed{\text{KE lost} = 2 \text{ J}}$

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5 [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

Two smooth uniform spheres A and B, of equal radius, have masses 2 kg and 1 kg respectively. They are moving on a smooth horizontal surface. Immediately before they collide, the velocity of A is $(5\mathbf{i} + 5\mathbf{j})$ m s^{-1} and the velocity of B is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At the instant of collision the line joining the centres of the spheres is parallel to $(3\mathbf{i} + 4\mathbf{j})$.

The coefficient of restitution between the spheres is $\frac{1}{2}$

a. Find the components of the velocities of A and B parallel to the line of centres immediately before the collision.

b. Find the velocity of each sphere immediately after the collision, giving your answers in the form $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}.

(9 marks)

a) Unit vector along the line of centres

$$n = \frac{1}{5}(3i + 4j) = 0.6i + 0.8j$$

A: $(5i + 5j)\cdot n = 3 + 4 = 7$

B: $(-i + 2j)\cdot n = -0.6 + 1.6 = 1$

A: 7 m s^{-1}, B: 1 m s^{-1}

b) Perpendicular parts are unchanged

A: $(5i + 5j) - 7(0.6i + 0.8j) = 0.8i - 0.6j$

B: $(-i + 2j) - 1(0.6i + 0.8j) = -1.6i + 1.2j$

Along n: conservation of momentum

$$2(7) + 1(1) = 2a + b$$

$$2a + b = 15$$

Newton's law of restitution

$$b - a = \frac{1}{2}(7 - 1) = 3$$

$$2a + a + 3 = 15$$

$$a = 4,\ b = 7$$

Recombine

A: $v = 4(0.6i + 0.8j) + (0.8i - 0.6j)$

A: $v = (3.2i + 2.6j)$ m s^{-1}

B: $v = 7(0.6i + 0.8j) + (-1.6i + 1.2j)$

B: $v = (2.6i + 6.8j)$ m s^{-1}

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6 A smooth uniform sphere A of mass m is moving on a smooth horizontal plane with speed u. It collides with a smooth uniform sphere B of mass km and the same radius, which is at rest on the plane. Immediately before the collision the direction of motion of A makes an angle α with the line of centres, where $0 < \alpha < 90°$.

The coefficient of restitution between the spheres is e. Immediately after the collision A moves at right angles to the line of centres.

a. Show that $e = \frac{1}{k}$

b. Explain why $k \geq 1$

Given that $k = 2$ and $\alpha = 45°$,

c. find the fraction of the total kinetic energy that is lost in the collision.

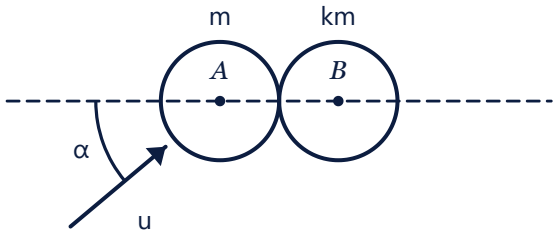


Diagram NOT accurately drawn

(7 marks)

a) Along the line of centres A has $u\cos\alpha$ before and 0 after; B has v after

Conservation of momentum

$$mu\cos\alpha = km\,v$$

$$v = \frac{u\cos\alpha}{k}$$

Newton's law of restitution

$$v - 0 = e(u\cos\alpha - 0)$$

$$\frac{u\cos\alpha}{k} = e\,u\cos\alpha$$

$$\boxed{e = \frac{1}{k}}$$

b) The coefficient of restitution is at most 1

$$\frac{1}{k} \leq 1$$

$$\boxed{k \geq 1}$$

c) $k = 2$, $\alpha = 45°$

A after: $u\sin 45° = \frac{u}{\sqrt{2}}$

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$$B \text{ after: } v = \frac{u\cos 45^\circ}{2} = \frac{u}{2\sqrt{2}}$$

$$\text{KE before} = \frac{1}{2}mu^2$$

$$\text{KE after} = \frac{1}{2}m\left(\frac{u^2}{2}\right) + \frac{1}{2}(2m)\left(\frac{u^2}{8}\right) = \frac{mu^2}{4} + \frac{mu^2}{8} = \frac{3mu^2}{8}$$

$$\text{KE lost} = \frac{1}{2}mu^2 - \frac{3}{8}mu^2 = \frac{1}{8}mu^2$$

$$\text{fraction} = \frac{1}{8} \div \frac{1}{2}$$

$$\boxed{\text{fraction lost} = \frac{1}{4}}$$

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7 Two smooth uniform spheres A and B, of equal radius, have masses 0.4 kg and 0.6 kg respectively. They are moving on a smooth horizontal surface when they collide.

Immediately before the collision, A is moving with speed 5 m s^{-1} in a direction making an angle of 35° with the line of centres, and B is moving with speed 3 m s^{-1} along the line of centres towards A, as shown in the diagram. The coefficient of restitution between the spheres is 0.4

a. Find the speed of B immediately after the collision.

b. Find the angle through which the direction of motion of A is deflected by the collision.

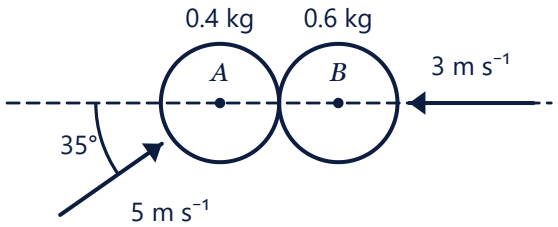


Diagram NOT accurately drawn

(8 marks)

a) Positive direction from A to B

A along: $5\cos 35^\circ = 4.0957...$

A perpendicular: $5\sin 35^\circ = 2.8678...$

B along: -3

Conservation of momentum along the line of centres

$$0.4(4.0957...) + 0.6(-3) = 0.4a + 0.6b$$

$$0.4a + 0.6b = -0.16169...$$

Newton's law of restitution

$$b - a = 0.4(4.0957... + 3) = 2.8383...$$

$$a = b - 2.8383...$$

$$0.4b - 1.1353... + 0.6b = -0.16169...$$

$$b = 0.97362...$$

speed of B = 0.974 m s^{-1}

b) Component of A along the line of centres

$$a = 0.97362... - 2.8383... = -1.8646...$$

Negative, so A now moves back towards the side it came from

$$\tan\phi = \frac{2.8678...}{1.8646...}$$

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$\phi = 56.968...^\circ$ (with the direction B to A)

angle with the direction A to B $= 180^\circ - 56.968...^\circ = 123.03...^\circ$

deflection $= 123.03...^\circ - 35^\circ$

deflection $= 88.0^\circ$

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8 [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

Two smooth uniform spheres A and B each have mass 0.16 kg and radius 0.1 m. They are moving on a smooth horizontal surface. Sphere B is at rest and sphere A is moving with velocity $(7\mathbf{i} + \mathbf{j})$ m s^{-1} when they collide.

At the instant of collision the centre of A has position vector $(\mathbf{i} + 2\mathbf{j})$ m and the centre of B has position vector $(1.12\mathbf{i} + 2.16\mathbf{j})$ m, relative to a fixed origin.

The coefficient of restitution between the spheres is $\frac{1}{2}$

a. Show that a unit vector along the line of centres, in the direction from A to B, is $(0.6\mathbf{i} + 0.8\mathbf{j})$.

b. Find the velocity of each sphere immediately after the collision.

c. Find the kinetic energy lost in the collision.

(9 marks)

a) Vector from the centre of A to the centre of B

$$\overrightarrow{AB} = (1.12\mathbf{i} + 2.16\mathbf{j}) - (\mathbf{i} + 2\mathbf{j}) = 0.12\mathbf{i} + 0.16\mathbf{j}$$

$$|\overrightarrow{AB}| = \sqrt{(0.12^2 + 0.16^2)} = 0.2$$

(= 0.1 + 0.1, the spheres are touching)

$$\mathbf{n} = \frac{0.12\mathbf{i} + 0.16\mathbf{j}}{0.2}$$

$$\boxed{\mathbf{n} = 0.6\mathbf{i} + 0.8\mathbf{j}}$$

b) Resolve A's velocity

$$(7\mathbf{i} + \mathbf{j}) \cdot \mathbf{n} = 4.2 + 0.8 = 5$$

$$\text{perpendicular part} = (7\mathbf{i} + \mathbf{j}) - 5(0.6\mathbf{i} + 0.8\mathbf{j}) = 4\mathbf{i} - 3\mathbf{j}$$

Along n: conservation of momentum (equal masses)

$$5 = a + b$$

Newton's law of restitution

$$b - a = \frac{1}{2} \times 5 = 2.5$$

$$2b = 7.5$$

$$b = 3.75,\ a = 1.25$$

$$A: \mathbf{v} = 1.25(0.6\mathbf{i} + 0.8\mathbf{j}) + (4\mathbf{i} - 3\mathbf{j})$$

$$\boxed{A: \mathbf{v} = (4.75\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}}$$

$$B: \mathbf{v} = 3.75(0.6\mathbf{i} + 0.8\mathbf{j})$$

$$\boxed{B: \mathbf{v} = (2.25\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}}$$

c) Speeds squared: 50 before; $4.75^2 + 2^2 = 26.5625$ and $2.25^2 + 3^2 = 14.0625$ after

$$\text{KE before} = \frac{1}{2}(0.16)(50) = 4$$

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$$KE\ after = \frac{1}{2}(0.16)(26.5625 + 14.0625) = 3.25$$

$$\boxed{KE\ lost = 0.75\ J}$$

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9 A smooth uniform sphere A of mass m is moving with speed u on a smooth horizontal floor. It collides with a smooth uniform sphere B of mass $3m$ and the same radius, which is at rest on the floor. Immediately before the collision the direction of motion of A makes an angle of 30° with the line of centres. The coefficient of restitution between the spheres is e.

a. Show that, immediately after the collision, the component of the velocity of A along the line of centres is $\frac{\sqrt{3}u(1 - 3e)}{8}$

The collision deflects the direction of motion of A through 90°.

b. Show that $e = \frac{7}{9}$

c. Find, in terms of u, the speed of B immediately after the collision.

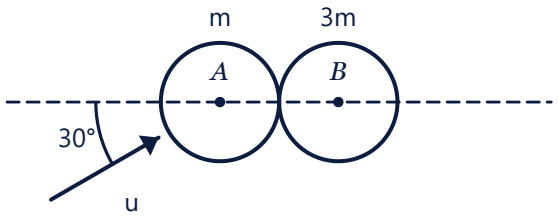


Diagram NOT accurately drawn

(9 marks)

a) Along the line of centres: A has $u\cos 30^\circ = \frac{\sqrt{3}u}{2}$ before

After: A has x and B has y along the line of centres

$$m \times \frac{\sqrt{3}u}{2} = mx + 3my$$

$$y - x = e \times \frac{\sqrt{3}u}{2}$$

$$y = x + \frac{\sqrt{3}eu}{2}$$

$$\frac{\sqrt{3}u}{2} = x + 3x + \frac{3\sqrt{3}eu}{2}$$

$$4x = \frac{\sqrt{3}u}{2}(1 - 3e)$$

$$\boxed{x = \frac{\sqrt{3}u(1 - 3e)}{8}}$$

b) Perpendicular component is unchanged: $u\sin 30^\circ = \frac{u}{2}$

Deflected through 90°: scalar product of old and new velocities is 0

$$x \times \frac{\sqrt{3}u}{2} + \frac{u}{2} \times \frac{u}{2} = 0$$

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$$x \times \frac{\sqrt{3u}}{2} = -\frac{u^2}{4}$$

$$x = -\frac{u}{2\sqrt{3}} = -\frac{\sqrt{3u}}{6}$$

$$\frac{\sqrt{3u}(1-3e)}{8} = -\frac{\sqrt{3u}}{6}$$

$$1 - 3e = -\frac{8}{6} = -\frac{4}{3}$$

$$3e = \frac{7}{3}$$

$$\boxed{e = \frac{7}{9}}$$

c) Use restitution

$$y = x + \frac{\sqrt{3eu}}{2} = -\frac{\sqrt{3u}}{6} + \frac{7}{9} \times \frac{\sqrt{3u}}{2}$$

$$y = -\frac{3\sqrt{3u}}{18} + \frac{7\sqrt{3u}}{18} = \frac{4\sqrt{3u}}{18}$$

$$\boxed{\text{speed of } B = \frac{2\sqrt{3u}}{9}}$$

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10 In a game of snooker, a white ball W strikes a red ball R which is at rest on the table. The balls are modelled as smooth uniform spheres of equal mass and equal radius, and the table as smooth and horizontal.

Immediately before the collision W is moving with speed 2.5 m s^{-1} in a direction making an angle of 25° with the line of centres, as shown in the diagram. The coefficient of restitution between the balls is 0.9

a. Find the speed of R immediately after the collision.

b. Find the speed of W immediately after the collision.

After the collision R moves along the line of centres and strikes a smooth straight cushion. The line of centres makes an angle of 40° with the cushion. The coefficient of restitution between R and the cushion is 0.6

c. Find the speed of R immediately after it rebounds from the cushion and the angle that its direction of motion then makes with the cushion.

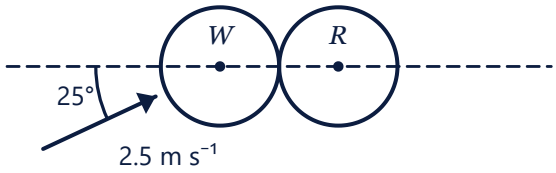


Diagram NOT accurately drawn

(10 marks)

a) Resolve the velocity of W

along: $2.5 \cos 25^\circ = 2.2657...$

perpendicular: $2.5 \sin 25^\circ = 1.0565...$ (unchanged)

Along the line of centres, mass m each: w for W and r for R after

$m(2.2657...) = mw + mr$

$w + r = 2.2657...$

Newton's law of restitution

$r - w = 0.9 \times 2.2657... = 2.0391...$

Add

$2r = 4.3049...$

$r = 2.1524...$

speed of R = 2.15 m s^{-1}

b) Component of W along the line of centres

$w = 2.2657... - 2.1524... = 0.11328...$

speed of W $= \sqrt{(0.11328...^2 + 1.0565...^2)}$

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speed of W = 1.0626...

$\boxed{1.06 \text{ m s}^{-1}}$

c) R meets the cushion at 40°: parallel component unchanged

parallel = $2.1524... \cos 40^\circ = 1.6488...$

Perpendicular: restitution with the cushion

perpendicular = $0.6 \times 2.1524... \sin 40^\circ = 0.83015...$

speed = $\sqrt{(1.6488...^2 + 0.83015...^2)} = 1.8460...$

$\boxed{1.85 \text{ m s}^{-1}}$

$$\tan \beta = \frac{0.83015...}{1.6488...}$$

$\boxed{\beta = 26.7^\circ \text{ to the cushion}}$

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11 [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

Two smooth uniform spheres A and B, of equal radius, have masses 0.2 kg and 0.3 kg respectively. They are moving on a smooth horizontal surface and collide.

Immediately before the collision the velocity of A is $(4\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of B is $(-\mathbf{i} + 3\mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of A is $(\mathbf{i} + 5\mathbf{j})$ m s^{-1}.

a. Find the impulse exerted on A by B in the collision.

b. Find the velocity of B immediately after the collision.

c. Explain why, at the instant of collision, the line of centres is parallel to $(\mathbf{i} - \mathbf{j})$.

d. Find the coefficient of restitution between the spheres.

e. Find the kinetic energy lost in the collision.

(10 marks)

a) Impulse on A = change in momentum of A

$$I = 0.2((i + 5j) - (4i + 2j))$$

$$I = 0.2(-3i + 3j)$$

$$\boxed{I = (-0.6i + 0.6j)\ \text{N s}}$$

b) Impulse on B is equal and opposite; let B have velocity v after

$$0.3(v - (-i + 3j)) = 0.6i - 0.6j$$

$$v - (-i + 3j) = 2i - 2j$$

$$\boxed{v = (i + j)\ \text{m s}^{-1}}$$

c) Smooth spheres: the impulse acts along the line of centres

$$-0.6i + 0.6j = -0.6(i - j)$$

So the line of centres is parallel to $i - j$

d) Components along $n = \frac{1}{\sqrt{2}}(i - j)$, the direction from A to B

A before: $\frac{4 - 2}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

B before: $\frac{-1 - 3}{\sqrt{2}} = -\frac{4}{\sqrt{2}}$

A after: $\frac{1 - 5}{\sqrt{2}} = -\frac{4}{\sqrt{2}}$

B after: $\frac{1 - 1}{\sqrt{2}} = 0$

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}$$

$$e = \frac{0 - (-4)}{2 - (-4)} = \frac{4}{6}$$

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$$\boxed{e = \frac{2}{3}}$$

e) Speeds squared

KE before $= \frac{1}{2}(0.2)(16 + 4) + \frac{1}{2}(0.3)(1 + 9) = 2 + 1.5 = 3.5$

KE after $= \frac{1}{2}(0.2)(1 + 25) + \frac{1}{2}(0.3)(1 + 1) = 2.6 + 0.3 = 2.9$

$$\boxed{\text{KE lost} = 0.6 \text{ J}}$$

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12 A smooth uniform sphere A of mass m is moving with speed u on a smooth horizontal surface when it collides with a smooth uniform sphere B of mass $4m$ and the same radius, which is at rest on the surface. Immediately before the collision the direction of motion of A makes an angle of 30° with the line of centres, as shown in the diagram. The coefficient of restitution between the spheres is e.

a. Find, in terms of u and e, the speed of B immediately after the collision.

b. Show that, immediately after the collision, the component of the velocity of A along the line of centres, in the direction from A to B, is $\frac{\sqrt{3}u(1 - 4e)}{10}$

c. Show that the direction of motion of A is deflected through more than 90° if and only if $e > \frac{2}{3}$

Given that $e = \frac{3}{4}$,

d. find, in terms of m and u, the kinetic energy lost in the collision.

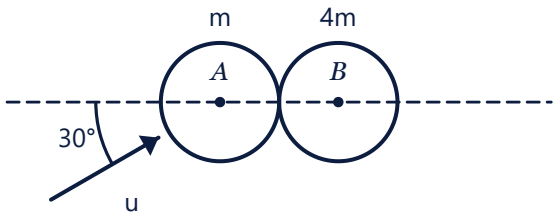


Diagram NOT accurately drawn

(12 marks)

a) Along the line of centres: A has $u\cos 30^\circ = \frac{\sqrt{3}u}{2}$ before; x (A) and y (B) after

Conservation of momentum

$$m \times \frac{\sqrt{3}u}{2} = mx + 4my$$

$$\frac{\sqrt{3}u}{2} = x + 4y$$

Newton's law of restitution

$$y - x = \frac{\sqrt{3}eu}{2}$$

Add the equations

$$5y = \frac{\sqrt{3}u}{2}(1 + e)$$

$$\boxed{y = \frac{\sqrt{3}u(1 + e)}{10}}$$

b) Use restitution

$$x = y - \frac{\sqrt{3}eu}{2} = \frac{\sqrt{3}u(1 + e)}{10} - \frac{5\sqrt{3}eu}{10}$$

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$$\boxed{x = \frac{\sqrt{3u(1-4e)}}{10}}$$

c) Perpendicular component unchanged: $\frac{u}{2}$

Deflection $> 90^\circ$ exactly when (old velocity)·(new velocity) < 0

$$\frac{\sqrt{3u}}{2} \times \frac{\sqrt{3u(1-4e)}}{10} + \frac{u}{2} \times \frac{u}{2} < 0$$

$$\frac{3u^2(1-4e)}{20} + \frac{u^2}{4} < 0$$

Multiply by $\frac{20}{u^2}$

$$3(1-4e) + 5 < 0$$

$$8 - 12e < 0$$

$$\boxed{e > \frac{2}{3}}$$

d) $e = 3/4$

$$x = \frac{\sqrt{3u(1-3)}}{10} = -\frac{\sqrt{3u}}{5}$$

$$y = \frac{\sqrt{3u \times 1.75}}{10} = \frac{7\sqrt{3u}}{40}$$

$$\text{KE of A after} = \frac{1}{2}m\left(\frac{3u^2}{25} + \frac{u^2}{4}\right) = \frac{1}{2}m \times \frac{37u^2}{100} = \frac{37mu^2}{200}$$

$$\text{KE of B after} = \frac{1}{2}(4m)\left(\frac{147u^2}{1600}\right) = \frac{147mu^2}{800}$$

$$\text{KE after} = \frac{148mu^2}{800} + \frac{147mu^2}{800} = \frac{295mu^2}{800} = \frac{59mu^2}{160}$$

$$\text{KE lost} = \frac{80mu^2}{160} - \frac{59mu^2}{160}$$

$$\boxed{\text{KE lost} = \frac{21mu^2}{160}}$$

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