# Oblique impact with a smooth plane

Worked solutions

**Further Maths topic practice**

This topic online

**1** A small smooth sphere is moving on a smooth horizontal floor when it strikes a fixed smooth vertical wall. Immediately before the impact the sphere has speed 6 m $s^{-1}$ and its direction of motion makes an angle of 40° with the wall, as shown in the diagram.

The coefficient of restitution between the sphere and the wall is 0.5

Find

a. the speed of the sphere immediately after the impact,

b. the angle between the wall and the direction of motion of the sphere immediately after the impact.

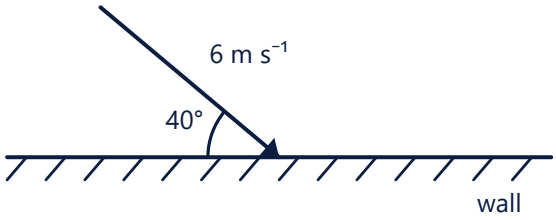



Diagram NOT accurately drawn

(5 marks)

a) Let the sphere leave with speed $v$ at angle $\beta$ to the wall

Smooth wall: the component parallel to the wall is unchanged

$$v\cos\beta = 6\cos 40^\circ = 4.5962...$$

Newton's law of restitution perpendicular to the wall

$$v\sin\beta = 0.5 \times 6\sin 40^\circ = 1.9283...$$

$$v^2 = 4.5962...^2 + 1.9283...^2$$

$$v = 4.9844...$$

$$\boxed{v = 4.98 \text{ m s}^{-1}}$$

b) Divide the two components

$$\tan\beta = \frac{1.9283...}{4.5962...}$$

$$\tan\beta = 0.41954...$$

$$\boxed{\beta = 22.8^\circ}$$


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**2** A small smooth sphere is moving on a smooth horizontal floor when it strikes a fixed smooth vertical wall. Immediately before the impact the direction of motion of the sphere makes an angle of 50° with the wall. Immediately after the impact the direction of motion of the sphere makes an angle of 30° with the wall, as shown in the diagram.

a. Find the coefficient of restitution between the sphere and the wall.

The speed of the sphere immediately before the impact is 5 m $s^{-1}$

b. Find the speed of the sphere immediately after the impact.

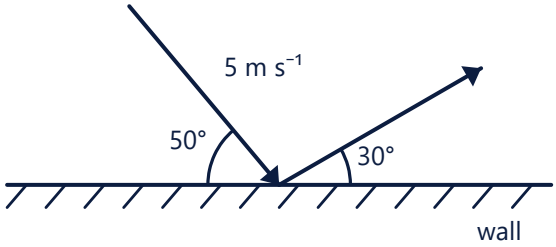



Diagram NOT accurately drawn

(5 marks)

a) Speed u before and v after

Parallel to the wall: unchanged

$$v\cos 30^\circ = u\cos 50^\circ$$

Perpendicular to the wall: restitution

$$v\sin 30^\circ = e\,u\sin 50^\circ$$

Divide the second equation by the first

$$\tan 30^\circ = e\tan 50^\circ$$

$$e = \frac{\tan 30^\circ}{\tan 50^\circ}$$

$$e = \frac{0.57735\ldots}{1.1917\ldots}$$

$$e = 0.48445\ldots$$

$$\boxed{e = 0.484}$$

b) Use the parallel component with u = 5

$$v\cos 30^\circ = 5\cos 50^\circ$$

$$v = \frac{3.2139\ldots}{0.86602\ldots}$$

$$v = 3.7111\ldots$$

$$\boxed{v = 3.71 \text{ m s}^{-1}}$$


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**3** A smooth ball of mass 0.3 kg is moving on a smooth horizontal surface with speed 8 m $s^{-1}$ when it strikes a fixed smooth vertical wall. Immediately before the impact the direction of motion of the ball makes an angle of 60° with the wall.

The coefficient of restitution between the ball and the wall is $\frac{1}{3}$

The ball is modelled as a particle.

a. Find the magnitude of the impulse exerted on the ball by the wall.

b. Find the kinetic energy lost by the ball in the impact.

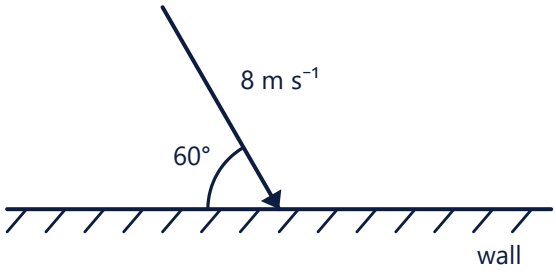



Diagram NOT accurately drawn

(6 marks)

a) Only the component perpendicular to the wall changes

before: $8 \sin 60^\circ = 4\sqrt{3}$

after: $\frac{1}{3} \times 4\sqrt{3} = \frac{4\sqrt{3}}{3}$ (away from the wall)

Impulse = change in momentum, taking away from the wall as positive

$$I = 0.3\left(\frac{4\sqrt{3}}{3} - (-4\sqrt{3})\right)$$

$$I = 0.3 \times \frac{16\sqrt{3}}{3}$$

$$I = 1.6\sqrt{3} = 2.7712\ldots$$

$$\boxed{I = 2.77 \text{ N s}}$$

b) Parallel component unchanged

$$\text{parallel} = 8 \cos 60^\circ = 4$$

$$\text{KE before} = \frac{1}{2} \times 0.3 \times 8^2 = 9.6$$

$$\text{speed after}^2 = 4^2 + \left(\frac{4\sqrt{3}}{3}\right)^2 = 16 + \frac{16}{3} = \frac{64}{3}$$

$$\text{KE after} = \frac{1}{2} \times 0.3 \times \frac{64}{3} = 3.2$$


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KE lost = 9.6 − 3.2

$$\boxed{\text{KE lost} = 6.4\ \text{J}}$$


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**4** A smooth sphere $S$ moving on a smooth horizontal plane strikes a fixed smooth vertical wall. Immediately before the impact, the direction of motion of $S$ makes an angle $\alpha$ with the wall. Immediately after the impact, the direction of motion of $S$ makes an angle $\beta$ with the wall, as shown in the diagram. The coefficient of restitution between $S$ and the wall is $e$, where $0 < e < 1$

a. Show that $\tan\beta = e\tan\alpha$

Given that $e = 0.6$ and $\beta = 25°$,

b. find the value of $\alpha$,

c. find the angle through which the direction of motion of $S$ is deflected by the impact.

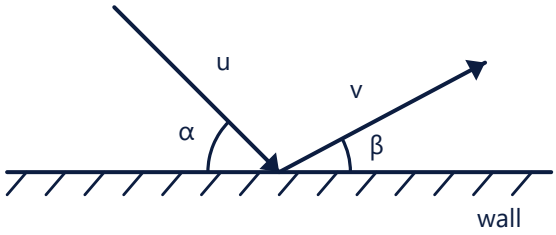



Diagram NOT accurately drawn

(7 marks)

a) Speeds u before and v after

Smooth wall: no impulse parallel to the wall

$$v\cos\beta = u\cos\alpha$$

Restitution perpendicular to the wall

$$v\sin\beta = e\,u\sin\alpha$$

Divide the second equation by the first

$$\frac{v\sin\beta}{v\cos\beta} = \frac{e\,u\sin\alpha}{u\cos\alpha}$$

$$\boxed{\tan\beta = e\tan\alpha}$$

b) Substitute $e = 0.6$, $\beta = 25°$

$$\tan 25° = 0.6\tan\alpha$$

$$\tan\alpha = \frac{0.46630\ldots}{0.6} = 0.77717\ldots$$

$$\boxed{\alpha = 37.9°}$$

c) The path turns through $\alpha$ on the way in and $\beta$ on the way out

$$\text{deflection} = \alpha + \beta = 37.853\ldots° + 25°$$

$$\boxed{\text{deflection} = 62.9°}$$


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**5** [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth puck of mass 0.5 kg is sliding on smooth horizontal ice. It strikes a fixed smooth vertical barrier. The barrier lies along a line parallel to **i**.

Immediately before the impact the velocity of the puck is $(5\mathbf{i} - 4\mathbf{j})$ m s$^{-1}$. The coefficient of restitution between the puck and the barrier is 0.75

a. Find the velocity of the puck immediately after the impact.

b. Find the impulse exerted on the puck by the barrier.

c. Find the kinetic energy lost by the puck in the impact. (7 marks)

a) i is parallel to the barrier, j is perpendicular to it

parallel: $5 \rightarrow 5$

Restitution reverses the j component

perpendicular: $-4 \rightarrow 0.75 \times 4 = 3$

$$\boxed{v = (5i + 3j) \text{ m s}^{-1}}$$

b) Impulse = change in momentum

$$I = 0.5((5i + 3j) - (5i - 4j))$$

$$I = 0.5 \times 7j$$

$$\boxed{I = 3.5j \text{ N s}}$$

c) Speeds squared

$$|\text{before}|^2 = 5^2 + 4^2 = 41$$

$$|\text{after}|^2 = 5^2 + 3^2 = 34$$

$$\text{KE lost} = \frac{1}{2} \times 0.5 \times (41 - 34)$$

$$\boxed{\text{KE lost} = 1.75 \text{ J}}$$


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**6** [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth sphere of mass 0.2 kg is moving on a smooth horizontal floor. It strikes a fixed smooth vertical wall which lies along a line parallel to **j**.

Immediately before the impact the velocity of the sphere is $(6\mathbf{i} + 5\mathbf{j})$ m s$^{-1}$. Immediately after the impact the velocity of the sphere is $(-4\mathbf{i} + 5\mathbf{j})$ m s$^{-1}$.

a. Find the coefficient of restitution between the sphere and the wall.

b. Find the magnitude of the impulse exerted on the sphere by the wall.

c. Find the angle through which the direction of motion of the sphere is deflected by the impact. (7 marks)

a) The i components are perpendicular to the wall

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}$$

$$e = \frac{4}{6}$$

$$\boxed{e = \frac{2}{3}}$$

b) Impulse = change in momentum

$$I = 0.2((-4i + 5j) - (6i + 5j))$$

$$I = 0.2(-10i) = -2i$$

$$\boxed{|I| = 2 \text{ N s}}$$

c) Angle between the two velocity vectors: scalar product

$$(6i + 5j)\cdot(-4i + 5j) = -24 + 25 = 1$$

$$\cos\theta = \frac{1}{\sqrt{61} \times \sqrt{41}} = \frac{1}{\sqrt{2501}}$$

$$\cos\theta = 0.019996...$$

$$\boxed{\theta = 88.9^\circ}$$


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**7** A small smooth sphere is moving on a smooth horizontal floor when it strikes a fixed smooth vertical wall. Immediately before the impact the sphere has speed $u$ and its direction of motion makes an angle $\alpha$ with the wall. The coefficient of restitution between the sphere and the wall is $\frac{1}{2}$

As a result of the impact, the sphere loses $\frac{1}{3}$ of its kinetic energy.

a. Show that $\sin\alpha = \frac{2}{3}$

b. Find the angle between the wall and the direction of motion of the sphere immediately after the impact.

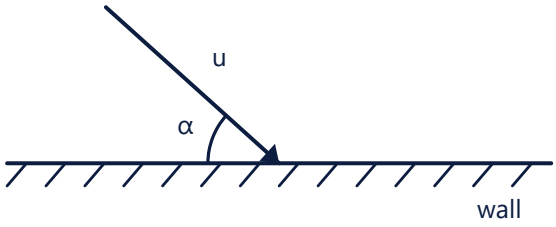



Diagram NOT accurately drawn

(7 marks)

a) Let the mass be m

parallel: $u\cos\alpha \rightarrow u\cos\alpha$ (unchanged)

perpendicular: $u\sin\alpha \rightarrow \frac{1}{2}u\sin\alpha$

Only the perpendicular component loses energy

$$KE\ lost = \frac{1}{2}m(u\sin\alpha)^2 - \frac{1}{2}m\left(\frac{1}{2}u\sin\alpha\right)^2$$

$$KE\ lost = \frac{1}{2}mu^2\sin^2\alpha\left(1-\frac{1}{4}\right)$$

$$KE\ lost = \frac{3}{8}mu^2\sin^2\alpha$$

This is one third of the original KE

$$\frac{3}{8}mu^2\sin^2\alpha = \frac{1}{3}\times\frac{1}{2}mu^2$$

$$\sin^2\alpha = \frac{1}{6}\times\frac{8}{3} = \frac{4}{9}$$

$$\boxed{\sin\alpha = \frac{2}{3}}$$

b) $\tan\beta = e\tan\alpha$

$$\cos\alpha = \sqrt{\left(1-\frac{4}{9}\right)} = \frac{\sqrt{5}}{3}$$


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$\tan\alpha = \frac{2}{\sqrt{5}}$

$\tan\beta = \frac{1}{2} \times \frac{2}{\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\boxed{\beta = 24.1^\circ}$


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**8** A small smooth sphere is moving on a smooth horizontal floor when it strikes a fixed smooth vertical wall. Immediately before the impact the direction of motion of the sphere makes an angle $\alpha$ with the wall. The coefficient of restitution between the sphere and the wall is $e$.

The impact deflects the direction of motion of the sphere through 90°.

a. Show that $e \tan^2\alpha = 1$

Given that $e = \frac{1}{3}$,

b. find the value of $\alpha$,

c. show that the sphere loses $\frac{2}{3}$ of its kinetic energy in the impact.

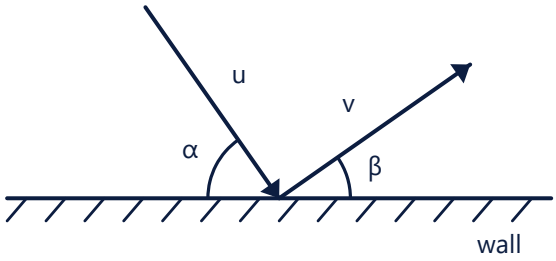



Diagram NOT accurately drawn

(8 marks)

a) Leaves at angle $\beta$ to the wall

$v\cos\beta = u\cos\alpha$

$v\sin\beta = e u \sin\alpha$

$\tan\beta = e\tan\alpha$

Deflection is $\alpha + \beta = 90^\circ$

$\beta = 90^\circ - \alpha$

$\tan\beta = \tan(90^\circ - \alpha) = \frac{1}{\tan\alpha}$

$\frac{1}{\tan\alpha} = e\tan\alpha$

$\boxed{e\tan^2\alpha = 1}$

b) Substitute $e = 1/3$

$\tan^2\alpha = 3$

$\tan\alpha = \sqrt{3}$

$\boxed{\alpha = 60^\circ}$

c) Only the perpendicular component loses energy


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$$\text{KE lost} = \frac{1}{2}m(u\sin\alpha)^2 - \frac{1}{2}m(eu\sin\alpha)^2$$

$$\text{KE lost} = \frac{1}{2}mu^2\sin^2\alpha(1-e^2)$$

$$\text{KE lost} = \frac{1}{2}mu^2 \times \frac{3}{4} \times \frac{8}{9}$$

$$\text{KE lost} = \frac{2}{3} \times \frac{1}{2}mu^2$$

$$\boxed{\text{Fraction of KE lost} = \frac{2}{3}}$$


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**9** A small smooth ball of mass 0.2 kg is falling vertically. It strikes a fixed smooth plane which is inclined at 30° to the horizontal, as shown in the diagram. Immediately before the impact the ball has speed 7 m $s^{-1}$. Immediately after the impact the ball is moving horizontally.

The coefficient of restitution between the ball and the plane is $e$.

a. Show that $e = \frac{1}{3}$

b. Find the speed of the ball immediately after the impact.

c. Find the magnitude of the impulse exerted on the ball by the plane.

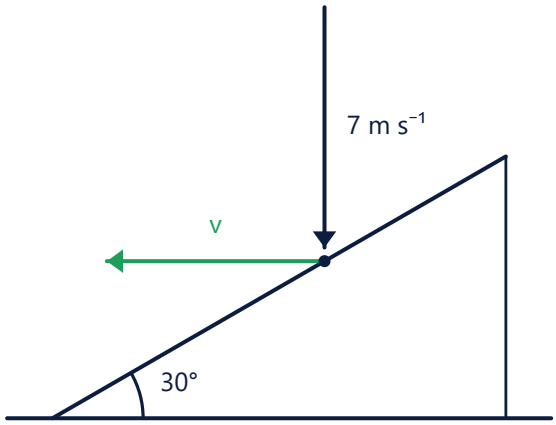



Diagram NOT accurately drawn

(8 marks)

a) Angles with the plane: 60° before, 30° after

Parallel to the plane: unchanged

$v \cos 30^\circ = 7 \cos 60^\circ$

Perpendicular to the plane: restitution

$v \sin 30^\circ = e \times 7 \sin 60^\circ$

Divide

$\tan 30^\circ = e \tan 60^\circ$

$\frac{1}{\sqrt{3}} = e\sqrt{3}$

$\boxed{e = \frac{1}{3}}$

b) Parallel component

$v \cos 30^\circ = 3.5$

$v = \frac{3.5}{\cos 30^\circ} = \frac{7}{\sqrt{3}}$

$v = 4.0414...$


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$v = 4.04 \text{ m s}^{-1}$

c) Impulse is perpendicular to the plane: change in momentum

before: $7 \sin 60^\circ = 6.0621\ldots$ into the plane

after: $v \sin 30^\circ = 2.0207\ldots$ out of the plane

$I = 0.2(2.0207\ldots + 6.0621\ldots)$

$I = 1.6165\ldots$

$I = 1.62 \text{ N s}$


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**10** [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

The corner of an ice rink is formed by two fixed smooth vertical barriers $P$ and $Q$ which meet at right angles. Barrier $P$ lies along a line parallel to **i** and barrier $Q$ lies along a line parallel to **j**, as shown in the diagram.

A puck of mass 0.17 kg slides on the smooth ice with velocity $(-6\mathbf{i} - 8\mathbf{j})$ m s$^{-1}$. It strikes $P$, then rebounds and strikes $Q$. The coefficient of restitution between the puck and each barrier is 0.5. The puck is modelled as a particle.

a. Find the velocity of the puck immediately after it strikes $P$.

b. Find the velocity of the puck immediately after it strikes $Q$.

c. Show that, after both impacts, the puck moves parallel to its original path but in the opposite direction.

d. Find the total kinetic energy lost in the two impacts.

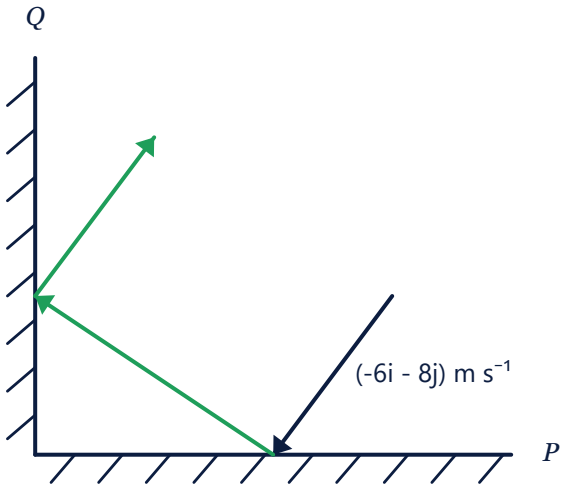



Diagram NOT accurately drawn

(9 marks)

a) At P (parallel to i) only the j component changes

i: $-6 \rightarrow -6$

j: $-8 \rightarrow 0.5 \times 8 = 4$

velocity after P = $(-6\mathbf{i} + 4\mathbf{j})$ m s$^{-1}$

b) At Q (parallel to j) only the i component changes

i: $-6 \rightarrow 0.5 \times 6 = 3$

j: $4 \rightarrow 4$

velocity after Q = $(3\mathbf{i} + 4\mathbf{j})$ m s$^{-1}$

c) Compare with the original velocity

$3\mathbf{i} + 4\mathbf{j} = -\frac{1}{2}(-6\mathbf{i} - 8\mathbf{j})$


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A negative multiple, so the puck moves parallel to its original path in the opposite direction

d) Speeds: $|(-6, -8)| = 10$ and $|(3, 4)| = 5$

$$\text{KE before} = \frac{1}{2} \times 0.17 \times 10^2 = 8.5$$

$$\text{KE after} = \frac{1}{2} \times 0.17 \times 5^2 = 2.125$$

$$\text{KE lost} = 8.5 - 2.125 = 6.375$$

$$\boxed{\text{KE lost} = 6.38 \text{ J}}$$


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**11** [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth sphere of mass 0.4 kg is moving on a smooth horizontal floor with velocity $(7\mathbf{i} + \mathbf{j})$ m s$^{-1}$. It strikes a fixed smooth vertical wall which lies along a line parallel to the vector $(\mathbf{i} + \mathbf{j})$. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$

a. Show that the velocity of the sphere immediately after the impact is $(3\mathbf{i} + 5\mathbf{j})$ m s$^{-1}$.

b. Find the magnitude of the impulse exerted on the sphere by the wall.

c. Find the angle through which the direction of motion of the sphere is deflected by the impact. (10 marks)

a) Unit vector along the wall

$$w = \frac{1}{\sqrt{2}}(i + j)$$

$$(7i + j)\cdot w = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

$$\text{parallel part} = 4\sqrt{2} \times \frac{1}{\sqrt{2}}(i + j) = 4i + 4j$$

$$\text{perpendicular part} = (7i + j) - (4i + 4j) = 3i - 3j$$

Parallel part unchanged; perpendicular part reversed and multiplied by e

$$\text{perpendicular after} = -\frac{1}{3}(3i - 3j) = -i + j$$

$$v = (4i + 4j) + (-i + j)$$

$$\boxed{v = (3i + 5j) \text{ m s}^{-1}}$$

b) Impulse = change in momentum

$$I = 0.4((3i + 5j) - (7i + j))$$

$$I = 0.4(-4i + 4j) = -1.6i + 1.6j$$

$$|I| = 1.6\sqrt{2} = 2.2627\ldots$$

$$\boxed{|I| = 2.26 \text{ N s}}$$

c) Scalar product of the velocities

$$(7i + j)\cdot(3i + 5j) = 21 + 5 = 26$$

$$\cos\theta = \frac{26}{\sqrt{50} \times \sqrt{34}} = 0.63059\ldots$$

$$\boxed{\theta = 50.9^\circ}$$


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**12** [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth ball of mass 0.25 kg is rolling on a smooth horizontal floor. It strikes a fixed smooth vertical wall. Immediately before the impact the velocity of the ball is $(7\mathbf{i} - \mathbf{j})$ m s$^{-1}$. Immediately after the impact the velocity of the ball is $(2.2\mathbf{i} + 5.4\mathbf{j})$ m s$^{-1}$. The ball is modelled as a particle.

a. Find the impulse exerted on the ball by the wall.

b. Explain why the wall lies along a line parallel to $(4\mathbf{i} + 3\mathbf{j})$.

c. Find the coefficient of restitution between the ball and the wall.

d. Find the percentage of the kinetic energy of the ball that is lost in the impact.

e. Find the angle between the wall and the direction of motion of the ball immediately after the impact.

(11 marks)

a) Impulse = change in momentum

$$I = 0.25((2.2i + 5.4j) - (7i - j))$$

$$I = 0.25(-4.8i + 6.4j)$$

$$\boxed{I = (-1.2i + 1.6j)\ \text{N s}}$$

b) A smooth wall gives an impulse perpendicular to the wall

$$(-1.2i + 1.6j)\cdot(4i + 3j) = -4.8 + 4.8 = 0$$

So the impulse is perpendicular to $4i + 3j$, and the wall lies parallel to $4i + 3j$

c) Components perpendicular to the wall, unit normal $n = \frac{1}{5}(-3i + 4j)$

$$\text{before: } (7i - j)\cdot n = \frac{1}{5}(-21 - 4) = -5$$

$$\text{after: } (2.2i + 5.4j)\cdot n = \frac{1}{5}(-6.6 + 21.6) = 3$$

$$e = \frac{3}{5}$$

$$\boxed{e = 0.6}$$

d) Speeds squared

$$|\text{before}|^2 = 7^2 + 1^2 = 50$$

$$|\text{after}|^2 = 2.2^2 + 5.4^2 = 34$$

$$\text{KE before} = \frac{1}{2} \times 0.25 \times 50 = 6.25$$

$$\text{KE after} = \frac{1}{2} \times 0.25 \times 34 = 4.25$$

$$\%\ \text{lost} = \frac{2}{6.25} \times 100$$


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32%

e) Component along the wall, unit vector $\frac{1}{5}(4\mathbf{i} + 3\mathbf{j})$

after: $\frac{1}{5}(8.8 + 16.2) = 5$

$$\tan\beta = \frac{3}{5}$$

$\beta = 31.0^\circ$


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