Power

Worked solutions

Further Maths topic practice

This topic online

1 A car moves along a straight horizontal road at a constant speed of 25 m s^{-1}. The resistance to motion is constant and of magnitude 720 N.

a. Find the rate at which the engine of the car is working, giving your answer in kW.

b. The engine now works at 24 kW. The resistance to motion is unchanged. Find the maximum speed of the car on this road.

(3 marks)

a) Constant speed: driving force = resistance

$F = 720$ N

$P = Fv = 720 \times 25$

$P = 18\,000$ W

18 kW

b) At maximum speed, $a = 0$ so $F = 720$ N

$24\,000 = 720v$

$v = 33.33...$

33.3 m s^{-1}

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2 A car of mass 1100 kg moves along a straight horizontal road. The engine of the car works at a constant rate of 30 kW. The resistance to motion is constant and of magnitude 800 N.

a. Find the maximum speed of the car.

b. Find the acceleration of the car at the instant when its speed is 20 m s^{-1}.

(4 marks)

a) Maximum speed: a = 0, driving force = resistance

$$\frac{30\,000}{v} = 800$$

$$v = 37.5$$

37.5 m s^{-1}

b) Driving force at 20 m s^{-1}

$$F = \frac{30\,000}{20} = 1500 \text{ N}$$

F = ma along the road

$$1500 - 800 = 1100a$$

$$a = 0.63636...$$

0.636 m s^{-2}

3 A lorry of mass 9000 kg is travelling along a straight horizontal road. The resistance to motion is constant and of magnitude 2400 N. At the instant when the speed of the lorry is 15 m s^{-1}, its acceleration is 0.25 m s^{-2}.

Find the rate at which the engine of the lorry is working at this instant.

(4 marks)

F = ma along the road, with driving force F

$$F - 2400 = 9000 \times 0.25$$

$$F - 2400 = 2250$$

$$F = 4650 \text{ N}$$

Power = driving force × speed

$$P = 4650 \times 15$$

$$P = 69\,750 \text{ W}$$

69.8 kW

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4 A car of mass 1250 kg moves up a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{14}$. The resistance to motion from non-gravitational forces is constant and of magnitude 600 N. The engine of the car works at a constant rate of 40 kW.

a. Find the maximum speed of the car up the road.

b. Find the acceleration of the car at the instant when its speed is 15 m s^{-1}.

(6 marks)

Weight component down the slope

$$1250g \sin\alpha = 1250 \times 9.8 \times \frac{1}{14} = 875 \text{ N}$$

a) Maximum speed: $a = 0$

$$\frac{40\,000}{v} = 600 + 875$$

$$\frac{40\,000}{v} = 1475$$

$$v = 27.118...$$

$$\boxed{27.1 \text{ m s}^{-1}}$$

b) Driving force at 15 m s^{-1}

$$F = \frac{40\,000}{15} = 2666.66...$$

$F = ma$ up the slope

$$2666.66... - 600 - 875 = 1250a$$

$$1191.66... = 1250a$$

$$a = 0.95333...$$

$$\boxed{0.953 \text{ m s}^{-2}}$$

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5 A car of mass 1000 kg moves down a straight road inclined at an angle β to the horizontal, where $\sin\beta = \frac{1}{20}$. The engine of the car works at a constant rate of 12 kW and the resistance to motion is constant and of magnitude 900 N.

a. Find the acceleration of the car when its speed is 20 m s^{-1}.

b. Find the maximum speed of the car down the road.

(5 marks)

Weight component down the slope (helps the motion)

$$1000g \sin\beta = 1000 \times 9.8 \times \frac{1}{20} = 490 \text{ N}$$

a) Driving force at 20 m s^{-1}

$$F = \frac{12\,000}{20} = 600 \text{ N}$$

F = ma down the slope

$$600 + 490 - 900 = 1000a$$

$$190 = 1000a$$

$$\boxed{a = 0.19 \text{ m s}^{-2}}$$

b) Maximum speed: a = 0

$$\frac{12\,000}{v} + 490 = 900$$

$$\frac{12\,000}{v} = 410$$

$$v = 29.268...$$

$$\boxed{29.3 \text{ m s}^{-1}}$$

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6 A van of mass 1500 kg travels along a straight horizontal road. The resistance to motion of the van has magnitude kv newtons, where v m s^{-1} is the speed of the van and k is a constant. The engine of the van works at a constant rate of 27 kW and the maximum speed of the van is 30 m s^{-1}.

a. Show that $k = 30$

b. Find the acceleration of the van at the instant when its speed is 20 m s^{-1}. (6 marks)

a) At maximum speed: driving force = resistance

$$\frac{27\,000}{30} = k \times 30$$

$$900 = 30k$$

$$\boxed{k = 30}$$

b) At 20 m s^{-1}

$$F = \frac{27\,000}{20} = 1350 \text{ N}$$

$$\text{resistance} = 30 \times 20 = 600 \text{ N}$$

$F = ma$

$$1350 - 600 = 1500a$$

$$a = 0.5$$

$$\boxed{0.5 \text{ m s}^{-2}}$$

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7 A car of mass 1200 kg tows a trailer of mass 400 kg along a straight horizontal road. The car and the trailer are connected by a light rigid tow bar, which is horizontal and parallel to the direction of motion. The resistances to motion of the car and the trailer are constant and of magnitude 500 N and 200 N respectively.

The engine of the car is working at 21 kW. At the instant when the speed is 15 m s^{-1}, find

a. the acceleration of the car,

b. the tension in the tow bar.

(7 marks)

Driving force at 15 m s^{-1}

$$F = \frac{21\,000}{15} = 1400 \text{ N}$$

a) F = ma for car and trailer together (tension is internal)

$$1400 - 500 - 200 = 1600a$$

$$700 = 1600a$$

$$a = 0.4375$$

$\boxed{0.438 \text{ m s}^{-2}}$

b) F = ma for the trailer alone

$$T - 200 = 400 \times 0.4375$$

$$T - 200 = 175$$

$$T = 375$$

$\boxed{375 \text{ N}}$

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8 A car of mass 900 kg has a maximum speed of 20 m s^{-1} when driving up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{30}$. Its maximum speed when driving down the same road is 32 m s^{-1}.

The engine works at the same constant rate P watts in both cases, and the resistance to motion from non-gravitational forces has the same constant magnitude R newtons in both cases.

a. Find the value of P and the value of R.

b. Find the maximum speed of the car on a straight horizontal road, with the same power and resistance.

(8 marks)

Weight component along the slope

$$900g \times \frac{1}{30} = 294 \text{ N}$$

a) Up the slope at max speed: driving force = R + 294

$$\frac{P}{20} = R + 294$$

Down the slope at max speed: driving force + 294 = R

$$\frac{P}{32} = R - 294$$

Subtract to eliminate R

$$\frac{P}{20} - \frac{P}{32} = 588$$

$$0.01875P = 588$$

$$P = 31\,360$$

$$R = \frac{31\,360}{20} - 294 = 1568 - 294$$

$$\boxed{P = 31\,360\ (31.4\text{ kW}),\ R = 1274}$$

b) On the level at max speed: driving force = R

$$\frac{31\,360}{v} = 1274$$

$$v = 24.615\ldots$$

$$\boxed{24.6\text{ m s}^{-1}}$$

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9 A car of mass 1400 kg tows a caravan of mass 800 kg up a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{16}$, as shown in the diagram. The car and the caravan are connected by a light rigid tow bar parallel to the road. The resistances to motion from non-gravitational forces are constant, of magnitude 450 N on the car and 300 N on the caravan.

At the instant when the speed is 15 m s^{-1}, the engine of the car is working at 45 kW. At this instant find

a. the acceleration of the car and caravan,

b. the tension in the tow bar.



Diagram NOT accurately drawn

(8 marks)

Driving force at 15 m s^{-1}

$$F = \frac{45\,000}{15} = 3000 \text{ N}$$

Weight components down the slope

$$\text{car: } 1400g \times \frac{1}{16} = 857.5 \text{ N}$$

$$\text{caravan: } 800g \times \frac{1}{16} = 490 \text{ N}$$

a) F = ma for the whole system up the slope

$$3000 - 450 - 300 - 857.5 - 490 = 2200a$$

$$902.5 = 2200a$$

$$a = 0.41022...$$

$\boxed{0.410 \text{ m s}^{-2}}$

b) F = ma for the caravan alone

$$T - 300 - 490 = 800 \times 0.41022...$$

$$T - 790 = 328.18...$$

$$T = 1118.18...$$

$\boxed{1120 \text{ N}}$

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10 A car of mass 1200 kg moves up a straight road inclined at an angle α to the horizontal, where sin α = $\frac{1}{40}$. The engine of the car works at a constant rate of 24 kW and the resistance to motion from non-gravitational forces is constant and of magnitude R newtons.

The car passes the point A with speed 10 m s^{-1}. It passes the point B, 300 m further up the road, with speed 20 m s^{-1}, 19 s after passing A.

a. Use the work-energy principle to find the value of R.

b. Find the acceleration of the car as it passes A.

c. Find the maximum speed of the car up this road.

(9 marks)

a) Work done by the engine = power × time

$24\,000 \times 19 = 456\,000$ J

Gain in KE

$\frac{1}{2}(1200)(20^2) - \frac{1}{2}(1200)(10^2) = 180\,000$ J

Gain in PE: height gained = $300 \sin \alpha = 7.5$ m

$1200 \times 9.8 \times 7.5 = 88\,200$ J

Work-energy: WD by engine = gain in KE + gain in PE + WD against R

$456\,000 = 180\,000 + 88\,200 + 300R$

$300R = 187\,800$

$\boxed{R = 626}$

b) At A: driving force

$$F = \frac{24\,000}{10} = 2400 \text{ N}$$

$$\text{weight component} = 1200g \times \frac{1}{40} = 294 \text{ N}$$

F = ma up the slope

$2400 - 626 - 294 = 1200a$

$1480 = 1200a$

$a = 1.2333...$

$\boxed{1.23 \text{ m s}^{-2}}$

c) Maximum speed: a = 0

$\frac{24\,000}{v} = 626 + 294 = 920$

$v = 26.086...$

$\boxed{26.1 \text{ m s}^{-1}}$

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11 A pump raises water from a lake and ejects it through a nozzle at a point 6 m above the surface of the lake. The water is ejected at a rate of 40 kg per second, with speed 9 m s^{-1}. The water is initially at rest at the surface of the lake. Resistances to the motion of the water are to be ignored.

a. Find the rate at which the pump is working, giving your answer in kW.

b. The pump is adjusted so that it works at 6 kW. The water is still raised 6 m and ejected at 40 kg per second. Find the new speed at which the water is ejected.

(7 marks)

a) Consider the water raised in 1 second (40 kg)

$PE \text{ gained} = 40 \times 9.8 \times 6 = 2352 \text{ J}$

$KE \text{ gained} = \frac{1}{2} \times 40 \times 9^2 = 1620 \text{ J}$

Power = work done per second

$P = 2352 + 1620 = 3972 \text{ W}$

3.97 kW

b) Work done per second is now 6000 J

$6000 = 2352 + \frac{1}{2}(40)v^2$

$20v^2 = 3648$

$v^2 = 182.4$

$v = 13.505...$

13.5 m s^{-1}

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12 A lorry of mass 8000 kg moves along a straight road. The resistance to motion of the lorry from non-gravitational forces has magnitude kv newtons, where v m s^{-1} is the speed of the lorry and k is a constant.

When the lorry is on a horizontal stretch of the road and its engine is working at 72 kW, its maximum speed is 30 m s^{-1}.

a. Show that $k = 80$

The road then goes up a hill inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{40}$.

b. The lorry arrives at the bottom of the hill moving at 30 m s^{-1}, with the engine still working at 72 kW. Find the deceleration of the lorry at this instant.

c. Find the maximum speed of the lorry up the hill with the engine working at 72 kW.

d. Find the rate at which the engine would need to work for the lorry to move up the hill at a constant speed of 30 m s^{-1}.

(11 marks)

a) Level road, maximum speed: driving force = resistance

$$\frac{72\,000}{30} = 30k$$

$$2400 = 30k$$

$$\boxed{k = 80}$$

b) Weight component down the hill

$$8000g \times \frac{1}{40} = 1960 \text{ N}$$

At 30 m s^{-1}: F = 2400 N, resistance = 80 × 30 = 2400 N

$$2400 - 2400 - 1960 = 8000a$$

$$a = -0.245$$

$$\boxed{\text{deceleration } 0.245 \text{ m s}^{-2}}$$

c) Maximum speed up the hill: a = 0

$$\frac{72\,000}{v} = 80v + 1960$$

$$72\,000 = 80v^2 + 1960v$$

$$v^2 + 24.5v - 900 = 0$$

$$v = \frac{-24.5 + \sqrt{(24.5^2 + 3600)}}{2}$$

$$v = \frac{-24.5 + 64.809\ldots}{2}$$

$$v = 20.154\ldots$$

$$\boxed{20.2 \text{ m s}^{-1}}$$

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d) Constant speed 30 m s^{-1}: driving force = resistance + weight component

$F = 80 \times 30 + 1960 = 4360$ N

$P = 4360 \times 30 = 130\,800$ W

131 kW

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