Successive collisions

Worked solutions

Further Maths topic practice

This topic online

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1 Two small smooth spheres A and B, of equal radius, have masses 2 kg and 1 kg respectively. They lie on a smooth horizontal floor on a straight line which is perpendicular to a fixed smooth vertical wall, with B between A and the wall. B is at rest and A is projected towards B with speed 6 m s^{-1}. The coefficient of restitution between A and B is 0.5 and the coefficient of restitution between B and the wall is 0.4

a. Find the speed of each sphere immediately after they collide.

b. Find the speed of B immediately after it rebounds from the wall.

c. Explain why there will be a second collision between A and B.

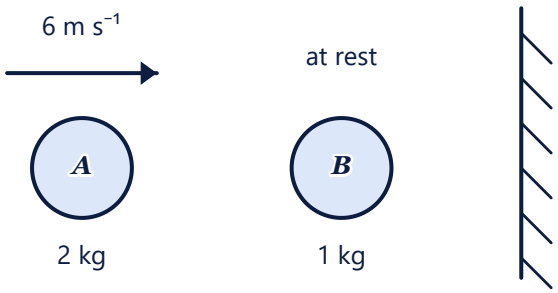


Diagram NOT accurately drawn

(7 marks)

a) Take the direction towards the wall as positive

Conservation of momentum

$2(6) + 1(0) = 2v_A + v_B$

$2v_A + v_B = 12$

Newton's law of restitution

$v_B - v_A = 0.5(6 - 0) = 3$

Substitute $v_B = v_A + 3$

$3v_A + 3 = 12$

$v_A = 3$

$v_B = 6$

A: 3 m s^{-1}, B: 6 m s^{-1}

b) Wall: speed after = e x speed before

$w = 0.4 \times 6$

2.4 m s^{-1}

c) After B rebounds

A moves towards the wall at 3 m s^{-1}

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B moves away from the wall, towards A, at 2.4 m s^{-1}

They move towards each other, so they collide again

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2 Three small smooth spheres A, B and C, each of mass 0.2 kg and of equal radius, lie at rest in a straight line on a smooth horizontal table, in that order. A is projected directly towards B with speed 4 m s^{-1}. The coefficient of restitution between each pair of spheres is 0.5

a. Find the speeds of A and B immediately after the first collision.

b. Find the speeds of B and C immediately after B collides with C.

c. Explain why there will be a further collision between A and B.

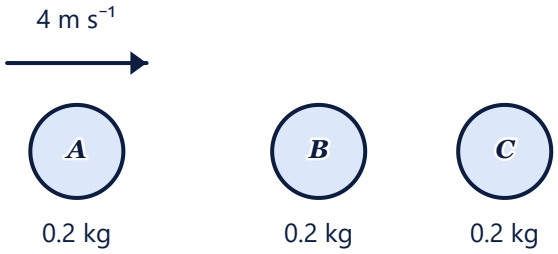


Diagram NOT accurately drawn

(7 marks)

a) A and B, A's direction positive

Conservation of momentum (equal masses, divide by 0.2)

$4 = v_A + v_B$

Newton's law of restitution

$v_B - v_A = 0.5 \times 4 = 2$

Add

$2v_B = 6$

$v_B = 3$

$v_A = 1$

A: 1 m s^{-1}, B: 3 m s^{-1}

b) B (3 m s^{-1}) hits C at rest

$3 = v_{B'} + v_C$

$v_C - v_{B'} = 0.5 \times 3 = 1.5$

$2v_C = 4.5$

$v_C = 2.25$

$v_{B'} = 0.75$

B: 0.75 m s^{-1}, C: 2.25 m s^{-1}

c) Compare A and B, both moving the same way

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A: $1\ m\ s^{-1}$, B: $0.75\ m\ s^{-1}$

$1 > 0.75$, so A catches up with B and they collide again

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3 A small smooth sphere P of mass 3 kg is moving with speed 5 m s^{-1} on a smooth horizontal floor towards a fixed smooth vertical wall. It collides directly with a small smooth sphere Q of mass 2 kg, which is at rest between P and the wall. The spheres have equal radius and move along a line perpendicular to the wall. Immediately after the collision P is moving towards the wall with speed 2 m s^{-1}.

a. Find the speed of Q immediately after the collision, and the coefficient of restitution between P and Q.

Q then hits the wall and rebounds with speed 1.8 m s^{-1}.

b. Find the coefficient of restitution between Q and the wall.

c. Find the velocities of P and Q immediately after their second collision.

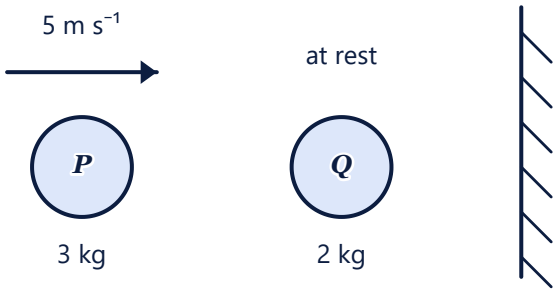


Diagram NOT accurately drawn

(9 marks)

Take the direction towards the wall as positive

a) Conservation of momentum

$$3(5) + 2(0) = 3(2) + 2v$$

$$15 = 6 + 2v$$

$$v = 4.5$$

Newton's law of restitution

$$e = \frac{4.5 - 2}{5 - 0}$$

$$e = 0.5$$

$\boxed{4.5 \text{ m s}^{-1};\ e = 0.5}$

b) Wall: speed after = f × speed before

$$1.8 = f \times 4.5$$

$\boxed{f = 0.4}$

c) Before: P velocity 2, Q velocity -1.8

Conservation of momentum

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$$3(2) + 2(-1.8) = 3x + 2y$$

$$3x + 2y = 2.4$$

Newton's law of restitution

$$y - x = 0.5(2 - (-1.8))$$

$$y - x = 1.9$$

Substitute $y = x + 1.9$

$$3x + 2x + 3.8 = 2.4$$

$$5x = -1.4$$

$$x = -0.28$$

$$y = -0.28 + 1.9 = 1.62$$

P: 0.28 m s^{-1} away from the wall; Q: 1.62 m s^{-1} towards the wall

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4 Two small smooth spheres A and B, of equal radius, have masses 1 kg and 3 kg respectively. They move on a smooth horizontal floor along a straight line perpendicular to a fixed smooth vertical wall. Both move towards the wall, B with speed 1 m s^{-1} and A, which is behind B, with speed 5 m s^{-1}. A collides directly with B. The coefficient of restitution between the spheres is 0.5 and between B and the wall it is 0.6

a. Find the speeds of A and B immediately after the first collision.

b. Show that there is a second collision between A and B, and find the velocities of the spheres immediately after it.

c. Explain why there are no further collisions.

d. Find the total kinetic energy lost in all the impacts.

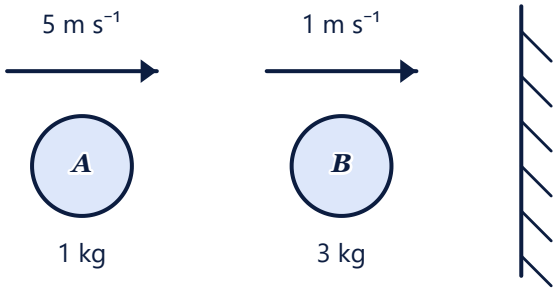


Diagram NOT accurately drawn

(10 marks)

Take the direction towards the wall as positive

a) Conservation of momentum

$1(5) + 3(1) = v_A + 3v_B$

$v_A + 3v_B = 8$

Newton's law of restitution

$v_B - v_A = 0.5(5 - 1) = 2$

Add

$4v_B = 10$

$v_B = 2.5$

$v_A = 0.5$

A: 0.5 m s^{-1}, B: 2.5 m s^{-1}

b) B rebounds from the wall

speed $= 0.6 \times 2.5 = 1.5$

A moves towards the wall, B away from it: they collide again

Velocities before: A 0.5, B -1.5

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$1(0.5) + 3(-1.5) = x + 3y$

$x + 3y = -4$

$y - x = 0.5(0.5 - (-1.5)) = 1$

Add

$4y = -3$

$y = -0.75$

$x = -0.75 - 1 = -1.75$

A: 1.75 m s^{-1}, B: 0.75 m s^{-1}, both away from the wall

c) Both move away from the wall

A is further from the wall and moving faster than B

The spheres separate and B never returns to the wall

d) KE at the start and at the end

KE start $= \frac{1}{2}(1)(5^2) + \frac{1}{2}(3)(1^2) = 12.5 + 1.5 = 14$

KE end $= \frac{1}{2}(1)(1.75^2) + \frac{1}{2}(3)(0.75^2) = 1.53125 + 0.84375 =$

2.375

KE lost $= 14 - 2.375 = 11.625$

11.6 J

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5 Three small smooth spheres A, B and C, of equal radius, have masses $3m$, m and $2m$ respectively. They lie in a straight line on a smooth horizontal surface, in that order, with B and C at rest. A is projected towards B with speed u. The coefficient of restitution between each pair of spheres is e.

a. Show that the speed of B immediately after it is hit by A is $\frac{3}{4}u(1 + e)$, and find the speed of A.

B then collides with C, and as a result B is brought to rest.

b. Find the value of e.

c. Show that A and B collide again.

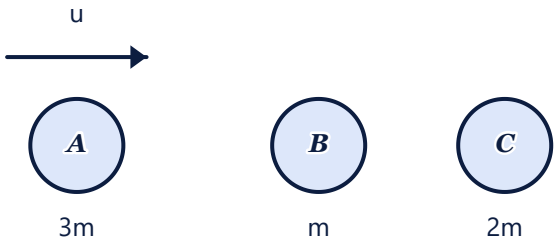


Diagram NOT accurately drawn

(8 marks)

a) A's direction positive

Conservation of momentum

$$3mu = 3mv_A + mv_B$$

$$3v_A + v_B = 3u$$

Newton's law of restitution

$$v_B - v_A = eu$$

Substitute $v_B = v_A + eu$

$$4v_A + eu = 3u$$

$$\boxed{v_A = \frac{1}{4}u(3 - e)}$$

$$v_B = \frac{1}{4}u(3 - e) + eu$$

$$\boxed{v_B = \frac{3}{4}u(1 + e)}$$

b) B, speed V, hits C (mass 2m) at rest

$$mV = mx + 2my$$

$$x + 2y = V$$

$$y - x = eV$$

Substitute $y = x + eV$

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$$3x + 2eV = V$$

$$x = \frac{1}{3}V(1 - 2e)$$

B is brought to rest: $x = 0$

$$1 - 2e = 0$$

$$\boxed{e = \frac{1}{2}}$$

c) Velocity of A with $e = \frac{1}{2}$

$$v_A = \frac{1}{4}u(3 - \frac{1}{2}) = \frac{5}{8}u$$

$\frac{5}{8}u > 0$: A still moves towards B, and B is at rest

So A and B collide again

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6 Two small smooth spheres A and B, each of mass 1 kg and of equal radius, lie on a smooth horizontal floor on a straight line perpendicular to a fixed smooth vertical wall. B is at rest at a distance of 1.2 m from the wall, and A is projected towards B with speed 4 m s^{-1}. The coefficient of restitution between A and B is 0.5 and between B and the wall it is also 0.5. The spheres are modelled as particles.

a. Find the speeds of A and B immediately after they first collide.

b. Find the distance from the wall of the point where A and B collide for the second time.

c. Find the time between the first and second collisions of A and B.

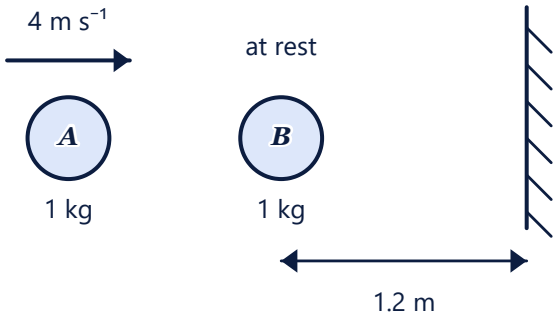


Diagram NOT accurately drawn

(9 marks)

a) Equal masses, towards the wall positive

$$4 = v_A + v_B$$

$$v_B - v_A = 0.5 \times 4 = 2$$

$$2v_B = 6$$

$$v_B = 3$$

$$v_A = 1$$

A: 1 m s^{-1}, B: 3 m s^{-1}

b) Time for B to reach the wall

$$t_1 = \frac{1.2}{3} = 0.4$$

A moves $1 \times 0.4 = 0.4$ m in that time

distance of A from the wall $= 1.2 - 0.4 = 0.8$

B rebounds from the wall

speed $= 0.5 \times 3 = 1.5$

A and B now move towards each other

closing speed $= 1 + 1.5 = 2.5$

$$t_2 = \frac{0.8}{2.5} = 0.32$$

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distance of B from the wall $= 1.5 \times 0.32$

0.48 m

c) Total time

$T = 0.4 + 0.32$

0.72 s

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7 A small smooth sphere A of mass m moves with speed u on a smooth horizontal floor and collides directly with a small smooth sphere B of mass $2m$, which is at rest. The spheres have equal radius and the coefficient of restitution between them is e. After the collision B moves towards a fixed smooth vertical wall, which is perpendicular to its direction of motion, and rebounds. The coefficient of restitution between B and the wall is $\frac{1}{3}$.

a. Show that, immediately after the collision between A and B, the velocity of A is $\frac{1}{3}u(1 - 2e)$ and the speed of B is $\frac{1}{3}u(1 + e)$.

b. Show that there will be a second collision between A and B if and only if $e < \frac{4}{5}$.

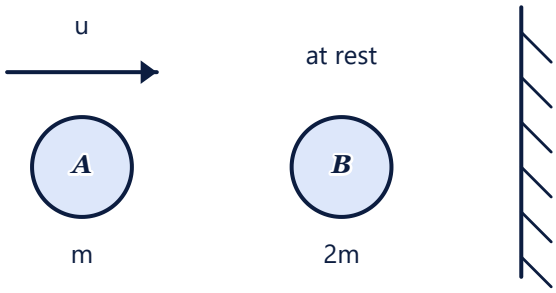


Diagram NOT accurately drawn

(10 marks)

a) A's original direction (towards the wall) positive

Conservation of momentum

$$mu = mv_A + 2mv_B$$

$$v_A + 2v_B = u$$

Newton's law of restitution

$$v_B - v_A = eu$$

Substitute $v_B = v_A + eu$

$$3v_A + 2eu = u$$

$$\boxed{v_A = \frac{1}{3}u(1 - 2e)}$$

$$v_B = \frac{1}{3}u(1 - 2e) + eu$$

$$\boxed{v_B = \frac{1}{3}u(1 + e)}$$

b) B rebounds from the wall

$$\text{speed of } B = \frac{1}{3} \times \frac{1}{3}u(1 + e) = \frac{1}{9}u(1 + e)$$

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Case 1: $e \leq \frac{1}{2}$, so $v_A \geqslant 0$

A is at rest or moving towards the wall, B moves back towards A: they collide

Case 2: $e > \frac{1}{2}$, A moves away from the wall

speed of A $= \frac{1}{3}u(2e - 1)$

B must be faster than A to catch it

$$\frac{1}{9}u(1 + e) > \frac{1}{3}u(2e - 1)$$

$$1 + e > 6e - 3$$

$$4 > 5e$$

$$e < \frac{4}{5}$$

Both cases together

Second collision if and only if $e < \frac{4}{5}$

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8 A small smooth sphere A of mass 2 kg moves with speed 3 m s^{-1} on a smooth horizontal floor towards a small smooth sphere B of mass 1 kg, which is at rest. The spheres have equal radius and the coefficient of restitution between them is 0.5. After the collision B hits a fixed smooth vertical wall, which is perpendicular to the line of motion, and rebounds. The coefficient of restitution between B and the wall is f. B then collides with A again, and this second collision brings A to rest.

a. Find the speeds of A and B immediately after the first collision.

b. Find the value of f.

c. Find the velocity of B immediately after the second collision, and state what happens to B next.

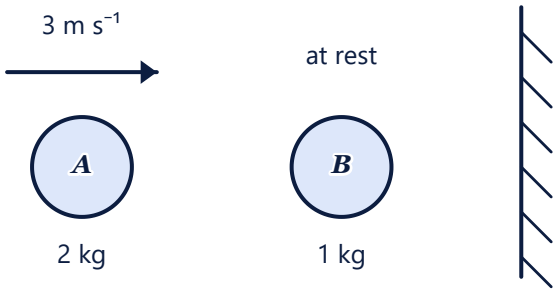


Diagram NOT accurately drawn

(8 marks)

Take the direction towards the wall as positive

a) Conservation of momentum

$2(3) = 2v_A + v_B$

Newton's law of restitution

$v_B - v_A = 0.5 \times 3 = 1.5$

Substitute $v_B = v_A + 1.5$

$3v_A + 1.5 = 6$

$v_A = 1.5$

$v_B = 3$

A: 1.5 m s^{-1}, B: 3 m s^{-1}

b) B rebounds with velocity $-3f$

Second collision: A ends at rest, B ends with velocity y

$2(1.5) + 1(-3f) = 2(0) + y$

$y = 3 - 3f$

Newton's law of restitution

$y - 0 = 0.5(1.5 - (-3f))$

$y = 0.75 + 1.5f$

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Equate

$3 - 3f = 0.75 + 1.5f$

$2.25 = 4.5f$

$\boxed{f = 0.5}$

c)

$y = 3 - 3(0.5) = 1.5$

$\boxed{1.5 \text{ m s}^{-1} \text{ towards the wall: B hits the wall again}}$

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9 Three small smooth spheres A, B and C, of equal radius, have masses m, m and $2m$ respectively. They lie at rest in a straight line on a smooth horizontal surface, in that order. A is projected towards B with speed u. The coefficient of restitution between each pair of spheres is e, where $0 < e \leq 1$

a. Find, in terms of u and e, the velocities of A and B immediately after they collide.

b. Find, in terms of u and e, the velocity of B immediately after it collides with C.

c. Show that, whatever the value of e, there will be a second collision between A and B.

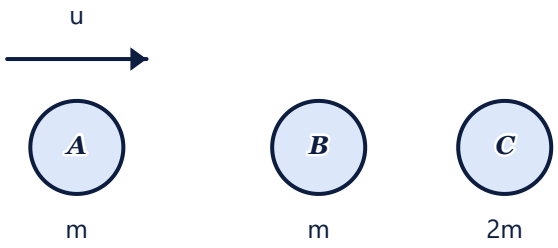


Diagram NOT accurately drawn

(9 marks)

a) A's direction positive, equal masses

$$u = v_A + v_B$$

$$v_B - v_A = eu$$

Subtract and add

$$\boxed{v_A = \frac{1}{2}u(1-e),\ v_B = \frac{1}{2}u(1+e)}$$

b) B, speed $v = \frac{1}{2}u(1+e)$, hits C (mass 2m)

$$mv = mx + 2my$$

$$x + 2y = v$$

$$y - x = ev$$

$$3x + 2ev = v$$

$$x = \frac{1}{3}v(1-2e)$$

$$\boxed{x = \frac{1}{6}u(1+e)(1-2e)}$$

c) A and B collide again if $v_A > x$

$$v_A - x = \frac{u}{6}[3(1-e) - (1+e)(1-2e)]$$

$$v_A - x = \frac{u}{6}[3 - 3e - 1 + e + 2e^2]$$

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$$v_A - x = \frac{u}{6}(2e^2 - 2e + 2)$$

$$v_A - x = \frac{u}{3}(e^2 - e + 1)$$

Complete the square

$$e^2 - e + 1 = (e - \frac{1}{2})^2 + \frac{3}{4}$$

$(e - \frac{1}{2})^2 + \frac{3}{4} > 0$ for all e

$v_A > x$ for every e, so A and B always collide again

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10 Three railway trucks P, Q and R stand on a straight horizontal track, in that order. Their masses are 4000 kg, 2000 kg and 4000 kg respectively. Q and R are at rest. P is moving towards Q with speed 3 m s^{-1} when it collides with Q. The coefficient of restitution between P and Q is 0.5. Q then collides with R, and the two trucks couple together. The trucks are modelled as particles and resistances to motion are ignored.

a. Find the speeds of P and Q immediately after they first collide.

b. Find the speed of the coupled trucks Q and R.

c. Show that P collides with Q again, and find the speeds of P and of the coupled trucks immediately after this collision.

d. Find the total kinetic energy lost in the three collisions.



Diagram NOT accurately drawn

(11 marks)

a) Direction of P's motion positive

$$4000(3) = 4000v_P + 2000v_Q$$

$$2v_P + v_Q = 6$$

$$v_Q - v_P = 0.5 \times 3 = 1.5$$

$$3v_P + 1.5 = 6$$

$$v_P = 1.5$$

$$v_Q = 3$$

P: 1.5 m s^{-1}, Q: 3 m s^{-1}

b) Q and R couple: one common velocity

$$2000(3) = 6000V$$

$V = 1$ m s^{-1}

c) P moves at 1.5 behind QR at 1 in the same direction

1.5 > 1, so P catches Q: second collision

Conservation of momentum, QR has mass 6000 kg

$$4000(1.5) + 6000(1) = 4000x + 6000y$$

$$2x + 3y = 6$$

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Newton's law of restitution

$$y - x = 0.5(1.5 - 1) = 0.25$$

$$2x + 3x + 0.75 = 6$$

$$x = 1.05$$

$$y = 1.3$$

P: 1.05 m s^{-1}, Q and R: 1.3 m s^{-1}

d) KE at the start and at the end

$$\text{KE start} = \frac{1}{2}(4000)(3^2) = 18\,000$$

$$\text{KE end} = \frac{1}{2}(4000)(1.05^2) + \frac{1}{2}(6000)(1.3^2)$$

$$\text{KE end} = 2205 + 5070 = 7275$$

$$\text{KE lost} = 18\,000 - 7275$$

10 725 J

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11 Three small smooth spheres A, B and C, of equal radius, have masses m, $2m$ and $4m$ respectively. They lie at rest in a straight line on a smooth horizontal surface, in that order. A is projected towards B with speed u. The coefficient of restitution between each pair of spheres is e, where $0 < e \leq 1$

a. Find, in terms of u and e, the velocities of A and B immediately after the first collision.

b. Find, in terms of u and e, the velocity of B immediately after it collides with C.

c. Show that there is a further collision between A and B if and only if $e < \frac{1}{2}$.

d. Describe the motion of A and B after all the collisions in the case $e = \frac{1}{2}$.

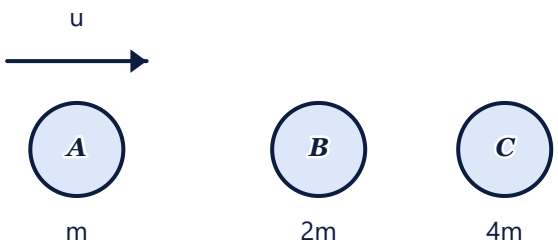


Diagram NOT accurately drawn

(10 marks)

a) A's direction positive

$$mu = mx + 2my$$

$$x + 2y = u$$

$$y - x = eu$$

$$3x + 2eu = u$$

$$\boxed{v_A = \frac{1}{3}u(1 - 2e)}$$

$$v_B = \frac{1}{3}u(1 - 2e) + eu$$

$$\boxed{v_B = \frac{1}{3}u(1 + e)}$$

b) B, speed $V = \frac{1}{3}u(1 + e)$, hits C (mass $4m$)

$$2mV = 2mp + 4mq$$

$$p + 2q = V$$

$$q - p = eV$$

$$3p + 2eV = V$$

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$$p = \frac{1}{3}v(1-2e)$$

$$\boxed{p = \frac{1}{9}u(1+e)(1-2e)}$$

c) A and B collide again if $v_A > p$

$$v_A - p = \frac{1}{3}u(1-2e) - \frac{1}{9}u(1+e)(1-2e)$$

$$v_A - p = \frac{u}{9}(1-2e)[3-(1+e)]$$

$$v_A - p = \frac{u}{9}(1-2e)(2-e)$$

$e \leq 1$, so $2 - e > 0$

$v_A - p > 0$ if and only if $1 - 2e > 0$

$$\boxed{\text{Further collision if and only if } e < \frac{1}{2}}$$

d) Put $e = \frac{1}{2}$

$$v_A = \frac{1}{3}u(1-1) = 0$$

$$p = \frac{1}{9}u\left(\frac{3}{2}\right)(0) = 0$$

$$\boxed{\text{A and B are both at rest; only C moves on}}$$

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12 A small smooth sphere A of mass m moves with speed u on a smooth horizontal floor and collides directly with a small smooth sphere B of mass km, which is at rest. The spheres have equal radius. B then moves to a fixed smooth vertical wall, which is perpendicular to its direction of motion, and rebounds. The coefficient of restitution between A and B is $\frac{1}{2}$ and the coefficient of restitution between B and the wall is $\frac{1}{2}$.

a. Show that, immediately after the first collision, the velocity of A is $\frac{u(2-k)}{2(1+k)}$ and the speed of B is $\frac{3u}{2(1+k)}$.

b. Find the range of values of k for which there is a second collision between A and B.

Given that $k = 3$,

c. find the speeds of A and B immediately after their second collision, and explain why there are no further collisions.

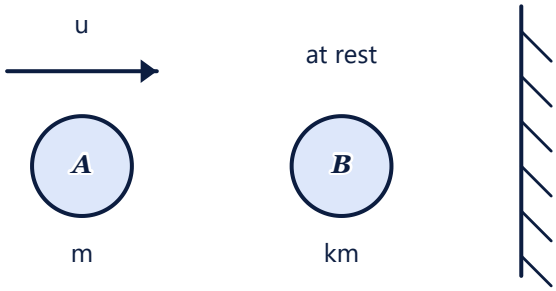


Diagram NOT accurately drawn

(12 marks)

a) Towards the wall positive

Conservation of momentum

$$mu = mv_A + kmv_B$$

$$v_A + kv_B = u$$

Newton's law of restitution

$$v_B - v_A = \frac{1}{2}u$$

Substitute $v_A = v_B - \frac{1}{2}u$

$$(1+k)v_B - \frac{1}{2}u = u$$

$$\boxed{v_B = \frac{3u}{2(1+k)}}$$

$$v_A = \frac{3u}{2(1+k)} - \frac{1}{2}u$$

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$$v_A = \frac{3u - u(1+k)}{2(1+k)}$$

$$\boxed{v_A = \frac{u(2-k)}{2(1+k)}}$$

b) B rebounds with speed $\frac{1}{2} \times \frac{3u}{2(1+k)}$

$$\text{speed of B} = \frac{3u}{4(1+k)}$$

Case 1: $k \leq 2$, so $v_A \geq 0$

A is at rest or moving towards the wall: they collide

Case 2: $k > 2$, A moves away from the wall

$$\text{speed of A} = \frac{u(k-2)}{2(1+k)}$$

B must be faster than A

$$\frac{3u}{4(1+k)} > \frac{u(k-2)}{2(1+k)}$$

$$3 > 2(k-2)$$

$$k < \frac{7}{2}$$

$$\boxed{0 < k < \frac{7}{2}}$$

c) $k = 3$: velocities before the second collision

$$v_A = \frac{u(2-3)}{2(4)} = -\frac{1}{8}u$$

$$v_B = -\frac{3u}{4(4)} = -\frac{3}{16}u$$

Conservation of momentum

$$m(-\frac{1}{8}u) + 3m(-\frac{3}{16}u) = mx + 3my$$

$$x + 3y = -\frac{11}{16}u$$

Newton's law of restitution (B approaches A from behind)

$$y - x = \frac{1}{2}(-\frac{1}{8}u - (-\frac{3}{16}u))$$

$$y - x = \frac{1}{32}u$$

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Substitute $x = y - \frac{1}{32}v$

$$4y - \frac{1}{32}v = -\frac{22}{32}v$$

$$4y = -\frac{21}{32}v$$

$$y = -\frac{21}{128}v$$

$$x = -\frac{21}{128}v - \frac{4}{128}v = -\frac{25}{128}v$$

A: $\frac{25}{128}v$, B: $\frac{21}{128}v$, both away from the wall

A is further from the wall and faster than B

B cannot catch A and never reaches the wall again: no more collisions

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