Work and energy

Worked solutions

Further Maths topic practice

This topic online

1 A crate is dragged 12 m in a straight line across a horizontal floor by a rope. The rope is inclined at 25° above the horizontal and the tension in the rope is 45 N.

a. Find the work done by the tension in the rope.

b. The crate moves at constant speed. State the work done against friction, giving a reason for your answer.

(3 marks)

a) Work done = component of force in direction of motion × distance

$WD = 45 \cos 25^\circ \times 12$

$WD = 489.40...$

489 J

b) Constant speed and level floor: no change in KE or PE

so all the work done by the tension is done against friction

489 J

2 A ball of mass 0.6 kg is projected vertically upwards with speed 14 m s^{-1} from a point 2 m above horizontal ground. The ball is modelled as a particle moving freely under gravity.

a. Find the kinetic energy of the ball at the instant it is projected.

b. Use the principle of conservation of mechanical energy to find the speed of the ball as it hits the ground.

(4 marks)

a) $KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2} \times 0.6 \times 14^2$

58.8 J

b) Energy at start = energy at ground (ground as zero PE)

$\frac{1}{2}(0.6)(14^2) + 0.6g(2) = \frac{1}{2}(0.6)v^2$

$58.8 + 11.76 = 0.3v^2$

$v^2 = 235.2$

$v = 15.336...$

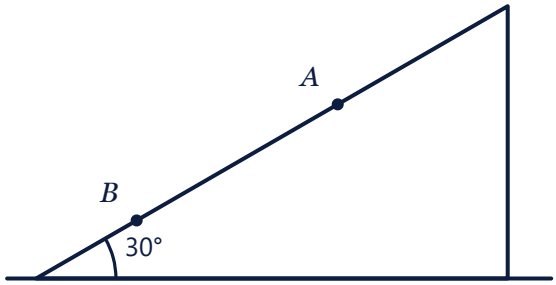
15.3 m s^{-1}

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3 The points A and B lie on a line of greatest slope of a rough plane inclined at 30° to the horizontal, with A above B and AB = 5 m, as shown in the diagram.

A particle of mass 3 kg is released from rest at A and slides down the line of greatest slope. It passes through B with speed 5.5 m s^{-1}.

Find the work done against friction as the particle moves from A to B.



(4 marks)

Vertical drop from A to B

$h = 5 \sin 30^\circ = 2.5$ m

Loss of PE

$mgh = 3 \times 9.8 \times 2.5 = 73.5$ J

Gain in KE

$\frac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 5.5^2 = 45.375$ J

Work-energy: WD against friction = loss of PE - gain in KE

$WD = 73.5 - 45.375$

$WD = 28.125$

28.1 J

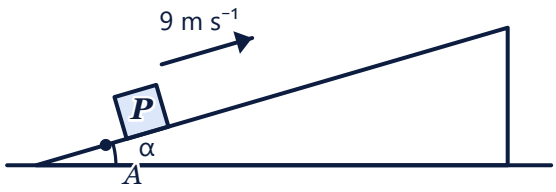
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4 A particle P of mass 2.5 kg is projected with speed 9 m s^{-1} from a point A up a line of greatest slope of a rough plane. The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{7}{24}$, as shown in the diagram. The coefficient of friction between P and the plane is 0.3

P comes to instantaneous rest at the point B.

a. Use the work-energy principle to find the distance AB.

b. Determine whether P remains at rest at B, giving a reason for your answer.



(7 marks)

tan α = 7/24, so a 7, 24, 25 triangle

$$\sin\alpha = \frac{7}{25}$$

$$\cos\alpha = \frac{24}{25}$$

a) Resolve perpendicular to the plane

$$R = 2.5g \times \frac{24}{25} = 23.52 \text{ N}$$

$$F = \mu R = 0.3 \times 23.52 = 7.056 \text{ N}$$

Let AB = d. Height gained = d sin α

Work-energy: loss of KE = gain in PE + WD against friction

$$\frac{1}{2}(2.5)(9^2) = 2.5g\left(\frac{7}{25}\right)d + 7.056d$$

$$101.25 = 6.86d + 7.056d$$

$$101.25 = 13.916d$$

$$d = 7.2758...$$

AB = 7.28 m

b) Compare the weight component with limiting friction

$$2.5g\sin\alpha = 2.5 \times 9.8 \times \frac{7}{25} = 6.86 \text{ N}$$

Limiting friction = 7.056 N

6.86 < 7.056

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P remains at rest at B

5 A cyclist and her bicycle have a total mass of 80 kg. She cycles up a straight road from A to B, where AB = 150 m and B is a vertical height of 6 m above A.

Her speed at A is 4 m s^{-1} and her speed at B is 7 m s^{-1}. The resistance to motion is modelled as a constant force of magnitude 18 N.

Find the work done by the cyclist as she moves from A to B.

(5 marks)

Gain in KE

$\frac{1}{2}(80)(7^2) - \frac{1}{2}(80)(4^2) = 1960 - 640 = 1320$ J

Gain in PE

$80 \times 9.8 \times 6 = 4704$ J

WD against resistance = force x distance

$18 \times 150 = 2700$ J

Work-energy: WD by cyclist = gain in KE + gain in PE + WD against resistance

$WD = 1320 + 4704 + 2700$

$WD = 8724$

8720 J (3 s.f.)

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6 At a swimming pool, a child of mass 28 kg starts from rest at the top of a water chute and slides down it. The bottom of the chute is 4.5 m vertically below the top. The child leaves the bottom of the chute with speed 8 m s^{-1}. The child is modelled as a particle.

a. Find the work done against the resistances to motion as the child moves down the chute.

b. The resistance to motion is now modelled as a constant force of magnitude 45 N. Using your answer to part (a), find the length of the chute.

(5 marks)

a) Loss of PE

$mgh = 28 \times 9.8 \times 4.5 = 1234.8$ J

Gain in KE

$\frac{1}{2}mv^2 = \frac{1}{2} \times 28 \times 8^2 = 896$ J

WO against resistances = loss of PE - gain in KE

$WO = 1234.8 - 896 = 338.8$

339 J

b) WO = resistance x distance along the chute

$45s = 338.8$

$s = 7.528...$

7.53 m

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7 A box of mass 15 kg rests on rough horizontal ground. The coefficient of friction between the box and the ground is 0.35. The box is pulled by a rope inclined at 20° above the horizontal. The tension in the rope is 70 N. The box starts from rest and moves 8 m in a straight line. The box is modelled as a particle.

a. Find the work done by the tension in the rope.

b. Find the work done against friction.

c. Use the work-energy principle to find the speed of the box after it has moved 8 m. (7 marks)

a) Horizontal component of tension × distance

$$WD = 70\cos 20^\circ \times 8$$

$$WD = 526.22...$$

526 J

b) Resolve vertically

$$R + 70\sin 20^\circ = 15g$$

$$R = 147 - 23.941... = 123.05...$$

$$F = 0.35 \times 123.05... = 43.070...$$

$$WD = 43.070... \times 8 = 344.56...$$

345 J

c) Work-energy: WD by tension − WD against friction = gain in KE

$$526.22... - 344.56... = \frac{1}{2}(15)v^2$$

$$181.66... = 7.5v^2$$

$$v^2 = 24.221...$$

$$v = 4.9215...$$

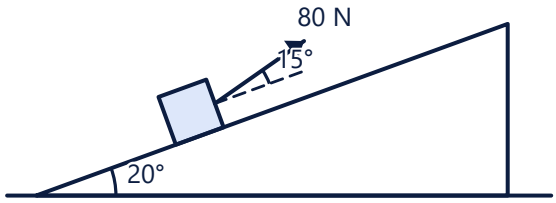
4.92 m s^{-1}

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8 A box of mass 10 kg is pulled up a line of greatest slope of a rough plane inclined at 20° to the horizontal. The pulling force has magnitude 80 N and acts at 15° to the line of greatest slope, in the vertical plane containing that line, as shown in the diagram.

The box starts from rest and, after moving 5 m up the plane, its speed is 5 m s^{-1}. The box is modelled as a particle.

Use the work-energy principle to find the coefficient of friction between the box and the plane.



(8 marks)

Resolve perpendicular to the plane

$$R + 80 \sin 15^\circ = 10g \cos 20^\circ$$

$$R = 92.089... - 20.705...$$

$$R = 71.384...$$

$$F = \mu R = 71.384...\mu$$

WD by the pulling force (component along the slope)

$$80 \cos 15^\circ \times 5 = 386.37...$$

Gain in KE and gain in PE

$$\frac{1}{2}(10)(5^2) = 125$$

$$10g \times 5 \sin 20^\circ = 167.58...$$

Work-energy: WD by force = gain in KE + gain in PE + WD against friction

$$386.37... = 125 + 167.58... + 5 \times 71.384...\mu$$

$$93.780... = 356.92...\mu$$

$$\mu = 0.26274...$$

$$\boxed{\mu = 0.263}$$

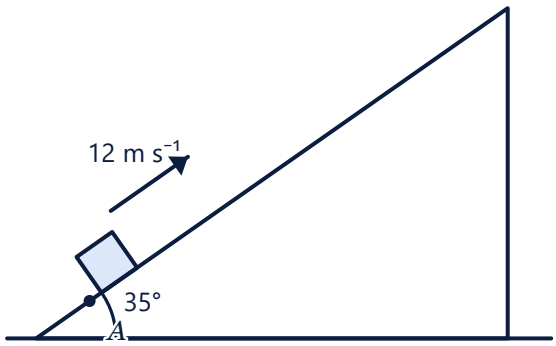
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9 A particle of mass 0.5 kg is projected with speed 12 m s^{-1} from a point A up a line of greatest slope of a rough plane inclined at 35° to the horizontal, as shown in the diagram. The coefficient of friction between the particle and the plane is 0.2

The particle comes to instantaneous rest at the point B and then slides back down to A.

a. Use the work-energy principle to find the distance AB.

b. Find the speed of the particle when it returns to A.



(8 marks)

a) Resolve perpendicular to the plane

$R = 0.5g \cos 35^\circ = 4.0138...$

$F = 0.2 \times 4.0138... = 0.80277...$

Let AB = d. Work-energy from A to B

$\frac{1}{2}(0.5)(12^2) = 0.5g \sin 35^\circ \times d + 0.80277...d$

$36 = 2.8105...d + 0.80277...d$

$36 = 3.6132...d$

$d = 9.9632...$

AB = 9.96 m

b) A to B and back to A: no change in PE

friction acts against the motion over a total distance 2d

$\frac{1}{2}(0.5)(12^2) - \frac{1}{2}(0.5)v^2 = 0.80277... \times 2 \times 9.9632...$

$36 - 0.25v^2 = 15.996...$

$0.25v^2 = 20.003...$

$v^2 = 80.015...$

$v = 8.9451...$

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$8.95\ m\ s^{-1}$

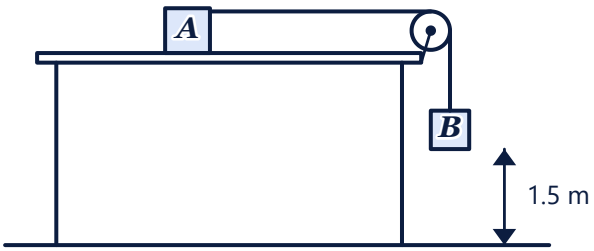
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10 A particle A of mass 3 kg lies on a rough horizontal table. It is attached to one end of a light inextensible string which passes over a small smooth pulley fixed at the edge of the table. The other end of the string is attached to a particle B of mass 2 kg, which hangs freely 1.5 m above the floor, as shown in the diagram. A is 4 m from the pulley. The coefficient of friction between A and the table is 0.25

The system is released from rest with the string taut.

a. Use the work-energy principle to find the speed of B as it hits the floor.

b. After B hits the floor it does not rebound. Find the further distance moved by A before it comes to rest, and show that A does not reach the pulley.



(8 marks)

a) A on the table: $R = 3g$

$F = 0.25 \times 3g = 7.35$ N

B falls 1.5 m and A moves 1.5 m along the table

Loss of PE $= 2g \times 1.5 = 29.4$ J

WD against friction $= 7.35 \times 1.5 = 11.025$ J

Work-energy for the whole system (both move at speed v)

$$\frac{1}{2}(3+2)v^2 = 29.4 - 11.025$$

$$2.5v^2 = 18.375$$

$$v^2 = 7.35$$

$$v = 2.7110...$$

$\boxed{2.71 \text{ m s}^{-1}}$

b) String slack: A's KE is used against friction

$$\frac{1}{2}(3)(7.35) = 7.35d$$

$$11.025 = 7.35d$$

$$d = 1.5$$

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1.5 m

Total distance moved by A

$1.5 + 1.5 = 3$ m

$3 \text{ m} < 4 \text{ m}$, so A does not reach the pulley

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11 A skier of mass 70 kg starts from rest at the point A at the top of a straight slope AB. The slope has length 120 m and is inclined at 15° to the horizontal. At B the skier moves on to horizontal snow BC, as shown in the diagram, without any change in speed.

The coefficient of friction between the skis and the snow is 0.08 on both the slope and the horizontal snow. In addition, the skier experiences a constant air resistance of magnitude 20 N throughout. The skier is modelled as a particle.

a. Use the work-energy principle to find the speed of the skier at B.

b. Find the distance the skier travels along BC before coming to rest.

c. Suggest one way in which the model for the air resistance could be improved.

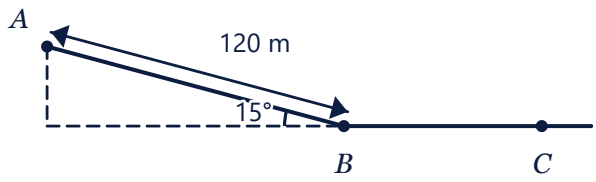


Diagram NOT accurately drawn

(9 marks)

a) On the slope: resolve perpendicular

$R = 70g \cos 15^\circ = 662.62...$

$F = 0.08 \times 662.62... = 53.010...$

Loss of PE from A to B

$70g \times 120 \sin 15^\circ = 21305.98...$

WD against friction and air resistance on AB

$53.010... \times 120 = 6361.2...$

$20 \times 120 = 2400$

Work-energy from A to B

$\frac{1}{2}(70)v^2 = 21305.98... - 6361.2... - 2400$

$35v^2 = 12544.78...$

$v^2 = 358.42...$

$v = 18.932...$

$\boxed{18.9 \text{ m s}^{-1}}$

b) On BC: $R = 70g$

total resistance $= 0.08 \times 70g + 20 = 74.88$ N

KE at B is all used against the resistances

$74.88s = 12544.78...$

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$s = 167.53...$

168 m

c) Refinement

Air resistance depends on speed, so model it as varying with v, not constant

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12 A stone of mass 0.2 kg is thrown from a point O on the top of a vertical cliff. O is 35 m above the sea. The stone is thrown with speed 15 m s^{-1} at an angle of 30° above the horizontal.

At first the stone is modelled as a particle moving freely under gravity.

a. Use the principle of conservation of mechanical energy to find the speed of the stone as it hits the sea.

b. Using the fact that the horizontal component of the velocity is constant, use energy to find the greatest height of the stone above the sea.

In fact, because of air resistance, the stone hits the sea with speed 26 m s^{-1}.

c. Find the work done against air resistance as the stone moves from O to the sea.

(9 marks)

a) Energy at O = energy at sea level (sea as zero PE)

$$\frac{1}{2}(0.2)(15^2) + 0.2g(35) = \frac{1}{2}(0.2)v^2$$

$$22.5 + 68.6 = 0.1v^2$$

$$v^2 = 911$$

$$v = 30.182...$$

30.2 m s^{-1}

b) At the highest point the velocity is horizontal

speed $= 15 \cos 30^\circ = 12.990...$

Energy from O to the top, h above O

$$\frac{1}{2}(0.2)(15^2) = \frac{1}{2}(0.2)(12.990...^2) + 0.2gh$$

$$22.5 = 16.875 + 1.96h$$

$$h = 2.8698...$$

height above sea $= 35 + 2.8698... = 37.869...$

37.9 m

c) WD against air resistance = loss of mechanical energy

energy at O $= 22.5 + 68.6 = 91.1$ J

KE at sea $= \frac{1}{2}(0.2)(26^2) = 67.6$ J

$WD = 91.1 - 67.6$

23.5 J

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