Discrete random variables: expectation and variance

AS Further Statistics 1 | Edexcel 8FM0 | 12 questions, 74 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 74**

1 The discrete random variable X has the probability distribution shown.

x	1	2	3	4
$\mathbf{P}(X = x)$	0.1	0.3	k	0.2

a. Find the value of k.

b. Find E(X).

(3 marks)

Answer _____

2 Priya records the number of goals, X, scored by her netball club's under-14 team in the first quarter of a match. The distribution of X is modelled as shown.

x	0	1	2	3
$\mathbf{P}(X = x)$	0.2	0.4	0.3	0.1

Find

a. E(X)

b. E(X^2)

c. the standard deviation of X.

(4 marks)

Answer _____

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3 The discrete random variable X has probability function

$P(X = x) = k(x + 2)$ for $x = 1, 2, 3, 4$

and $P(X = x) = 0$ otherwise, where k is a constant.

a. Show that $k = \frac{1}{18}$

b. Find $P(X > 2)$.

c. Find the exact values of $E(X)$ and $Var(X)$.

(5 marks)

Answer

4 The discrete random variable X has the probability distribution shown, where a and b are constants.

x	−1	0	2	5
$P(X = x)$	a	0.3	b	0.1

Given that $E(X) = 1.1$

a. find the value of a and the value of b

b. find $Var(X)$.

(6 marks)

Answer

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5 The discrete random variable X has the probability distribution shown.

x	1	2	4	5
$\mathbf{P}(X = x)$	0.2	0.25	0.4	0.15

a. Find $\mathrm{E}\left(\frac{1}{X}\right)$.

b. Find $\mathrm{P}(X^2 > 3X + 2)$.

(5 marks)

Answer

6 The discrete random variable X has the probability distribution shown.

x	−1	0	1	2
$\mathbf{P}(X = x)$	0.2	0.1	0.3	0.4

a. Find $\mathrm{Var}(X)$.

b. Find $\mathrm{Var}(X^2)$.

(6 marks)

Answer

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7 The discrete random variable X can take only the values 1, 2, 3 and 4, with

$P(X = 1) = p$, $P(X = 2) = q$, $P(X = 3) = r$, $P(X = 4) = 0.2$

Given that $E(X) = 2.5$ and $Var(X) = 1.05$, find the values of p, q and r.

(7 marks)

Answer

8 At a school fair, a player pays £1 to roll three fair six-sided dice. The number of sixes rolled is X. The prize, W pounds, depends on X as shown.

Number of sixes, x	0	1	2	3
Prize, w (£)	0	1	5	20

a. Show that $P(X = 2) = \frac{15}{216}$

b. Find $E(W)$.

c. The game is played 200 times. Find the profit the school should expect to make.

d. Find $Var(W)$.

(7 marks)

Answer

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9 A bakery sells a type of loaf to customers. The number of these loaves, X, bought by a customer has the distribution shown.

x	1	2	3
$P(X = x)$	0.5	0.3	0.2

Two customers are chosen at random, independently. They buy X_1 and X_2 loaves. Let $M = \frac{X_1 + X_2}{2}$ be the mean number of loaves they buy.

a. Find $P(X_1 + X_2 = 4)$.

b. Find $P(M \geq 2.5)$.

c. Find $E(M)$, showing your method.

(6 marks)

Answer

10 The discrete random variable X has the probability distribution shown.

x	0	1	2	3
$P(X = x)$	0.1	0.4	0.3	0.2

a. Find $E(X)$ and $Var(X)$.

The random variable Y is defined by $Y = (X - 1)^2$

b. Find the probability distribution of Y.

c. Find $E(Y)$ and verify that $E(Y) = Var(X) + (E(X) - 1)^2$

d. Find $Var(Y)$.

e. Find $P(Y > X)$.

(8 marks)

Answer

11 Mo designs a spinner for a board game. The score, X, on the spinner has probability function

$P(X = x) = \frac{k}{x}$ for $x = 1, 2, 3, 6$

and $P(X = x) = 0$ otherwise, where k is a constant.

a. Show that $k = \frac{1}{2}$

b. Show that $E(X) = Var(X)$.

c. Find the exact value of $E(\frac{1}{X})$.

(7 marks)

Answer

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12 Ravi runs a bike-hire stall at a country park. The number of bikes, X, hired by a customer has probability function

$P(X = x) = a + bx$ for $x = 1, 2, 3, 4$

where a and b are constants. Given that $E(X) = 2.8$

a. find the value of a and the value of b.

Ravi gives a discount for groups, so a customer who hires x bikes is charged £C, where $C = 12X - X^2$

b. Find $E(C)$.

c. Find $Var(C)$.

d. Two customers arrive independently. Find the probability that Ravi charges them a total of at least £59

(10 marks)

Answer

End of questions | Total 74 marks

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